Duke University
Department of Physics

Physics 361 Spring Term 2020

WUN2K FOR LECTURE 8

These are notes summarizing the main concepts you need to understand and be able to apply.

Forced Oscillations

- The equation of motion for a damped oscillator (damping constant β, natural frequency ω_0) with a sinusoidal (in time, driving frequency ω) applied force is $\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = f_0 \cos\omega t$. The solution describing the motion has two parts, $x(t) = x_h(t) + x_p(t)$, where

  1. A **homogeneous** or **complementary** $x_h(t)$ solution, which represents the solution with forcing function set to zero; this is just the solution for the damped oscillator, $x_h(t) = e^{-\beta t}[C_1 e^{\sqrt{\beta^2-\omega_0^2}t} + C_2 e^{-\sqrt{\beta^2-\omega_0^2}t}]$, where the constants are determined by initial conditions. This represents a **transient** motion.
  2. A **particular solution** $x_p(t)$, which reproduces the right hand side (forcing function). The particular solution is: $x_p(t) = \frac{f_0}{\sqrt{(\omega_0^2-\omega^2)^2+4\omega^2\beta^2}}\cos(\omega t - \delta)$, where $\delta = \tan^{-1}\left(\frac{2\omega\beta}{\omega_0^2-\omega^2}\right)$ is the *phase shift* with respect to the forcing function. This represents a **steady-state** motion (also known as an **attractor**.)
  3. The general solution is then the sum of the particular and homogeneous solutions.

  Both the amplitude and phase shift depend on the frequency ω of the forcing (driving), and the properties of the oscillator (ω_0 and β.)

- **Resonance** phenomena occur at particular forcing frequencies.

– **Amplitude resonance** occurs at $\omega = \omega_R$, the frequency at which the amplitude of the forced oscillation is maximized. $\omega_R^2 = \omega_0^2 - 2\beta^2$, which depends on the damping. We define the **quality factor** $Q \equiv \frac{\omega_R}{2\beta}$, which determines how sharp the resonance is. The smaller the damping, the sharper the resonance.