Duke University
Department of Physics

Physics 361 Spring Term 2020

WUN2K FOR LECTURE 6

These are notes summarizing the main concepts you need to understand and be able to apply.

- In spherical coordinates r, θ, ϕ, the gradient of a scalar function $f(\vec{r})$ can be written $\nabla f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\hat{\phi}$. For other useful expressions in spherical coordinates, see the inside back cover of your textbook.

- A **central force** always acts in the direction towards or away from some "force center". If this is the origin, $\vec{F}(\vec{r}) = f(\vec{r})\hat{r}$. A a central force is spherically symmetric $f(\vec{r}) = f(r)$ if the magnitude of the force depends only on distance from the origin. A central force is spherically symmetric if and only if it is conservative.

- If all forces on a multiparticle system are conservative, the total potential energy can be written as a sum of external and internal contributions: $U = U^{\text{int}} + U^{\text{ext}} = \sum_\alpha \sum_{\beta>\alpha} U_{\alpha\beta} + \sum_\alpha U_\alpha^{\text{ext}}$, and the net force on particle α can be written $-\nabla_\alpha U$. In this situation the total kinetic and potential energy is constant, $T + U = E = \text{const}$.

- A **restoring force** acts to restore the position of a particle towards an equilibrium point origin. In 1D, expanding to first order, we have $F(x) = -kx$, where the spring constant $k = -\left(\frac{dF}{dx}\right)_0$. This expression is known as "Hooke's Law" (although it's not a law, but a first-order approximation). The corresponding potential energy is $U = \frac{1}{2}kx^2$.

- Simple Harmonic Oscillator:

  - In 1D the equation of motion corresponding to $F(x) = -kx$ is $\ddot{x} + \omega^2 x = 0$, where $\omega^2 \equiv k/m$. The period is $\tau = 2\pi/\omega$.

– The general solution for the motion is $x(t) = A\cos(\omega t - \delta)$, where the amplitude A and the phases δ, ϕ are determined by the initial conditions. ω is the angular frequency of the oscillation. This solution can also be written as the real part of a complex exponential, $x(t) = \Re[Ae^{i(\omega t - \delta)}]$.