Duke University
Department of Physics

Physics 361 Spring Term 2020

WUN2K FOR LECTURE 5

These are notes summarizing the main concepts you need to understand and be able to apply.

Conservation Theorems: these follow from Newton's Laws.

- **Work** done on a particle by a force $\vec{F}$ along a path from 1 to 2 is defined to be $W_{12} \equiv \int_1^2 \vec{F} \cdot d\vec{r}$.
- **Kinetic energy** is $T \equiv \frac{1}{2}mv^2$.
- The **Work-kinetic energy theorem** says that $W_{12} = \Delta T$, i.e. the work done by the net force is the change in kinetic energy.
- A force $\vec{F}$ is **conservative** if (and only if) the following conditions hold:
  - $\vec{F} = \vec{F}(\vec{r})$, i.e., the force depends only on position, and not velocity, time, etc.
  - The work done from 1 to 2 is independent of path.

  In this case, one can define a **potential energy**, U, according to $\vec{F} = -\nabla U$, and $\nabla \times \vec{F} = 0$ (the **curl** of $\vec{F}$ is zero). We then get that $\int_1^2 \vec{F} \cdot d\vec{r} = U_1 - U_2$, the (negative) difference in potential energy between the points.
- One can always add a constant to a potential energy definition, as physical effects depend only on differences in potential energy. There is also no such thing as an absolute kinetic energy, as velocities will be different in different inertial reference frames.

- The **total energy** of a system is $E \equiv T + U$. If conservative forces are acting, and $\frac{\partial U}{\partial t} = 0$, then **the total energy E of a particle in a conservative force field is a constant in time.** This is the principle of conservation of energy.

- The concept of energy can be useful in problem-solving. If you have a system in which energy is conserved, $E = \frac{1}{2}mv^2 + U(x)$, the solution for the motion, $x(t)$, can be found as $t - t_0 = \int_{x_0}^{x} \frac{\pm dx'}{\sqrt{\frac{2}{m}[E - U(x')]}}$, where $x = x_0$ at $t = t_0$.

- The form of $U(x)$ is also useful for describing motion. For a particle of energy E moving under the influence of a force associated with $U(x)$: if $E > U(x)$ for all x, the particle is "unbound". Otherwise a particle's motion may be bounded if $E > U(x)$ within some bounded region; it may oscillate between *turning points*. (A particle cannot exist with $E < U(x)$, since $T > 0$.)

  - An **equilibrium point** exists where $\left(\frac{dU}{dx}\right)_0 = 0$.
  - Equilibrium is *stable* if $\left(\frac{d^2U}{dx^2}\right)_0 > 0$ and *unstable* if $\left(\frac{d^2U}{dx^2}\right)_0 < 0$.