Duke University
Department of Physics

Physics 361 Spring Term 2020

WUN2K FOR LECTURE 4

These are notes summarizing the main concepts you need to understand and be able to apply.

- (Also in WUN2K 1) The principle of **conservation of momentum** states that *if* the net external force on a system of N particles is zero, then the system's total momentum $\vec{P} = \sum m_\alpha \vec{v}_\alpha$ is constant.

- The **center of mass** (CM) of a system is at a position given by $\vec{R} = \frac{1}{M}\sum_{\alpha=1}^{N} m_\alpha \vec{r}_\alpha$. For a continuous mass distribution, this is $\vec{R} = \frac{1}{M}\int \rho \vec{r} dV$, where ρ is the density. We have $\vec{F}^{\text{ext}} = M\ddot{\vec{R}}$, which means that the center of mass behaves like a single particle of total mass M.

- The **angular momentum** of a system defined with respect to some origin $\mathcal{O}$ (fixed in some inertial frame) is $\vec{\ell} = \vec{r} \times \vec{p}$. The **torque** defined with respect to $\mathcal{O}$ corresponding to a force applied at position $\vec{r}$ is $\vec{\Gamma} \equiv \vec{r} \times \vec{F}$. The "rotational analog of Newton's second law" $\vec{\Gamma} = \dot{\vec{\ell}}$. More generally if $\vec{L}$ is the sum of angular momenta of N particles, $\vec{L} = \sum_\alpha^N \vec{r}_\alpha \times \vec{p}_\alpha$, then $\dot{L} = \vec{\Gamma}$.

- The principle of **conservation of angular momentum** states that *if* the net external torque of an N-particle system is zero, then the system's total angular momentum $\vec{L}$ is a constant.

- For a rigid body rotating around a fixed axis, we can write $L_z = I\omega$, where I is the **moment of inertia**, $I = \sum m_\alpha \rho_\alpha^2$, where ρ_α is the distance of each particle of mass m_α from the axis of rotation.

- It turns out that if we take the origin to be the center of mass of a system (even if not fixed in an inertial frame), then we can write $\frac{d}{dt}\vec{L}(\text{about CM}) = \vec{\Gamma}(\text{about CM})$.