Duke University
Department of Physics

Physics 361 Spring Term 2020

WUN2K FOR LECTURE 3

These are notes summarizing the main concepts you need to understand and be able to apply.

- A **retarding force** is a force which always acts in the direction opposite to the direction of motion. For the case of **air resistance** or **drag**, it can be modeled reasonably by a function of the form $\vec{f} = -f(v)\hat{v}$. At low speeds (less than the speed of sound), a reasonable model is $f(v) = bv + cv^2$, where b and c are constants. These constants depend on the shape and size of the object, and the density and viscosity of the medium.

- One can determine the motion of objects with retarding forces acting on them by writing the relevant second law expression for the system ($m\ddot{\vec{r}} = -\vec{f} + m\vec{g}$ for a projectile under the influence of gravity), writing equations of motion in each coordinate, and solving.

- For linear drag, one gets uncoupled equations for x and y, which are straightforward to solve. For projectiles:

  - For horizontal linear drag, with $k = b/m$, initial position zero and initial velocity v_{x0}, we find $v_x(t) = v_{x0}e^{-t/\tau}$ and $x(t) = v_{x0}\tau(1 - e^{-t/\tau})$, , where $\tau = 1/k = m/b$.
  - For vertical linear drag, for y direction down, we find $v_y(t) = v_{\rm ter} + (v_{y0} - v_{\rm ter})e^{-t/\tau}$, $y(t) = v_{\rm ter}t + (v_{y0} - v_{\rm ter})\tau(1 - e^{-t/\tau})$. Here $v_{\rm ter}$ is the **terminal velocity**, which physically is the constant velocity reached when the drag force balances the force of gravity, so that net force is zero and velocity is constant.

  (Do not memorize these solutions, but understand where they came from, their behavior and meaning.)

- For quadratic drag, the x and y equations of motion are in general *coupled*, and this may require numerical solution.
- The **range** of a projectile can be found by finding the y value when x is zero.