Duke University
Department of Physics

Physics 361 Spring Term 2020

WUN2K FOR LECTURE 25

These are notes summarizing the main concepts you need to understand and be able to apply.

- The example of two coupled oscillating masses is an instance of a much more general description of coupled oscillations. In general if there n degrees of freedom (n coupled 1D oscillators or $n/3$ coupled 3D oscillators), described by $i = 1, ..., n$ generalized coordinates q_i, and if we assume *small* oscillations from equilibrium, we can write the coupled equations of motion in matrix form, $M\ddot{\vec{q}} = -K\vec{q}$. M is the $n \times n$ "mass" matrix and K is the $n \times n$ "spring constant" matrix. By assuming an oscillatory solution, the problem turns into one of finding the eigenfrequencies for which solutions are the eigenvectors.

- The *normal modes* are solutions for the motion in which all coordinates oscillate with the same frequency ω.

- The general motion can also be described as a superposition of normal-mode motions.

- A "how-to" for solving for the motion of coupled oscillatros in n dimensions:

  - Write generalized coordinates for the usual Lagrangian treatment, with some coordinate transformation $\vec{r}_\alpha = \vec{r}_\alpha(q_1, ..., q_n)$.
  - Write the kinetic energy in the form $T = \frac{1}{2}\sum_{j,k} M_{jk}\dot{q}_j\dot{q}_k$.
  - Write the potential energy in the form $U = \frac{1}{2}\sum_{j,k} K_{jk}q_jq_k$, where $K_{jk} \equiv \frac{\partial^2 U}{\partial q_j \partial q_k}|_0$ (derivative evaluated at equilibrium).
  - Write out the M and K matrices as $n \times n$ arrays.

- Solve the **characteristic equation** $|K - \omega^2 M| = 0$ for ω to find the n **eigenfrequencies** ω_i.
- For each ω_i find the corresponding **eigenvectors** by finding the ratios $a_{1i} : a_{2i} : ... : a_{ni}$, by substituting ω_i into $(K - \omega_i^2 M)\vec{a} = 0$.
- You can also find the normal coordinates $\xi_i(t)$ by finding linear combinations of the q_j that have oscillations at a single frequency.
- The general motion of the system is a superposition of normal modes, $\vec{q}(t) = \sum_i \xi_i(t)\vec{a}_{(i)}$.