Duke University
Department of Physics

Physics 361 Spring Term 2020

WUN2K FOR LECTURE 24

These are notes summarizing the main concepts you need to understand and be able to apply.

- An example of a *coupled harmonic oscillator system* with two degrees of freedom is two masses m_1 and m_2 connected to walls with springs of force constant k_1 and k_2, respectively, and connected by a spring of force constant k_{12}.

  The equations of motion form two coupled differential equations,

$$M\ddot{x}_1 = -(k_1 + k_2)x_1 + k_2 x_2 \quad (1)$$
$$M\ddot{x}_2 = k_2 x_1 - (k_2 + k_3)x_2. \quad (2)$$

  This system of equations can be written as a matrix equation, $(K - \omega^2 M)\vec{a} = 0$. The solutions are oscillatory with two **characteristic frequencies** (or **eigenfrequencies**) which are solutions for $|K - \omega^2 M| = 0$. The general solution can be written in terms of the corresponding eigenvectors.

  – The problem can be simplified by defining new coordinates: for our two-mass example, $\xi_1 = \frac{1}{2}(x_1 + x_2)$, and $\xi_2 = \frac{1}{2}(x_1 - x_2)$. In these coordinates, the equations of motion are *uncoupled*. These are **normal coordinates**.

    For appropriate initial conditions, $\xi_1(t) = 0$, and particles oscillate *out of phase* with frequency ω_2; this is the **antisymmetrical** mode of oscillation. For other initial conditions, $\xi_1(t) = 0$, and particles oscillate *in phase* with frequency ω_1; this is the **symmetrical** mode of oscillation.

- For the example of **weak coupling**, we take $k_2 << k$. In this case we can write $\omega_0 \sim \frac{\omega_1+\omega_2}{2}$, $\omega_1 = \omega_0 - \epsilon$, $\omega_2 = \omega_0 + \epsilon$. The solution works out, for appropriate constants, to $x_1(t) = A\cos\epsilon t\cos\omega_0 t$, $x_2(t) = A\sin\epsilon t\sin\omega_0 t$, which describes a fast oscillation modulated by a slow one. This corresponds physically to energy slowly sloshing back and forth between the masses, in a pattern of *beats*.