Duke University
Department of Physics

Physics 361 Spring Term 2020

WUN2K FOR LECTURE 23

These are notes summarizing the main concepts you need to understand and be able to apply.

- An axially symmetric spinning top pivoted at its tip *precesses* due to a weak gravitational torque, according to $\dot{\vec{L}} = \vec{\Gamma}$. Assuming the magnitude of angular velocity ω remains constant, and $\vec{\omega}$ remains along $\hat{e}_3$, a principal axis of inertia, the *direction* of the angular velocity changes according to $\dot{\hat{e}}_3 = \frac{MgR}{\lambda_3 \omega} \hat{z} \times \hat{e}_3$, where $\vec{R}$ is the vector from the pivot to the CM of the top and λ_3 is the principal moment along $\hat{e}_3$.
- **Euler's equations** for a rotating rigid body are $\dot{\vec{L}} + \vec{\omega} \times \vec{L} = \vec{\Gamma}$, and can be written in terms of components,

$$\begin{aligned} \lambda_1 \dot{\omega}_1 - (\lambda_2 - \lambda_3)\omega_2 \omega_3 &= \Gamma_1 \\ \lambda_2 \dot{\omega}_2 - (\lambda_3 - \lambda_1)\omega_3 \omega_1 &= \Gamma_2 \\ \lambda_3 \dot{\omega}_3 - (\lambda_1 - \lambda_2)\omega_1 \omega_2 &= \Gamma_3 \end{aligned} \tag{1}$$

  Some consequences for the case of zero torque:

  - If $\lambda_1 \neq \lambda_2 \neq \lambda_3$, if $\vec{\omega}$ is along one of the principal axes at some time, then it will remain constant along that axis indefinitely. Conversely, if $\vec{\omega}$ is not along one of the principal axes at some time, $\vec{\omega}$ will not be constant.
  - If $\lambda_1 = \lambda_2$, the body will exhibit *free precession*: for the "body frame" (frame with principal axes fixed in the body), $\hat{e}_3$, $\vec{\omega}$ and $\vec{L}$ lie in the same plane, and $\vec{\omega}$ and $\vec{L}$ precess about $\hat{e}_3$, forming the *body cone*. As seen in the "space frame" (inertial frame), $\vec{\omega}$ and $\vec{e}_3$ precess about a fixed $\vec{L}$, forming the *space cone*.