Duke University
Department of Physics

Physics 361 Spring Term 2020

WUN2K FOR LECTURE 22

These are notes summarizing the main concepts you need to understand and be able to apply.

- Coordinate system axes for which $I_{ij} = 0$ for $i \neq j$, i.e., for which $\{\mathbf{I}\}$ is *diagonal*, are known as **principal axes** of the body. The diagonal elements λ_i are known as the **principal moments of inertia**. These quantities are the eigenvectors and eigenvalues of the inertia tensor (treated as a matrix.)

- The principal moments of inertia can be found by finding the three roots $\lambda_1, \lambda_2, \lambda_3$ of the *secular equation* $|\{\mathbf{I}\} - \lambda I| = 0$; the principal axes can then be found by solving the system of equations, $\{\mathbf{I}\}\vec{\omega} - \lambda\vec{\omega} = 0$ for each of the given eigenvalues λ. The process of finding the principal moments and axes of inertia (more generally, the eigenvalues and eigenvectors) is known as *diagonalization*. If a body rotates about one of its principal axes, then $\vec{L}||\vec{\omega}$.

- Kinetic energy for a rotating rigid body is $T = \frac{1}{2}\vec{\omega} \cdot \vec{L}$. If we use principal axes for the coordinate system, $T = \frac{1}{2}(\lambda_1\omega_1^2 + \lambda_2\omega_2^2 + \lambda_3\omega_3^2)$, where the λ_i are the eigenvalues (moments of inertia with respect to the principal axes.)