Duke University
Department of Physics

Physics 361 Spring Term 2020

WUN2K FOR LECTURE 21

These are notes summarizing the main concepts you need to understand and be able to apply.

- A **rigid body** is defined as a collection of particles arranged so that the relative distances between the particles remains fixed. In our notation the particle indices are $\alpha = 1, 2, 3, ..., N$ for finite N particles.
- The total angular momentum can be decomposed as $\vec{L} = \vec{L}$(motion of CM)+ $\vec{L}$(motion relative to CM). Similarly, the kinetic energy can be written as $T = T$(motion of CM) + T(motion relative to CM).
- We can write the elements of the **inertia tensor** $\{\mathbf{I}\}$ as $I_{ij} = \sum_\alpha m_\alpha \left(\delta_{ij} \sum_k x_{\alpha,k}^2 - x_{\alpha,i} x_{\alpha,j}\right)$. The inertia tensor is defined for a body with respect to a particular coordinate system.
- The diagonal elements of the inertia tensor, I_{11}, I_{22}, I_{33} are the **moments of inertia** and the off-diagonal elements are the **products of inertia**. The products of inertia are symmetric, i.e., $I_{ij} = I_{ji}$. Therefore $\{\boldsymbol{I}\}$ has six independent elements.
- For a continuous mass distribution, $I_{ij} = \int_V \rho(\vec{r}) \left(\delta_{ij} \sum_k x_k^2 - x_i x_j\right) dv$, where $dv = dxdydz$ is a volume element at the position $\vec{r}$, and V is the volume of the body.
- The angular momentum of a body with respect to a point O (usually chosen as a fixed point of the body in a fixed coordinate system, or else the center of mass of the body) can be written $L_i = \sum_j I_{ij} \omega_j$ (or $\vec{L} = \{\mathbf{I}\}\vec{\omega}$). The $\vec{L}$ vector is not parallel to $\vec{\omega}$ if the products of inertia are non-zero!