Duke University
Department of Physics

Physics 361 Spring Term 2020

WUN2K FOR LECTURE 2

These are notes summarizing the main concepts you need to understand and be able to apply.

- A **unit vector** is a vector of unit length, often denoted with a "hat". The unit vector in the direction of $\vec{a}$ is $\hat{a} = \vec{a}/|\vec{a}|$. Unit vectors in the Cartesian coordinate axis directions are written $(\hat{x}, \hat{y}, \hat{z})$, or $(\hat{i}, \hat{j}, \hat{k})$ or $(\hat{e}_1, \hat{e}_2, \hat{e}_3)$.

- For description of a point on a plane in **plane polar** coordinates r, ϕ, we write $x = r\cos\phi$, $y = r\sin\phi$, $r = \sqrt{x^2 + y^2}$, $\phi = \tan^{-1}(y/x)$.

- The unit vectors for polar coordinates $\hat{r}$, $\hat{\phi}$ are *not constant in time*, which must be taken into account when finding velocity and acceleration in polar coordinates. The time derivatives of the polar unit vectors are:

$$\dot{\hat{r}} = \dot{\phi}\hat{\phi}$$
$$\dot{\hat{\phi}} = -\dot{\phi}\hat{r}$$

Position, velocity and acceleration in polar coordinates are:

$$\begin{aligned} \vec{r} &= r\hat{r} \\ \vec{v} &= \dot{r}\hat{r} + r\dot{\phi}\hat{\phi} \\ \vec{a} &= (\ddot{r} - r\dot{\phi}^2)\hat{r} + (r\ddot{\phi} + 2\dot{r}\dot{\phi})\hat{\phi} \end{aligned} \tag{1}$$

- For cylindrical coordinates (your homework), (ρ, ϕ, z), the velocity and acceleration are:

$$
\begin{aligned}
\vec{r} &= r\hat{r} \\
\vec{v} &= \dot{r}\hat{r} + r\dot{\phi}\hat{\phi} + \dot{z}\hat{z} \\
\vec{a} &= (\ddot{r} - r\dot{\phi}^2)\hat{r} + (r\ddot{\phi} + 2\dot{r}\dot{\phi})\hat{\phi} + \ddot{z}\hat{z}
\end{aligned}
\tag{2}
$$