Duke University
Department of Physics

Physics 361 Spring Term 2020

WUN2K FOR LECTURE 19

These are notes summarizing the main concepts you need to understand and be able to apply.

- Newton's Laws are valid only in inertial reference frames. However, it is often possible to treat motion in *noninertial* reference frames, i.e., accelerating ones.
- For a frame with rectilinear acceleration $\vec{A}$ with respect to an inertial one, we can write the equation of motion as $m\ddot{\vec{r}} = \vec{F} - m\vec{A}$, where $\vec{F}$ is the force in the inertial frame, and $-m\vec{A}$ is the *inertial force*. The inertial force is a "fictitious" force: it's a quantity that one adds to the equation of motion to make it look as if Newton's second law is valid in a noninertial reference frame.
- For a *rotating* frame: Euler's theorem says that the most general motion of any body with respect to an origin is rotation about some axis through the origin. We define an axis direction $\hat{u}$, a rate of rotation about that axis $\omega = \frac{d\theta}{dt}$, and an instantaneous angular velocity $\vec{\omega} = \omega\hat{u}$, with direction given by the right hand rule (fingers curl in the direction of rotation, thumb in the direction of $\vec{\omega}$.) We'll generally use $\vec{\omega}$ to refer to the angular velocity of a body, and $\vec{\Omega}$ to refer to the angular velocity of a noninertial, rotating reference frame.
- We have for the velocity of a point at $\vec{r}$ on the rotating body, $\vec{v} = \vec{\omega} \times \vec{r}$.
- Generally, for vectors $\vec{Q}$, one can write $\left(\frac{d\vec{Q}}{dt}\right)_{S_0} = \left(\frac{d\vec{Q}}{dt}\right)_{S} + \vec{\omega} \times \vec{Q}$, for S_0 an inertial reference frame and S a rotating reference frame.
- Considering forces in a rotating frame: Newton's second law is valid only in the fixed frame, and can be written using the expressions above as $m\ddot{\vec{r}} = \vec{F} + m(\vec{\Omega} \times \vec{r}) \times \vec{\Omega} + 2m\dot{\vec{r}} \times \vec{\Omega}$.

We can *define* an effective force in the rotating frame (like that of the Earth) as $\vec{F}_{\text{eff}} \equiv m\ddot{\vec{r}}$, so that it satisfies and expression *like* Newton's second law. We have $\vec{F}_{\text{eff}} = \vec{F} + m(\vec{\Omega} \times \vec{r}) \times \vec{\Omega} + 2m\dot{\vec{r}} \times \vec{\Omega}$.

Here $\vec{F}$ is the real force applied to the system. The other terms are are the noninertial "forces":

- the **centrifugal** "force", $\vec{F}_{\text{cf}} = m(\vec{\Omega} \times \vec{r}) \times \vec{\Omega}$, and
- the **Coriolis** "force", $\vec{F}_{\text{cor}} = 2m\dot{\vec{r}} \times \vec{\Omega}$.

Both of these are *fictitious* or *effective* forces; they are not real forces applied to the system, but are quantities introduced so that it *looks* like Newton's second law applies in a rotating system.