Duke University
Department of Physics

Physics 361 Spring Term 2020

WUN2K FOR LECTURE 18

These are notes summarizing the main concepts you need to understand and be able to apply.

- One can write the radial equation equivalently as an orbit equation, i.e., a differential equation to determine $r = r(\phi)$, by making the variable change $u = 1/r$. This yields $u''(\phi) = -u(\phi) - \frac{\mu}{l^2 u(\phi)^2} F$, where $F(r) = -dU/dr$.

- For the Kepler problem (gravity), $F = -Gm_1m_2/r^2 = -\gamma/r^2$, and the relation between r and ϕ (i.e., the orbit shape) can be written as $r(\phi) = \frac{c}{1+\epsilon\cos(\phi-\delta)}$, where $c = \frac{l^2}{\gamma\mu}$, and ϵ is the **eccentricity**. Usually we choose the orbit to start at $\delta = 0$. This equation corresponds to a *conic section* with one focus at the origin, and the shape depends on E, the total energy. We have $E = \frac{\gamma^2\mu}{2l^2}(\epsilon^2 - 1)$.

  - For $\epsilon > 1$, $E > 0$, the shape is a hyperbola (unbounded motion)
  - For $\epsilon = 1$, $E = 0$, the shape is a parabola (also unbounded).
  - For $0 < \epsilon < 1$, $U_{\text{eff,min}} < E < 0$, the shape is an ellipse (bounded).
  - For $\epsilon = 0$, $E = U_{\text{eff,min}}$, the shape is a circle (bounded).

  Here $U_{\text{eff,min}}$ is the minimum of the effective potential energy curve. The minimum r from the focus is the **pericenter** and the maximum r is the **apocenter**. (For orbits around the Sun, these are perihelion and aphelion respectively; for the Earth, they are perigee and apogee, respectively.)

- For the elliptical orbit case, the semimajor axis is given by $a = \frac{c}{1-\epsilon^2}$. The semiminor axis is given by $b = \frac{c}{\sqrt{1-\epsilon^2}}$.

- Kepler's Laws follow from these, and are:

  I. Planets move in elliptical orbits about the Sun with the Sun at one focus.

  II. The area per unit time swept out by a radius vector from the Sun to a planet is constant (see homework 3).

  III. The square of a planet's period is proportional to the cube of the major axis of the planet's orbit, i.e. $\tau^2 = \frac{4\pi^2\mu}{\gamma}a^3$, where τ is the period.

- One can move from one orbit to another by changing velocity (preserving continuity of orbit). A *Hohmann transfer* moves from one circular orbit to another via an elliptical transfer orbit. The required velocity changes can be calculated by finding the energy of each orbit in terms of the semimajor axes.