Duke University
Department of Physics

Physics 361 Spring Term 2020

WUN2K FOR LECTURE 14

These are notes summarizing the main concepts you need to understand and be able to apply.

- The **generalized momentum** p_i for a system described by a Lagrangian $\mathcal{L}(q_i, \dot{q}_i, t)$ corresponding to coordinate q_i is $p_i = \frac{\partial \mathcal{L}}{\partial \dot{q}_i}$. The **generalized force** corresponding to q_i is $F_i = \frac{\partial \mathcal{L}}{\partial q_i}$. A consequence of Lagrange's equations is that the generalized force equals the rate of change of the generalized momentum.

- **Conservation Theorems**

  - A coordinate q_i is **cyclic** or **ignorable** if it does not appear in the Lagrangian. A consequence of Lagrange's equations is that the generalized momentum associated with a cyclic coordinate will be conserved, i.e., constant in time.
  - *Conservation of Linear Momentum:* Total linear momentum $\vec{P}$ of N particles is conserved, i.e., $\sum \vec{p}_\alpha =$ constant, if the Lagrangian is invariant under translation, i.e., if $\delta\mathcal{L} = 0$ for $\vec{r}_\alpha \rightarrow \vec{r}_\alpha + \vec{\epsilon}$.
  - *Conservation of Energy:* The **Hamiltonian** of a system is $\mathcal{H} = \sum_{i=1}^{n} p_i \dot{q}_i - \mathcal{L}$. If $\mathcal{L}$ does not depend explicitly on the time (i.e., $\frac{\partial \mathcal{L}}{\partial t} = 0$), then $\mathcal{H}$ is conserved.
    The Hamiltonian is equivalent to the total energy of the system $E = T + U$ if the coordinate transformation $\vec{r}_\alpha = \vec{r}_\alpha(q_1, ...q_n)$ is time-independent (or "natural").
    If $\frac{\partial \mathcal{L}}{\partial t} = 0$ and the coordinate transformation is natural, then the total energy is conserved.
  - *Conservation of Angular Momentum:* conservation of angular momentum is a consequence of the invariance of the Lagrangian under rotation.

These relations for closed systems are all manifestations of **Noether's Theorem**, which connects symmetries (invariances) of the Lagrangian to conservation laws.