Duke University
Department of Physics

Physics 361 Spring Term 2020

WUN2K FOR LECTURE 11

These are notes summarizing the main concepts you need to understand and be able to apply.

- Calculus of variations: the basic problem is to determine the function $y(x)$ such that the integral $S = \int_{x_1}^{x_2} f[y(x), y'(x), x]dx$ is an *extremum* (usually a minimum). Here $y'(x) = dy/dx$. If S has its minimum value, any *neighboring function* $y(\alpha, x) = y(0, x) + \alpha\eta(x)$ must make S increase; at the endpoints $\eta(x_1) = \eta(x_2) = 0$. A necessary condition that S has a *stationary values* (i.e., has an extremum) is that $\frac{\partial S}{\partial \alpha}|_{\alpha=0} = 0$ for all $\eta(x)$.

- The **Euler-Lagrange equation** is a relation which much be satisfied in order for S to have a stationary point: $\frac{\partial f}{\partial y} - \frac{d}{dx}\frac{\partial f}{\partial y'} = 0$.

- If there are several variables $f[x_i(u), x_i'(u), u]$, $i = 1, 2, ...n$, then Euler's equation must be satisfied for each: $\frac{\partial f}{\partial x_i} - \frac{d}{du}\frac{\partial f}{\partial x_i'} = 0$.