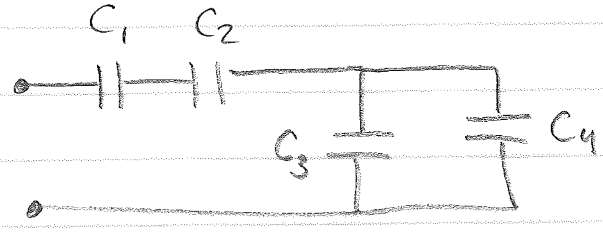
Homework # 3 Solutions

1)



$C_1 = 2.7\,\mu F$
$C_2 = 2.7\,\mu F$
$C_3 = 100\,\mu F$
$C_4 = 10\,\mu F$

Capacitors in parallel: add

$$C_3 \parallel C_4 = C_3 + C_4$$

Capacitors in series: add reciprocals

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3 + C_4}$$

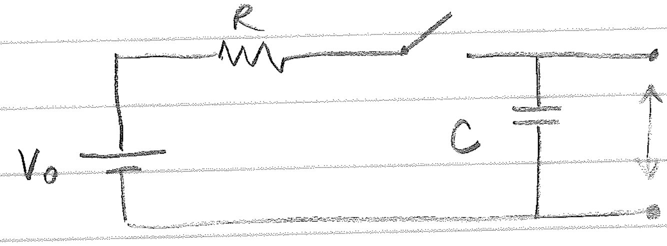
$$\frac{1}{C_{eq}} = \frac{C_1 + C_2}{C_1 C_2} + \frac{1}{C_3 + C_4} = \frac{(C_1 + C_2)(C_3 + C_4) + C_1 C_2}{C_1 C_2 (C_3 + C_4)}$$

$$\Rightarrow C_{eq} = \frac{C_1 C_2 (C_3 + C_4)}{(C_1 + C_2)(C_3 + C_4) + C_1 C_2}$$

$$= \frac{(2.7)(2.7)(110)}{(5.4)(110) + (2.7)(2.7)}\ \mu F$$

$$= \underline{\underline{1.3^3\,\mu F}}$$

2)



$V_0 = 12\,V$

$R = 100\,k\Omega$

$C = 10\,\mu F$

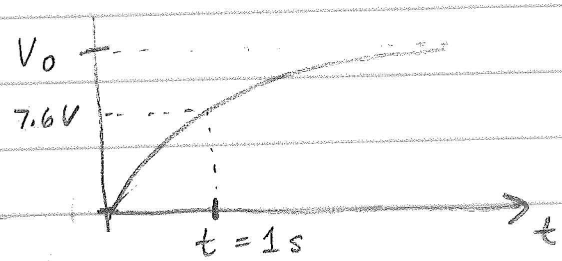
Want: V_{out} vs time after switch is closed

@ $t = 0$ $Q = 0$ on cap, $V_{out} = 0$ @ $t = 0$

RC circuit, charging capacitor

Time constant $\tau = RC = (100 \times 10^3)(10 \cdot 10^{-6}) = 1\,s$

Exponential



$$V_{out}(t) = V_0 \left(1 - e^{-t/RC}\right)$$

@ $t = RC$, $V = V_0(1 - 1/e)$

$= V_0(0.63) = 7.6\,V$

3) $V_0 = 100\,V$, $R = 1\,M\Omega$

What is C for $V_{out} = 70\,V$ @ $t = 10.0\,s$?

$$V_{out} = V_0\left(1 - e^{-t/RC}\right)$$

$$1 - e^{-t/RC} = \frac{V_{out}}{V}$$

$$e^{-t/RC} = 1 - V_{out}/V$$

$$-t/RC = \ln(1 - V_{out}/V)$$

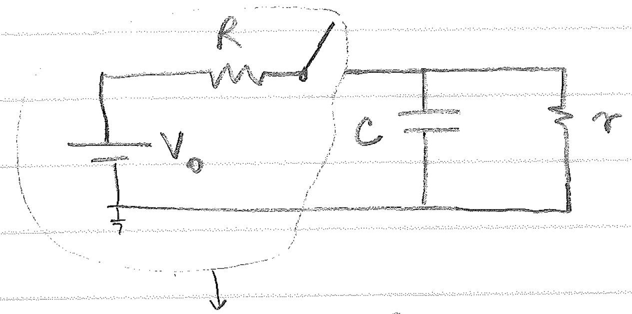
$$C = \frac{-t}{R}\,\frac{1}{\ln(1 - V_{out}/V)} = \frac{-10.0}{10^6}\,\frac{1}{\ln(1 - 70/100)}$$

$$C = \underline{\underline{8.3\,\mu F}}$$

4) Fully charge capacitor, then open switch

$V_0 = 100\ V$ $\quad$ $C = 8.3\ \mu F$

$R = 1\ M\Omega$ $\quad$ $V(t=0) = V_0$



$r = 10\ k\Omega$

ignore this if switch opened

It's an RC discharge situation

$$V(t) = V_0\, e^{-t/rC}$$

Solve for t s.t. $V = 1.0\ V$

$$e^{-t/rC} = \frac{V}{V_0}$$

$$-t/rC = \ln V/V_0$$

$$t = -rC \ln V/V_0 = (10\times10^3)(8.3\mu F)\ln(0.01)$$

$$\underline{\underline{t = 0.38\ s}}$$

5) Sine wave, amplitude = 20 V_{pp}

$$R = 10\,k\Omega$$

$$V(t) = (10\,V)\sin(\omega t + \phi)$$

$$I(t) = \frac{V(t)}{R} = \left(\frac{10\,V}{10^4\,\Omega}\right)\sin(\omega t + \phi)$$

$$I(t) = (1\,mA)\sin(\omega t + \phi)$$

Peak current is $\underline{\underline{1\,mA}}$ (or 2 mA pp)

Average current for a sine wave is $\underline{\underline{0}}$

$$I_{rms} = \frac{1\,mA}{\sqrt{2}} = \underline{\underline{0.707\,mA}}$$

Power rating: use rms

$$P = I^2_{rms} R$$

$$= (0.707 \times 10^{-3})^2 (10 \times 10^3)$$

$$P = \underline{\underline{5\,mW}}$$

(or greater)