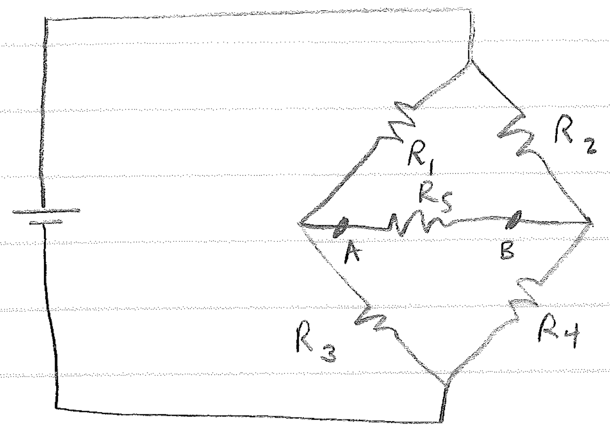
Homework # 2 Solutions

1.13)

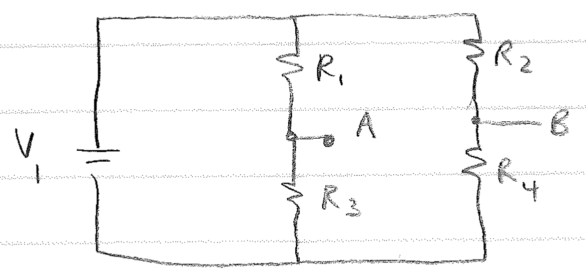


$V_1 = 10\ V$
$R_1 = 100\ \Omega$
$R_2 = 1\ k\Omega$
$R_3 = 99\ \Omega$
$R_4 = 1\ k\Omega$
$R_S = 10\ \Omega$

Find V_{Th}, R_{Th} w/ R_S removed

This problem was done in class:

To find V_{Th}, find V_{AB} (open)



We have two voltage dividers

$$V_A = V_1\left(\frac{R_3}{R_1+R_3}\right) = (10)\left(\frac{99}{100+99}\right)$$

$$\therefore V_A = 4.97\ V$$

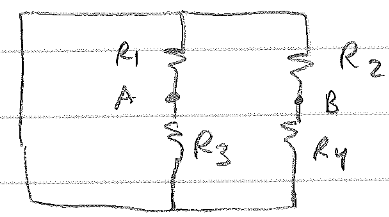
$$V_B = V_1\left(\frac{R_2}{R_2+R_4}\right) = \left(10 \cdot \frac{1000}{1000+1000}\right)$$

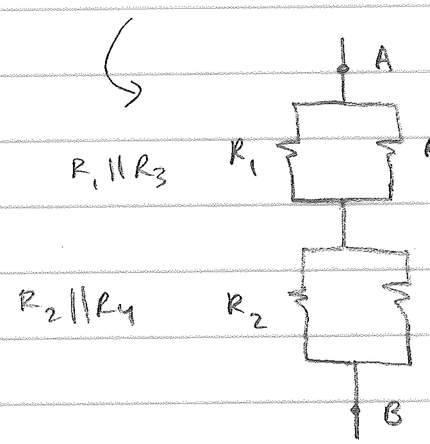
$$= 5\ V$$

$$\Rightarrow V_B - V_A = 5 - 4.97\ V$$

$$V_{Th} = V_B - V_A = \underline{\underline{0.025\ V}}$$

Now the thevenin resistance is the equivalent resistance with V_1 shorted





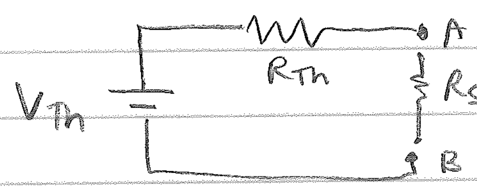
$$R_{Th} = \frac{R_1 R_3}{R_1 + R_3} + \frac{R_2 R_4}{R_2 + R_4}$$

$$= \frac{(100)(99)}{100 + 99} + \frac{(1000)(1000)}{2000}$$

$$= \underline{\underline{549.7\ \Omega}}$$

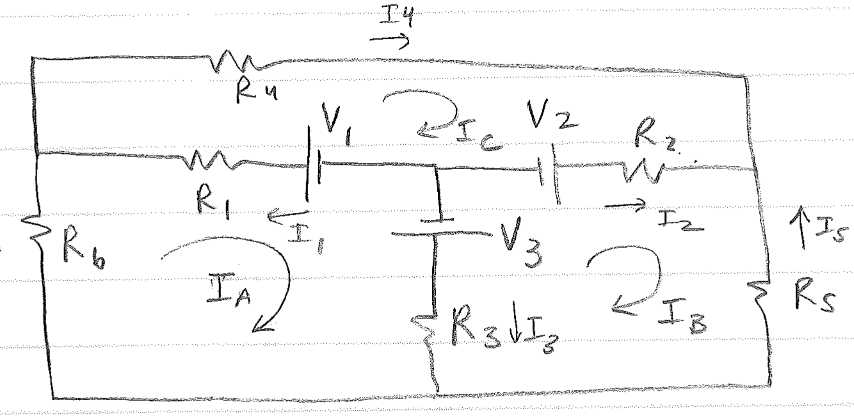
1.14) Now find current through R_S

The circuit without R_S is equivalent to



So $I_{R_S} = \dfrac{V_{Th}}{R_{Th} + R_S} = \dfrac{0.025\ V}{(549.7\ \Omega + 10)} = \underline{\underline{0.049\ mA}}$

1.17)



$V_1 = 5V$
$V_2 = 10V$
$V_3 = 15V$
$R_1 = 2\Omega$
$R_2 = 4\Omega$
$R_3 = 6\Omega$
$R_4 = 7\Omega$
$R_5 = 5\Omega$
$R_6 = 3\Omega$

Use "mesh loop" method for easier algebra

Walk around each loop

$$V_3 - (I_A - I_B)R_3 - I_A R_6 - (I_A - I_C)R_1 - V_1 = 0$$

$$V_2 - (I_B - I_C)R_2 - I_B R_5 - (I_B - I_A)R_3 - V_3 = 0$$

$$V_1 - (I_C - I_A)R_1 - I_C R_4 - (I_C - I_B)R_2 - V_2 = 0$$

Rearrange: put voltages on RHS

$$(-R_3 - R_6 - R_1)I_A + R_3 I_B + R_1 I_C = V_1 - V_3$$
$$R_3 I_A + (-R_2 - R_5 - R_3)I_B + R_2 I_C = V_3 - V_2$$
$$R_1 I_A + R_2 I_B + (-R_1 - R_4 - R_2)I_C = V_2 - V_1$$

Now plug this set of simultaneous linear equations into a computer - any method OK.

See attached Mathematica notebook

Mathematica notebook for simultaneous equation solution

```
In[39]:= ClearAll["Global`*"]

In[40]:=

In[41]:= Isol =
  Solve[{(-R3 - R1 - R6) Ia + R3 Ib + R1 Ic == V1 - V3, R3 Ia + (-R2 - R5 - R3) Ib + R2 Ic == V3 -
          V2, R1 Ia + R2 Ib + (-R1 - R4 - R2) Ic == V2 - V1}, {Ia, Ib, Ic}] /. V1 → 5 /.
      V2 → 10 /. V3 → 15 /. R1 → 2 /. R2 → 4 /. R3 → 6 /. R4 → 7 /. R5 → 5 /. R6 → 3.

Out[41]= {{Ia → 0.810409, Ib → -0.0855019, Ic → -0.286245}}
```

$$I_1 = I_C - I_a = \underline{\underline{-1.1\,A}}$$

$$I_2 = I_B - I_C = \underline{\underline{0.20}}\,A$$

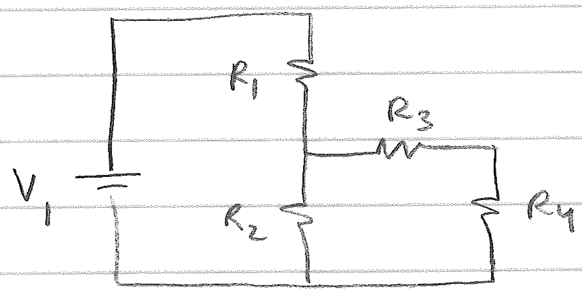
$$I_3 = I_A - I_B = \underline{\underline{0.90}}\,A$$

$$I_4 = I_C = \underline{\underline{-0.29}}\,A$$

$$I_5 = -I_B = \underline{\underline{0.086}}\,A$$

$$I_6 = I_A = \underline{\underline{0.81}}\,A$$

1.18)



$V_1 = 10\ V$

$R_1 = 12\ \Omega$

$R_2 = 8\ \Omega$

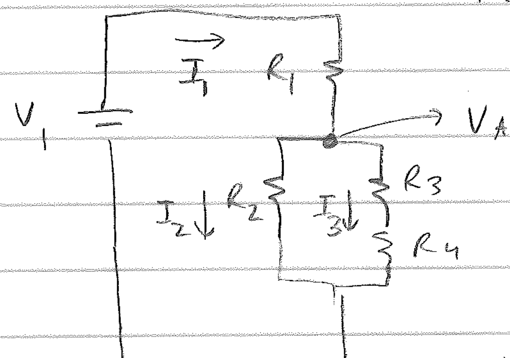
$R_3 = 5\ \Omega$

$R_4 = 10\ \Omega$

a) Compute current through R_4, first w/o using Thevenin & Norton equivalents

Just treat as voltage divider. want I_3

Kirchoff's Rule



$$I_1 = I_2 + I_3$$

$$V_A = \frac{R_2 \| (R_3 + R_4)}{R_1 + (R_2 \| R_3 + R_4)} \cdot V_1$$

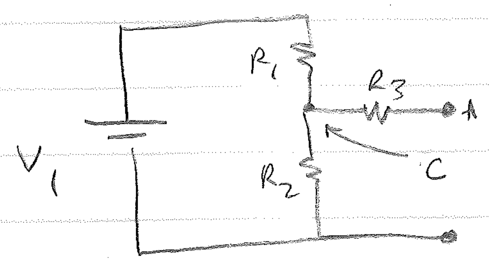
$$R_2 \| (R_3 + R_4) = \frac{R_2(R_3 + R_4)}{R_2 + (R_3 + R_4)}$$

$$= \frac{(8)(15)}{8 + 15} = 5.217\ \Omega$$

$$V_A = (10V)\left(\frac{5.217}{12 + 5.217}\right) = 3.03\ V$$

$$I_3 = \frac{V_A}{R_3 + R_4} = \frac{3.03\ V}{(15\ \Omega)} = \underline{0.202\ A}$$

b) $V_{Th} = V(\text{open})$ for circuit with R_4 removed

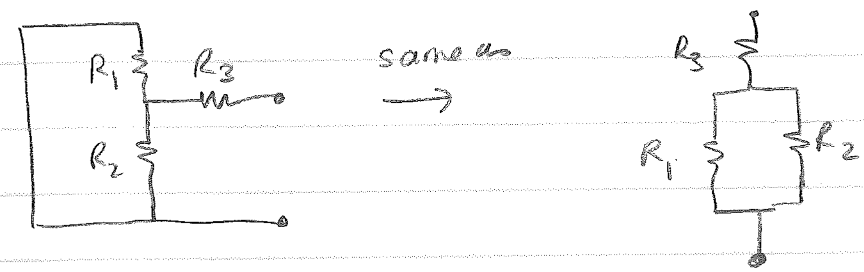


No current through R_3
so $V_A = V$.

Use voltage divider

$$V_A = V_1 \frac{R_2}{R_1 + R_2} = (10)\left(\frac{8}{8+12}\right) = \underline{\underline{4\ V}}$$

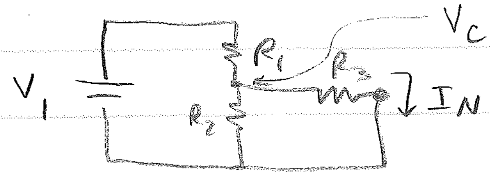
R_{Th} is the equivalent resistance with V_1 shorted



$$R_{eq} = R_3 + R_1 \parallel R_2$$
$$= R_3 + \frac{R_1 R_2}{R_1 + R_2}$$

$$R_{Th} = 5 + \frac{8 \cdot 12}{8+12} = \underline{\underline{9.8\ \Omega}}$$

Norton current : current with terminals shorted



V_C from voltage divider

$$\therefore V_C = V_1 \cdot \frac{R_2 \parallel R_3}{R_2 \parallel R_3 + R_1}$$

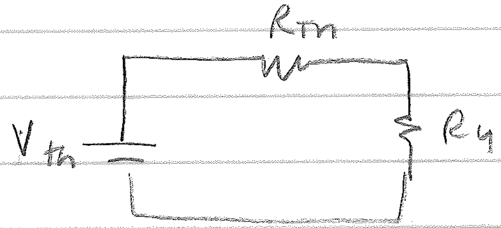
$$R_2 \parallel R_3 = \frac{R_2 R_3}{R_2 + R_3}$$

$$V_C = (10V) \cdot \frac{\left(\frac{8 \cdot 5}{8+5}\right)}{\left(\frac{8 \cdot 5}{8+5}\right) + 12} = 2.04V$$

$$I_N = \frac{V_C}{R_3} = \frac{2.04}{5} = \underline{\underline{0.408\ A}}$$

c) If $R_4 = 10\,\Omega$ is connected across the Thevenin equivalent, we should get the answer from part a

Thevenin equivalent



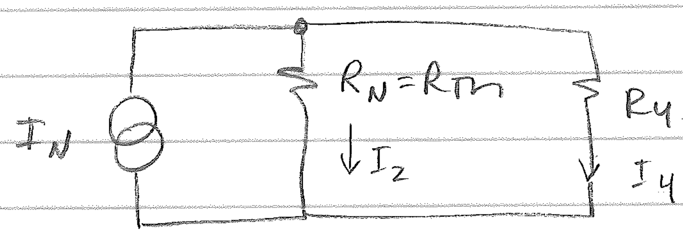
$$I_4 = \frac{V_{Th}}{R_{Th} + R_4} = \frac{4\,V}{(9.8+10)}$$

$$I_4 = \underline{\underline{0.202\,A}}$$

✓ same as part (a)

Norton equivalent

This is a current divider



Both resistors have the same voltage across them

$$I_4 = I_N - I_2 \qquad I_4 R_4 = I_2 R_N$$

$$I_4 = I_N - \frac{I_4 R_4}{R_N}$$

$$I_4\left(1 + \frac{R_4}{R_N}\right) = I_N$$

$$I_4 = I_N\left(\frac{R_N}{R_N + R_4}\right) = (0.408)\left(\frac{9.8}{9.8+10}\right) = \underline{\underline{0.202\,A}}$$

Again, same answer, as expected