Homework #1 Solutions

1.1) Nichrome wire $d = 1mm$, $L = 1\ m$

$$R = \rho \frac{L}{A}, \quad \rho = 100 \times 10^{-8}\ \Omega m \quad \text{(Table 1.2)}$$

$$= \frac{(100 \times 10^{-8})(1)}{\pi (0.5 \times 10^{-3})^2}\ \Omega$$

$$\underline{\underline{R = 1.27\ \Omega}}$$

1.2) Maximum allowable current through $R = 10\ k\Omega$, for $P = 10\ W$:

Power dissipated by an ohmic resistor

$$P = I^2 R$$

$$I_{max} = \sqrt{P/R} = \sqrt{10/(10 \times 10^3)} = \underline{\underline{0.032\ A}}$$

For $R = 10\ k\Omega$, $P = 0.25\ W$

$$I_{max} = \sqrt{0.25/(10 \times 10^3)} = \underline{\underline{0.005\ A}}$$

1.3) a) $R = 100\,\Omega$ Ohmic resistor $V = IR$

$V = 100\,V$

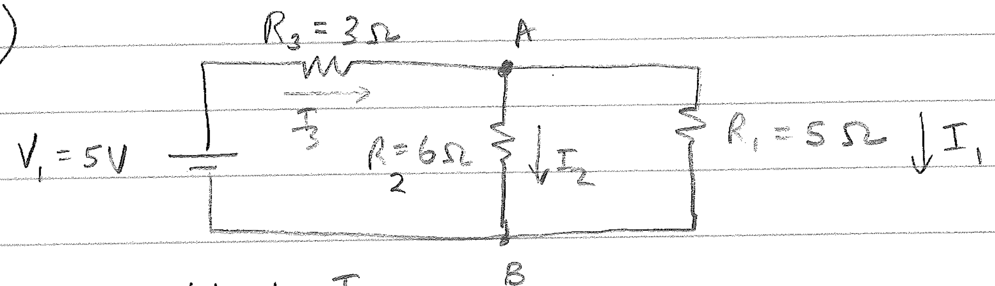
Power is $P = IV = \frac{V^2}{R}$

$$P = \frac{(100)^2}{100} = \underline{\underline{100\,W}}$$

b) For $R = 100\,k\Omega$

$$P = \frac{(100)^2}{10^5} = \underline{\underline{0.1\,W}}$$

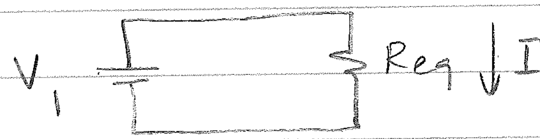
1.4)



Want I_3

Apply Kirchoff:
$$\left.\begin{aligned} I_3 &= I_2 + I_1 \\ V_1 - I_3R_3 - I_2R_2 &= 0 \\ V_1 - I_3R_3 - I_1R_1 &= 0 \end{aligned}\right\}$$
3 eqns, 3 unknowns, can solve

But an easier way for this problem: consider as



The current through "Req" will be the same as through R_3

$$R_{eq} = R_3 + R_1 \| R_2 = R_3 + \frac{R_1R_2}{R_1+R_2} = 3 + \frac{(6)(5)}{6+5} = 5.73\,\Omega$$

$$I = V_1/R_{eq} = 5V/5.73V = \underline{\underline{0.8731A}}$$

1.5) Using voltage divider equation, voltage at point A is

$$V_A = \frac{R_2 \| R_1}{R_3 + R_2 \| R_1} V_1$$

$$R_2 \| R_1 = \frac{R_1 R_2}{R_1 + R_2} = \frac{(5)(6)}{5+6} = 2.727\ \Omega$$

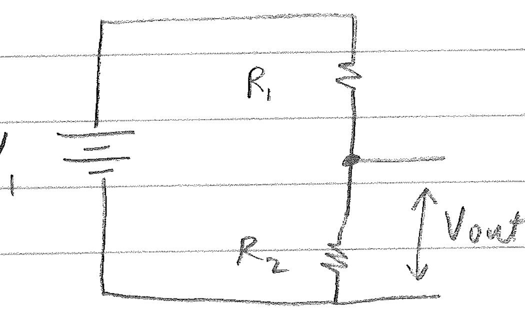
$$V_A = \frac{2.727}{2.727 + 3} (5\text{ V}) = 2.38\text{ V}$$

$$V_B = 0$$

so $I_1 = V_A / R_1 = 2.38 / 5 = \underline{\underline{0.476}}\text{ A}$

$I_2 = V_A / R_2 = 2.38/6 = \underline{\underline{0.397}}\text{ A}$

1.6) Voltage divider

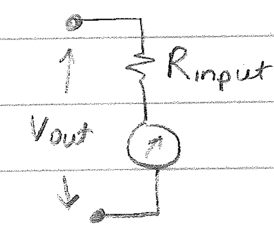


$V_1 = 3\,V$

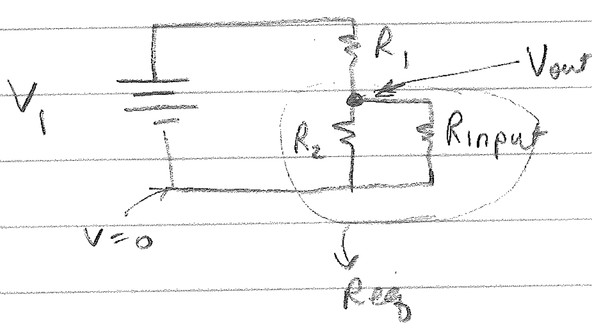
$R_1 = 2\,k\Omega$

$R_2 = 1\,k\Omega$

Voltmeter: need to consider its internal resistance



so looks like



$$V_{out} = \frac{R_{eq}}{R_1 + R_{eq}} V_1$$

$$R_{eq} = R_2 \parallel R_{input}$$

$$R_{eq} = \frac{R_2 R_{input}}{R_2 + R_{input}}$$

What the meter reads: $$V_{out} = V_1 \frac{R_2 R_{input} / (R_2 + R_{input})}{R_1 + R_2 R_{input} / (R_2 + R_{input})} = \frac{R_2 R_{input} V_1}{R_1 R_2 + R_1 R_{input} + R_2 R_{input}}$$

For $R_{input} = 100\,\Omega$: $$V_{out} = \frac{(10^3)(10^2)(3)}{(2\times10^3)(10^3) + (2\times10^3)(100) + (10^3)(100)} = \underline{0.130\,V}$$

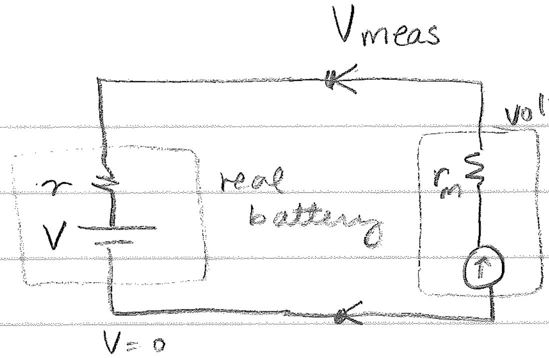
Similarly for $R_{input} = 1\,k\Omega$, $V_{out} = \underline{0.600\,V}$

$R_{input} = 50\,k\Omega$, $V_{out} = \underline{0.987\,V}$

$R_{input} = 1\,M\Omega$, $V_{out} = \underline{0.999\,V}$

Note that the higher the meter internal resistance, the closer Vout is to the value without the meter

1.7)



r = battery internal resistance

V = ideal source voltage

r_m = voltmeter internal resistance

$r_m = 1000\ \Omega$

$V = 1.5\ V$

$V_{meas} = 0.9\ V$

Treat as voltage divider $V_{meas} = \dfrac{r_m}{r + r_m} V$

solve for r

$$r + r_m = r_m \frac{V}{V_{meas}}$$

$$r = r_m \frac{V}{V_{meas}} - r_m = 10^3\left(\frac{1.5}{0.9}\right) - 10^3$$

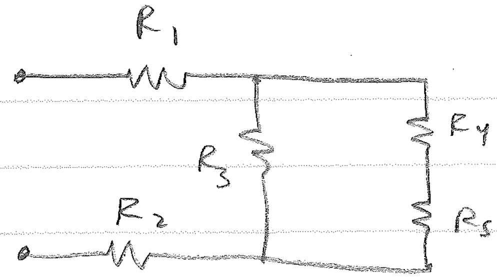
$$r = \underline{\underline{667\ \Omega}}$$

1.8) Same relation $V_{meas} = \dfrac{r_m}{r + r_m} V$, but $r_m = 10 \times 10^6\ \Omega$

$$V_{meas} = \frac{10 \times 10^6}{(10 \times 10^6) + 667} \cdot 1.5 = \underline{\underline{1.499\ V}}$$

Note: Large meter resistance → closer to "ideal" measurement

1.9)



$R_1 = 3\,\Omega$

$R_2 = 2\,\Omega$

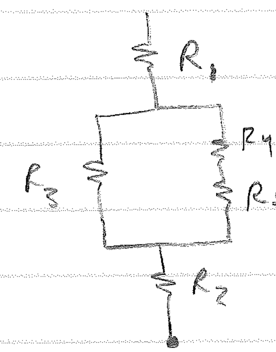
$R_3 = 15\,\Omega$

$R_4 = 10\,\Omega$

$R_5 = 5\,\Omega$

Want equivalent resistance of this network

Redraw:



$$R_{eq} = R_1 + R_3 \| (R_4 + R_5) + R_2$$

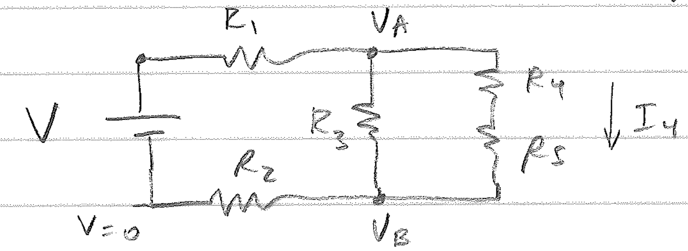
$$= R_1 + \frac{R_3(R_4 + R_5)}{R_3 + R_4 + R_5} + R_2$$

$$= 3 + \frac{15(10+5)}{15+10+5} + 2$$

$$R_{eq} = \underline{\underline{12.5\,\Omega}}$$

1.10) Consider a battery across the terminals

$V = 25\ V$



What is current in R_4? (I_4) $\quad I = \frac{V}{R_{eq}}$ through whole circuit

$$I = \frac{25\ V}{12.5\ \Omega} = 2A$$

Kirchoff Loop Rule

$$V - IR_1 - I_4(R_4 + R_5) - IR_2 = 0$$

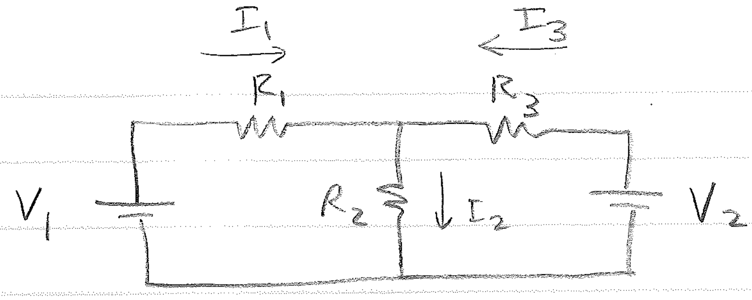
Solve for I_4

$$I_4 = \frac{V - IR_1 - IR_2}{(R_4 + R_5)}$$

$$= \frac{25 - (2)(3) - 2(2)}{15}$$

$$\underline{\underline{I_4 = 1A}}$$

1.11)



$V_1 = 5\,V$

$V_2 = 15\,V$

$R_1 = 3\,\Omega$

$R_2 = 5\,\Omega$

$R_3 = 8\,\Omega$

Want I_2, I_3

Use Kirchoff's Rules

$$\left.\begin{aligned} V_1 - I_1 R_1 - I_2 R_2 &= 0 \\ V_2 - I_3 R_3 - I_2 R_2 &= 0 \\ I_1 &= I_2 - I_3 \end{aligned}\right\}$$

3 eqns, 3 unknowns

Solve:

$$I_3 = \frac{V_2 - I_2 R_2}{R_3}$$

$$V_1 - (I_2 - I_3) R_1 - I_2 R_2 = 0$$

$$V_1 - I_2(R_1 + R_2) + I_3 R_1 = 0$$

$$V_1 - I_2(R_1 + R_2) + \left(\frac{V_2 - I_2 R_2}{R_3}\right) R_1 = 0$$

Solve for I_2 $\quad V_1 - I_2\left(R_1 + R_2 + \frac{R_2 R_1}{R_3}\right) + \frac{V_2 R_1}{R_3} = 0$

$$I_2 = \frac{V_1 + V_2 R_1 / R_3}{(R_1 + R_2 + R_2 R_1 / R_3)} = \frac{(5 + 15 \cdot 3/8)}{(3 + 5 + 3 \cdot 5/8)} = \underline{\underline{1.076\,A}}$$

Solve for I_3 $\quad I_3 = \frac{V_2 - I_2 R_2}{R_3} = \frac{15\,V - (1.076)(5)}{8} = \underline{\underline{1.20\,A}}$