Homework # 10 Solutions

5.1) a) The V_{gs} voltages range for large I_d at $V_{gs} = 0V$ to ~zero I_d at $V_{gs} = -2.5V$

This corresponds to **JFET** operation

b)

V_{gs} (V)	$I_d(sat)$ (mA)	$\sqrt{I_d(sat)}$ $(mA)^{1/2}$
0	9	3
−0.5	6.25	2.5
−1.0	4	2
−1.5	2.25	1.5
−2.0	1	1
−2.5	0.25	0.5

c) slope of $\sqrt{I_d(sat)}$ vs. V_{gs} : $-1\ \sqrt{mA}/V$

Intercept : $3\ \sqrt{mA}$ @ $V_{gs} = 0$

Model equations

$$I_d(sat) = K(V_{gs} - V_t)^2$$

$$\sqrt{I_d(sat)} = \sqrt{K}\,(V_{gs} - V_t)$$

Intercept : $\boxed{V_t = -3V}$

$$\sqrt{K} = 1\ \sqrt{mA}/V$$

$$\Rightarrow \boxed{K = 1\ mA/V^2}$$

IP[y]: Notebook (/tree/Documents/phy271/homework) hw10 Last Checkpoint: Mar 29 17:39 (autosaved)

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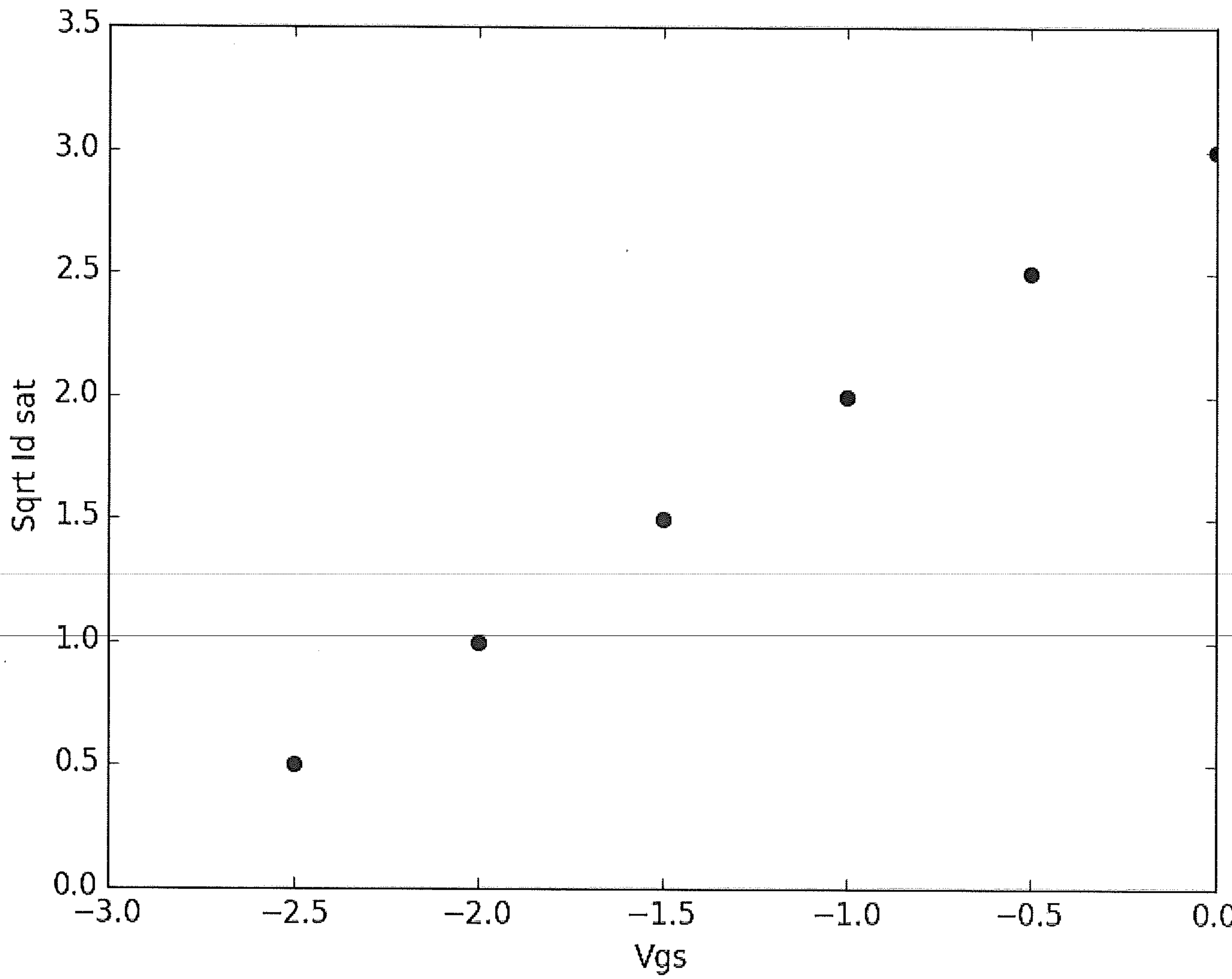
In [*]:
```
import numpy as np
import matplotlib.pyplot as plt
plt.xlabel('Vgs')
plt.ylabel('Sqrt Id sat')
plt.plot([0,-0.5,-1.0,-1.5,-2.0,-2.5], np.sqrt([9.,6.25,4.,2.25,1.,0.25]), 'ro')
plt.axis([-3., 0, 0, 3.5])
plt.show()
```

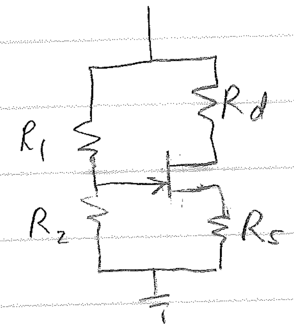
In [*]:

In [ ]:


3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
Sqrt Id sat
−3.0
−2.5
−2.0
−1.5
−1.0
−0.5
0.0
Vgs

5.2) Operating Point (V_{gs}, I_d, V_{ds})

DC bias circuit



$V_{dd} = 15\,V$

$R_1 = 1\,M\Omega$

$R_2 = 100\,k\Omega$

$R_d = 3000\,\Omega$

$R_s = 1000\,\Omega$

$V_t = -3V$

$k = 1\,mA/V^2$

First find V_{gs} from the quadratic

$$k(V_{gs} - V_t)^2 = \frac{V_g - V_{gs}}{R_s} \qquad \text{(see attachment)}$$

$$V_{gs} = \underline{\underline{-1.35\,V}}$$

$$I_d = \frac{V_g - V_{gs}}{R_s} = \underline{\underline{2.71\,mA}}$$

$$V_{ds} = V_{dd} - I_d(R_s + R_d) = \underline{\underline{4.14\,V}}$$

```
In[60]:= Ans = Solve[K (Vgs - Vt) ^2 == ((Vdd R2 / (R1 + R2)) - Vgs) / Rs, Vgs]
```

Out[60]=
$$\left\{\left\{Vgs \to \frac{-\frac{1}{Rs} + 2\,K\,Vt - \frac{\sqrt{R1+R2+4\,K\,R2\,Rs\,Vdd-4\,K\,R1\,Rs\,Vt-4\,K\,R2\,Rs\,Vt}}{\sqrt{R1\,Rs^2+R2\,Rs^2}}}{2\,K}\right\},\right.$$
$$\left.\left\{Vgs \to \frac{-\frac{1}{Rs} + 2\,K\,Vt + \frac{\sqrt{R1+R2+4\,K\,R2\,Rs\,Vdd-4\,K\,R1\,Rs\,Vt-4\,K\,R2\,Rs\,Vt}}{\sqrt{R1\,Rs^2+R2\,Rs^2}}}{2\,K}\right\}\right\}$$

```
In[61]:= Ans /. Vdd → 15 /. R1 → 10^6 /. R2 → 10^5 /. Rd → 3000 /. Rs → 1000. /. Vt → -3 /.
         K → 10^(-3)
```

```
Out[61]= {{Vgs → -5.64794}, {Vgs → -1.35206}}
```

→ pick this solution
since can't have $V_{gs} < V_t$

```
In[62]:= Vg = Vdd R2 / (R1 + R2);

In[63]:= Id = (Vg - Vgs) / Rs /. Ans[[2]] /. Vt → -3 /. K → 10^(-3) /. Vdd → 15 /. R1 → 10^6 /.
             R2 → 10^5 /. Rd → 3000 /. Rs → 1000.

Out[63]= 0.0027157

In[64]:= Vds = Vdd - Id (Rs + Rd) /. Vdd → 15 /. Rd → 3000 /. Rs → 1000.

Out[64]= 4.13721

In[65]:=
```

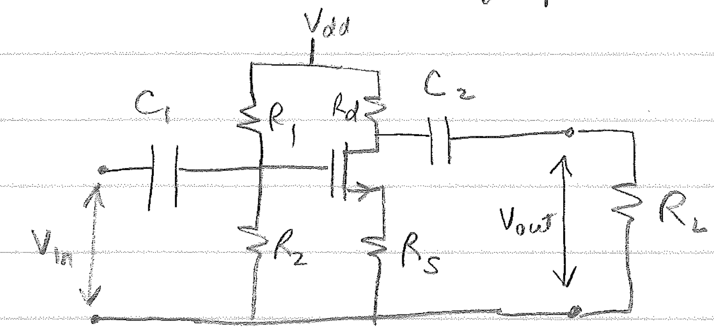
5.3) $g_m = 2K(V_{gs} - V_t)$

$$i_{ii} = 2(1)\left(-1.35 - (-3)\right)$$

$$g_m = \underline{3.3 \times 10^{-3}} \ A/V$$

In the saturation regime, the slope of I_d vs V_{gs} is zero, so we take $\underline{r_{out} \to \infty}$

5.4) Common source amplifier



$R_L = 10\ k\Omega$

$g_m = 3.3 \times 10^{-3} \frac{A}{V}$

$R_S = 1\ k\Omega$

$R_d = 3\ k\Omega$

Voltage gain $\quad a = \dfrac{-g_m R_L'}{1 + g_m R_S}$

$$R_L' = \frac{R_d R_L}{R_d + R_L} = \frac{(3)(10)}{3+10} = 2.31\ k\Omega$$

$$a = \frac{-(3.3\times10^{-3})(2.31\times10^{3})}{1+(3.3\times10^{-3})(1000)} = \underline{\underline{-1.77}}$$

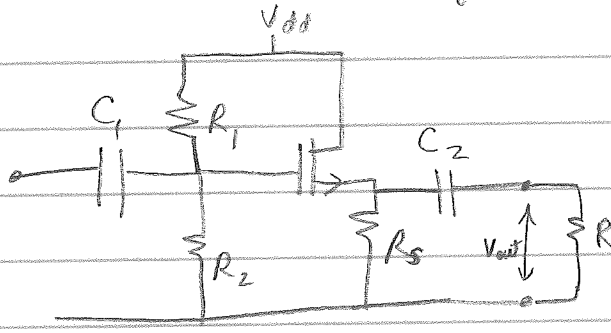
Current gain $\quad g = \dfrac{-g_m}{1+g_m R_S}\left(\dfrac{R_d R_G}{R_d + R_L}\right)$

$$R_G = R_1 \parallel R_2 = \frac{(1)(0.1)}{1.1} = 90.9\ k\Omega$$

$$g = \frac{-(3.3\times10^{-3})}{(1+(3.3\times10^{-3})(1000))}\left(\frac{3000 \cdot 90.9\times10^{3}}{3000+10000}\right)$$

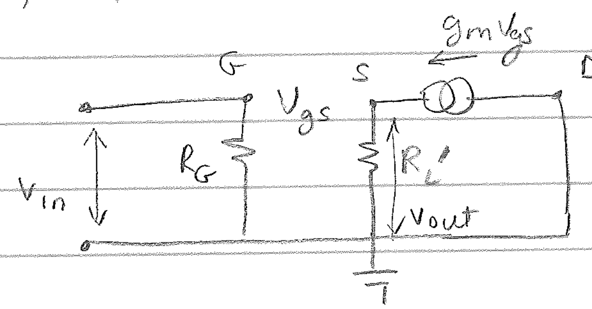
$$g = \underline{\underline{-16.1}}$$

5.5) Common drain amplifier



Derive: g_1, Z_{in}, Z_{out}

AC model:



$R_G = R_1 \| R_2$

$R_L' = R_S \| R_L$

$$g = \frac{i_{out}}{i_{in}} \qquad i_{in} = \frac{V_{in}}{R_G}$$

$$i_{out} = \frac{V_{out}}{R_L} \Rightarrow \frac{i_{out}}{i_{in}} = a\,\frac{R_G}{R_L}$$

$$a = \frac{V_{out}}{V_{in}} \qquad V_{out} = (\underbrace{g_m V_{gs}}_{i_{out}})\, R_L'$$

$$V_{in} - V_{gs} - i_{out} R_L' = 0 \Rightarrow V_{in} = V_{gs} + g_m V_{gs} R_L'$$

$$a = \frac{g_m \cancel{V_{gs}} R_L'}{\cancel{V_{gs}} + g_m \cancel{V_{gs}} R_L'} = \frac{g_m R_L'}{1 + g_m R_L'}$$

Current gain

$$g = \frac{g_m R_L'}{1 + g_m R_L'}\,\frac{R_L}{R_G} = \frac{g_m \left(\frac{R_S \cancel{R_L}}{R_S + R_L}\right)}{1 + g_m R_L'}\,\frac{R_G}{\cancel{R_L}} = \frac{g_m}{1 + g_m R_L'}\,\frac{R_S R_G}{R_S + R_L}$$

as required

Input impedance

$$Z_{in} = \frac{v_{in}}{i_{in}} = \frac{v_{in}}{v_{in}/R_G} = \underline{\underline{R_G}}$$

$R_L' \to R_S$ for $R_L \to \infty$

$$Z_{out} = \frac{v_{out}(R_L \to \infty)}{i_{out}(R_L \to 0)} = \frac{g_m v_{gs} R_S}{g_m v_{gs}} = \underline{\underline{R_S}}$$

$R_L' \to R_L$ for $R_L \to \infty$