Duke University
Department of Physics

Physics 271 Spring Term 2017

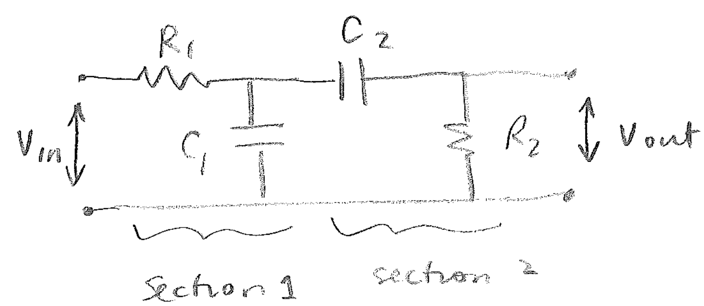
PRACTICE MIDTERM SOLUTIONS

I will abide by the Duke Community Standard. Name: ____________________

This is a closed book exam, with one side of one page cheat sheet allowed. Calculators are allowed, but only for basic calculations: you may not use special memory, graphing etc. functions. You must always show your work for credit; all answers must be justified. **You must hand in your cheat sheet with your test.**

Problem 1: (10 points)

Using only capacitors and resistors, make a sketch of a two-pole band-pass filter. Roughly sketch the resulting Bode plot, labeling the slopes of any lines. Don't use specific values for R and C but indicate relative sizes of them.

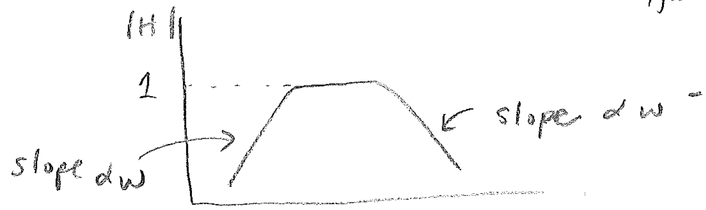


Want section 2 to draw little current from section 1, so choose $R_2 \gg R_1$

$C_2 \ll C_1$

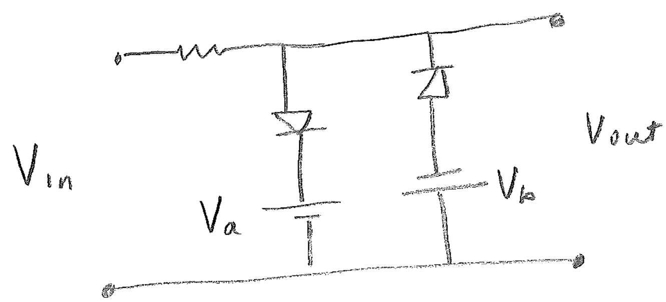
$$\hat{H}(\omega) \sim \hat{H}_1(\omega)\,\hat{H}_2(\omega) = \left(\frac{1/j\omega C_1}{1/j\omega C_1 + R_1}\right)\left(\frac{R_2}{1/j\omega C_2 + R_2}\right)$$

Bode plot (log-log)



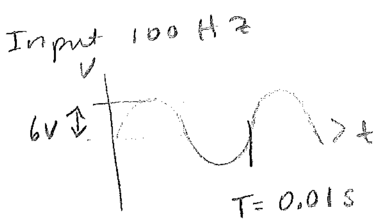
Problem 2: (10 points)

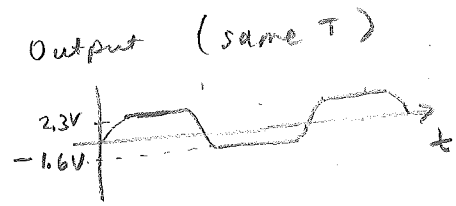
Design a diode clipper circuit that limits the output to no more than +2.3 V and -1.6 V. Sketch the output for a 100 Hz sine wave with 6 V amplitude, and for a 50 Hz square wave with 2 V amplitude.



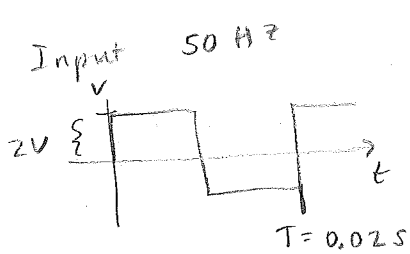
Choose $V_a = 1.7$ V, $V_b = -1$ V

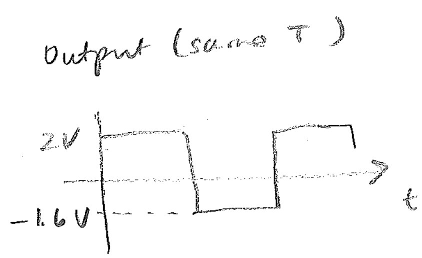
Then the output will never exceed 2.3 V or be less than −1.6 V





clipped

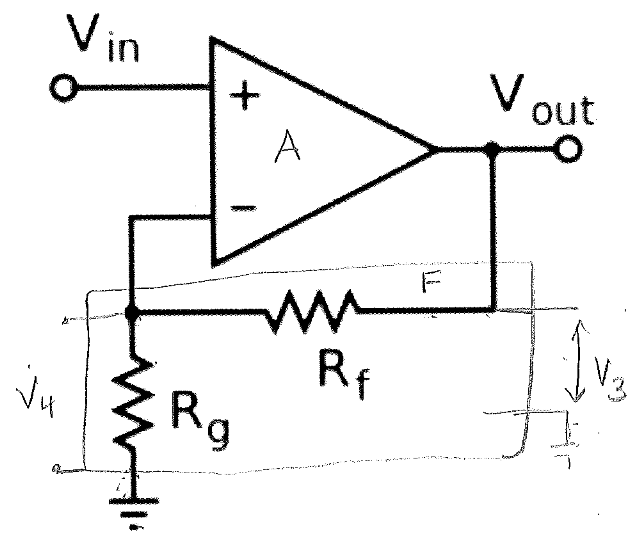




only negative part is clipped

Problem 3: (10 points)

Determine the gain of the given op-amp circuit, assuming that the op-amp's open-loop DC gain is A.



Note: Vout and Vin are with respect to ground

Voltage divider : $V_4 = \underbrace{\left(\frac{R_g}{R_f+R_g}\right)}_{F} V_3$

$$|\hat{H}| = \left|\frac{\hat{A}}{1+\hat{A}\hat{F}}\right| = \left|\frac{\hat{A}}{1+\frac{R_g}{R_f+R_g}\hat{A}}\right|$$

Assuming $|\hat{A}\hat{F}| \gg 1$, $|\hat{H}| = G = \frac{1}{F} = \underline{\frac{R_f+R_g}{R_g}}$

Problem 4: (10 points)

Determine the locations of the poles and zeroes of the transfer function $\hat{\mathbf{H}}(\omega) = \frac{\omega - 3}{\omega^2 - 4j\omega - 13}$. Express your answers in the form of values of $\hat{\mathbf{s}}$, and indicate their positions (in standard notation) on the complex frequency plane.

$$\hat{s} = j\omega$$

zero for $\omega = 3$

so $\boxed{\hat{s} = 3j}$ → □

$$\omega^2 = -\hat{s}^2$$

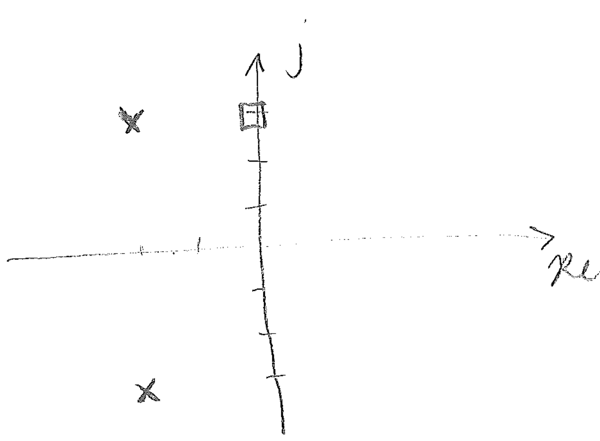
Poles for $-\hat{s}^2 - 4\hat{s} - 13 = 0$

$$\hat{s}^2 + 4\hat{s} + 13 = 0$$

$$\hat{s} = \frac{-4 \pm \sqrt{16 - 4 \cdot 13}}{2}$$

$$\hat{s} = \frac{-4 \pm 2\sqrt{4 - 13}}{2}$$

$\boxed{\hat{s} = -2 \pm 3j}$ → ×



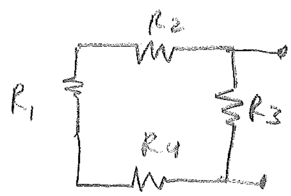
Problem 5: (20 points)

For the given circuit

a. Determine the Thevenin resistance R_T. → a) Open sources, short supplies

b. Find the Norton equivalent current I_N.

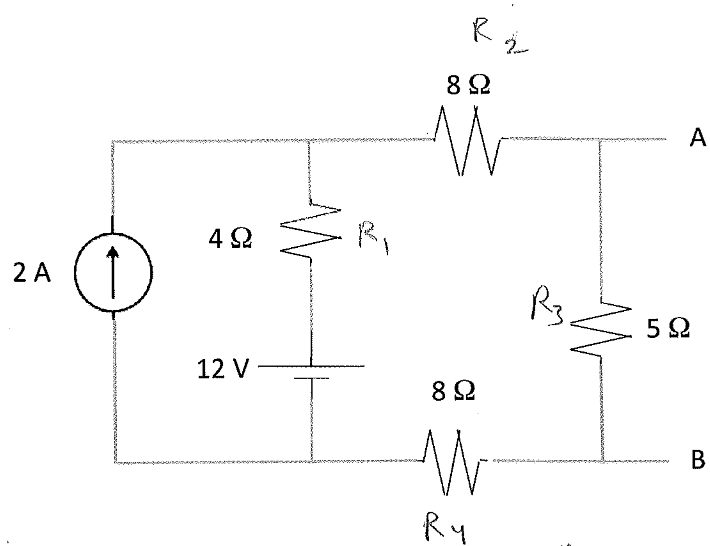
c. Find the Thevenin equivalent voltage V_T.



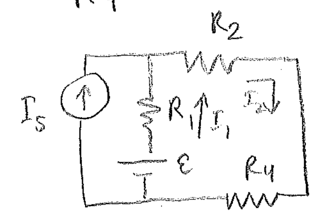
$$R_{Th} = \frac{R_3(R_1+R_2+R_4)}{R_1+R_2+R_3+R_4}$$

$$= \frac{5(20)}{25}$$

$$R_{Th} = \underline{4\ \Omega}$$



b) I_N is for AB shorted (ignore R_3)



Use Kirchhoff ($I_2 = I_N$)

$$I_S + I_1 = I_2$$

$$\mathcal{E} - I_1 R_1 - I_2(R_2+R_4) = 0$$

$$\mathcal{E} - I_1 R_1 - (I_1 + I_S)(R_2+R_4) = 0$$

$$I_1(R_1+R_2+R_4) = \mathcal{E} - I_S(R_2+R_4)$$

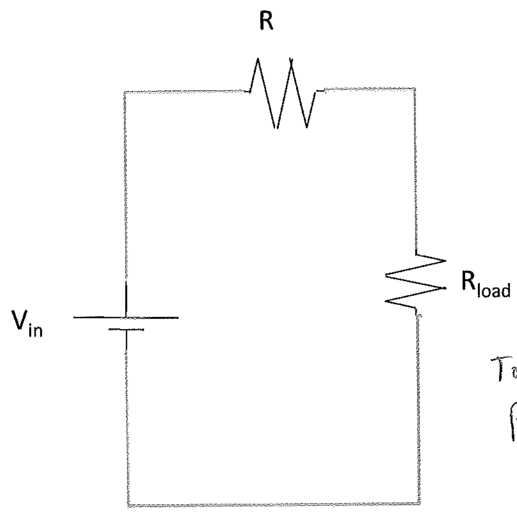
$$\Rightarrow I_1 = \frac{12 - 2(16)}{20} = -1\ \text{A}$$

$$I_N = (2-1)\ \text{A} = \underline{1\ \text{A}}$$

c) $V_{Th} = I_N R_{Th} = (1)(4\text{A}) = \underline{4\ \text{V}}$

Problem 6: (20 points)

For the given circuit, show that the maximum power will be dissipated in R_{load} if $R = R_{\text{load}}$. (This is known as impedance matching, and is a useful result.)



$P = VI$

$V = I^2 R$

$\Rightarrow P = I^2 R = \frac{V^2}{R}$

Same I through each R

Total power dissipated:

$P_{tot} = I^2 R + I^2 R_{load}$

Loop Rule

$$V_{in} - IR - IR_{load} = 0$$

$$I = \frac{V_{in}}{R + R_{load}}$$

Power dissipated in R_{load} is $P_{load} = \frac{V_{in}^2}{(R+R_{load})^2} R_{load}$

Maximize $\frac{dP}{dR_{load}} = V_{in}^2 \left[\frac{1}{(R+R_{load})^2} + R_{load} \left[\frac{-2}{(R+R_{load})^3} \right] \right] = 0$

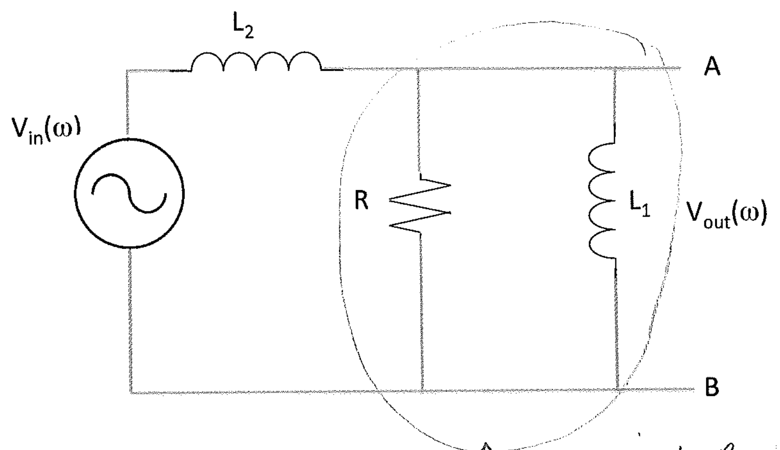
$$1 - \frac{2R_{load}}{R+R_{load}} = 0$$

$$(R + R_{load}) - 2R_{load} = 0$$

$\Rightarrow \boxed{R_{load} = R}$ at maximum

Problem 7: (25 points)

For the given filter



$$\hat{Z}_{eq} = \frac{j\omega L_1 R}{j\omega L_1 + R}$$

a) $$\hat{H}(\omega) = \frac{j\omega L_1 R/(j\omega L_1 + R)}{j\omega L_2 + j\omega L_1 R/(j\omega L_1 + R)} = \frac{j\omega L_1 R}{j\omega L_2 (j\omega L_1 + R) + j\omega L_1 R}$$

a. What is $\hat{\mathbf{H}}(\omega)$? Answer in terms of L_1, L_2, R and ω.

b. Is this a low-pass, high-pass, band-pass or band-rejection filter? Explain. Sketch the Bode plot.

c. Determine the corner frequency or frequencies.

d. Determine the values of the zeroes and poles and sketch them on the complex frequency plane.

e. Write down $|\hat{\mathbf{H}}(\omega)|$ using the results of part d.

$$\boxed{\hat{H}(\omega) = \frac{j\omega L_1 R}{j\omega R(L_1 + L_2) - \omega^2 L_1 L_2}} = \frac{j L_1 R}{jR(L_1 + L_2) - \omega L_1 L_2}$$

$$= \frac{L_1 R}{R(L_1 + L_2) + j L_1 L_2 \omega}$$

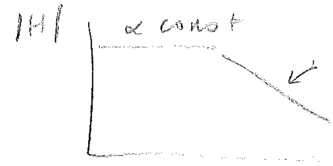
b) Low ω behavior: neglect 2^{nd} term in denominator

$$|\hat{H}| \approx \frac{j\omega L_1 R}{j\omega R(L_1+L_2)} = \frac{L_1}{L_1+L_2}$$

High ω behavior:
2^{nd} term in denominator dominates

$$|\hat{H}| \approx \frac{j\omega L_1 R}{-\omega^2 L_1 L_2} \propto \omega^{-1}$$

Bode plot



$\boxed{\text{low pass}}$

c) Corner frequency: $\dfrac{\cancel{L_1}}{L_1+L_2} = \dfrac{\cancel{L_1} R}{\omega_c L_1 L_2}$

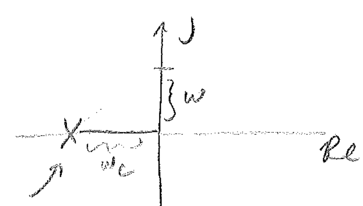
$$\boxed{\omega_c = \frac{(L_1+L_2)R}{L_1 L_2}}$$

d) $$H(\hat{s}) = \frac{L_1 R}{\hat{s} L_1 L_2 + R(L_1+L_2)}$$

No zeroes

Poles: $\hat{s} L_1 L_2 = -R(L_1+L_2)$

$$\boxed{\hat{s} = -\frac{R(L_1+L_2)}{L_1 L_2} = -\omega_c}$$



$\hat{s} = \dfrac{-(L_1+L_2)R}{L_1 L_2} = -\omega_c$

e) $$|\hat{H}| \propto \frac{|\overset{\text{none}}{\text{distance to zeroes}}|}{|\text{distance to poles}|} = \frac{1}{\sqrt{\omega^2+\omega_c^2}}$$