Duke University
Department of Physics

Physics 271 Spring Term 2017

MIDTERM SOLUTIONS

I will abide by the Duke Community Standard. Name: ________________________

This is a closed book exam, with one side of one page cheat sheet allowed. Calculators are allowed, but only for basic calculations: you may not use special memory, graphing etc. functions. You must always show your work for credit; all answers must be justified. **You must hand in your cheat sheet with your test.**

Pay attention to units and significant figures.

This midterm has six problems.

Problem 1 ________________

Problem 2 ________________

Problem 3 ________________

Problem 4 ________________

Problem 5 ________________

Problem 6 ________________

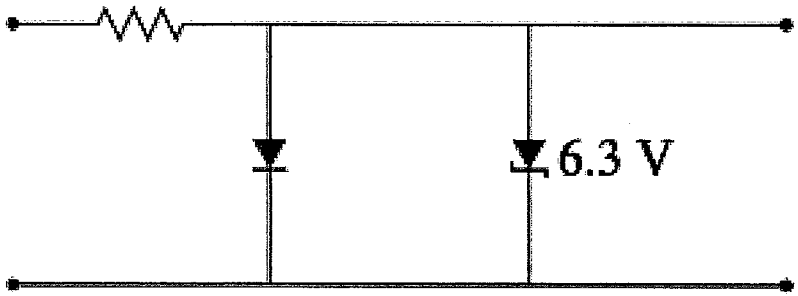
Total ________________

Problem 1: (12 points)

Sketch the output waveforms expected for the following applied signals:

- A 500 Hz, 8 V amplitude sinusoid.
- A 50 Hz, 4 V_{pp} square wave.

Specify important voltage levels and time scales. The input is on the left and the output is on the right. Explain your answers in a few sentences.



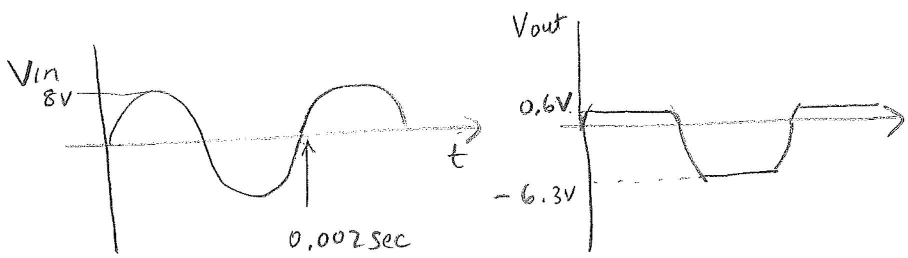
This is a clipper – for $V_{in} > 0.6$, left diode conducts and holds V to V = 0.6 V (zener reverse-biased and not conducting)

For $V_{in} < -6.3$ V, Zener in breakdown mode and holds output to −6.3 V

In between, neither diode conducts and output follows input

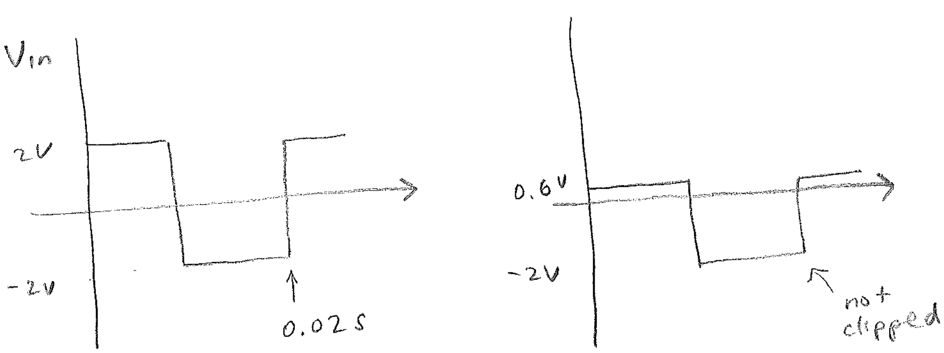
In each case, output has same period as input

Input: 500 Hz, 8 V amplitude sinusoid



input

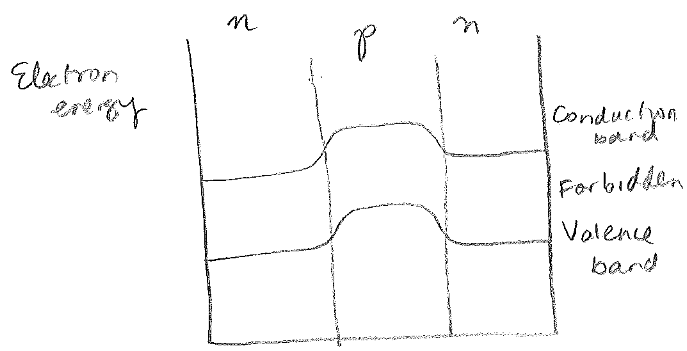
Input: 50 Hz, 4 Vpp square wave

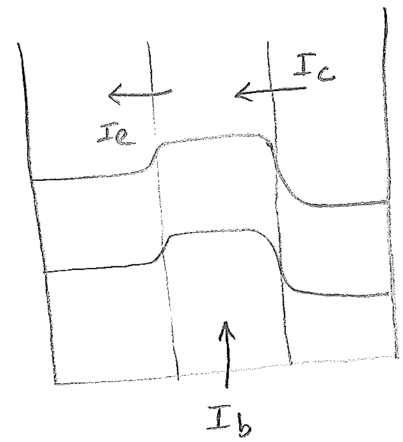


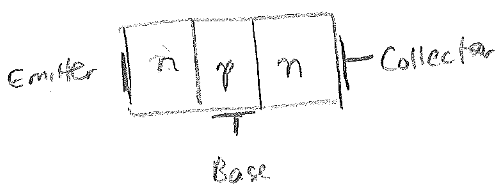
Problem 2: (6 points)

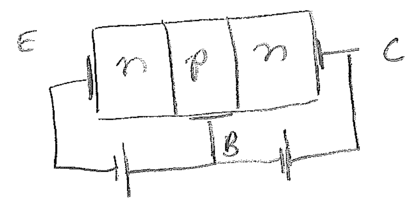
Sketch electron energy diagrams for a bipolar npn transistor, for unbiased and for normal biased operation cases. For the biased case, what is the direction of current flow when the transistor is operating in the linear active region?

Bipolar npn transistor









Unbiased

Biased for normal active-forward operation

emitter made negative, collector made positive

Problem 3: (10 points)

A complex voltage and current are related by $\hat{v}(t) = (-1 + j\sqrt{3})\hat{\imath}(t)$.

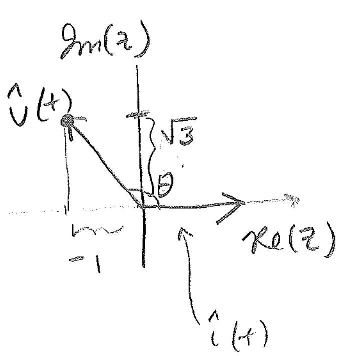
a. What is the phase angle between voltage and current?

b. Sketch the observed current and voltage as a function of time, assuming zero voltage at $t = 0$, and frequency $\omega = 10$ rad/s.

$\hat{z} = -1 + j\sqrt{3} = Ae^{j\theta}$

$\hat{v} = \hat{z}\hat{\imath}$

a)



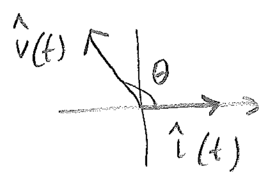
$\theta = 180° + \tan^{-1}\left(\frac{-\sqrt{3}}{1}\right) = 120°$

($\tan^{-1}\left(\frac{-\sqrt{3}}{1}\right) = -60°$)

(Note: $\theta = \tan^{-1}\left(\frac{\sqrt{3}}{-1}\right)$ gives wrong quadrant)

b) $\hat{v}(t) = |A| e^{j\theta} \hat{\imath}(t)$, where $|A| = \sqrt{1+3} = 2$

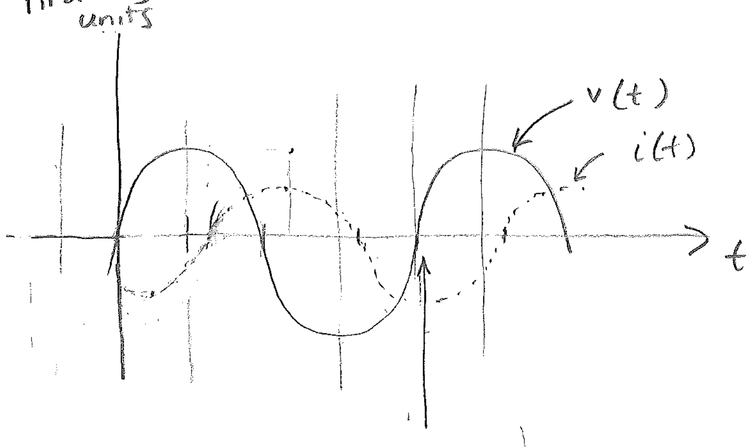
As phasors,



$\hat{v}(t)$ <u>leads</u> $\hat{\imath}(t)$ by 120°

$\Rightarrow$ $v(t)$ peaks <u>earlier</u> in time (smaller t)

$v(t)$, $i(t)$ are the real parts of the phasors

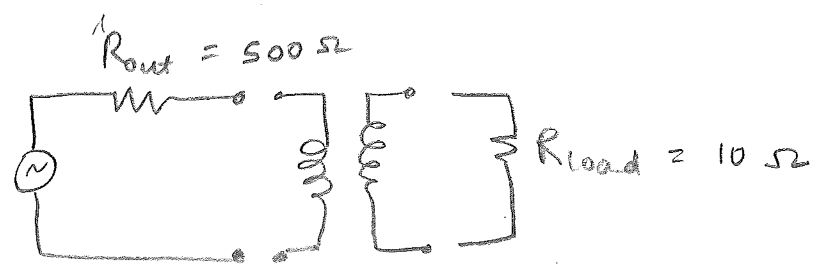


120° is between 1/4 & 1/2 of a full cycle; the $i(t)$ curve is shifted by that much along t axis

$t = T = \frac{1}{f} = \frac{2\pi}{\omega} = 0.62$ s

Problem 4: (10 points)

An audio signal has an output impedance of 500 Ω. To drive a 10 Ω speaker with maximum power transfer. an impedance-matching transformer is used between the generator and the speaker. What is the necessary turns ratio for such a transformer?



Impedance matching

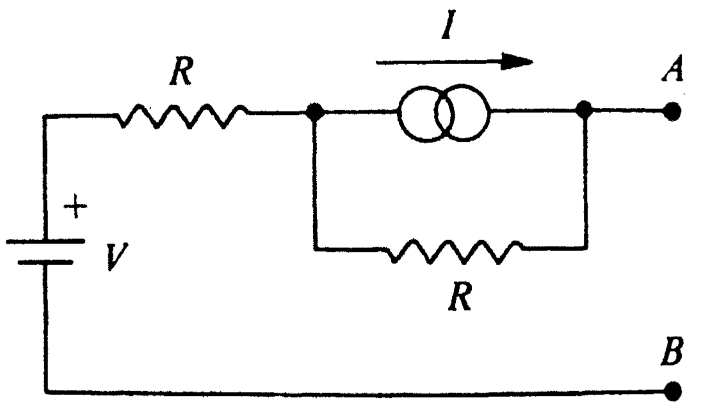
$$R_{eff} = \left(\frac{n_1}{n_2}\right)^2 R_L$$

want $R_{eff} = R_L$ for maximum power transfer

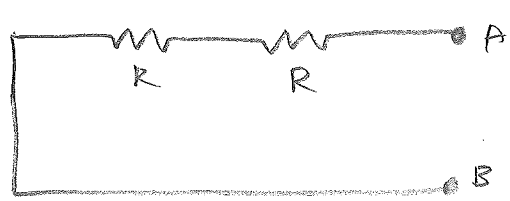
$$\text{Want } \left(\frac{n_1}{n_2}\right) = \sqrt{\frac{R_{eff}}{R_L}} = \sqrt{\frac{500}{10}} = \sqrt{50} = 7.07$$

Problem 5: (10 points)

Find the Norton equivalents I_N and R_N for the given circuit.

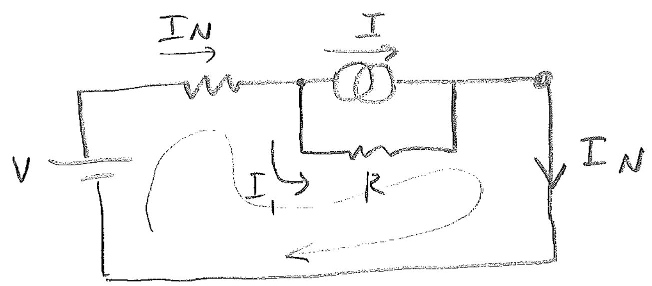


R_{Th} : open sources & short supplies



$R_{Th} = R_N = \underline{\underline{2R}}$

I_N : find current with shorted output



Kirchoff's Rules

$I_N = I + I_1 \quad , \quad I_1 = I_N - I$

$V - I_N R - I_1 R = 0$

$I_N = \dfrac{V - I_1 R}{R}$

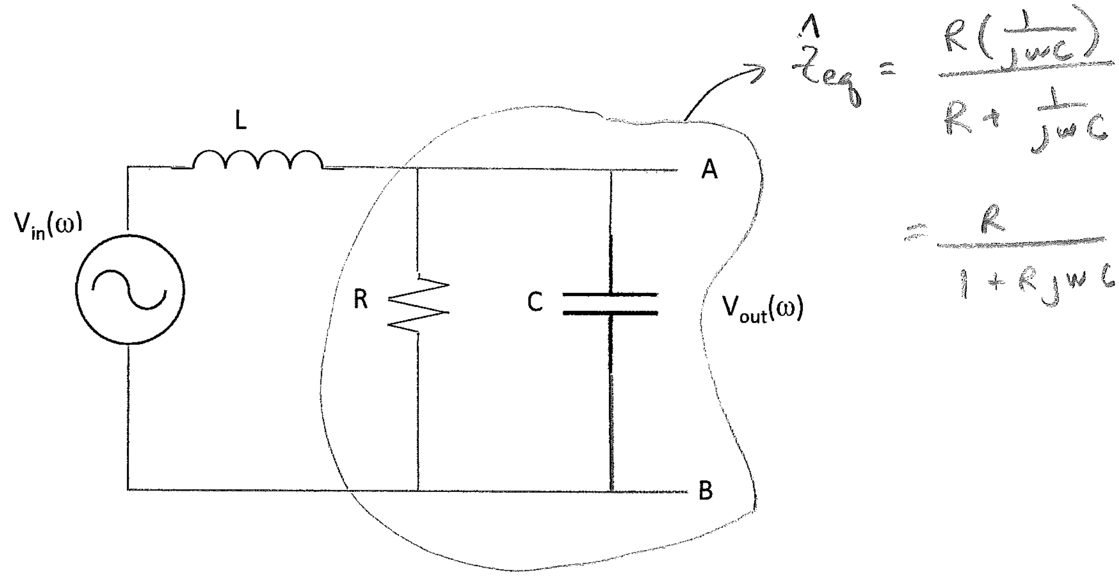
solve for I_N

$I_N = \dfrac{V - (I_N - I)}{R}$

$2 I_N = \dfrac{V}{R} + I$

$\Rightarrow \boxed{I_N = \frac{1}{2}\left(I + \frac{V}{R}\right)}$

Problem 6: (20 points)



a. Determine the transfer function of the given circuit. Answer in terms of s, L, R, and C.

b. Does it behave as a low-pass, high-pass, band-pass or band-rejection filter? Explain.

c. Determine the values of any zeroes and poles.

d. If $R = \infty$ draw the zeroes and poles on the complex frequency plane.

e. Determine the value of R in terms of L and C in order that the poles are all together on the x-axis (i.e., have the same value and are real) and draw them on the complex frequency plane.

a) $\hat{H}(j\omega) = \dfrac{R/(1+Rj\omega C)}{\frac{R}{(1+Rj\omega C)} + j\omega L} = \dfrac{R}{R + j\omega L - \omega^2 RLC}$ (voltage divider)

$s \to j\omega$ $\quad \boxed{\hat{H}(\hat{s}) = \dfrac{R}{R + \hat{s}L + \hat{s}^2 CLR}}$

f. Sketch the Bode plot for $R = \infty$. Find any corner frequencies in terms of L and C and indicate them on the diagram, and also specify the slopes of any lines.

$$\hat{H} = \frac{1}{1 + \hat{s}L/R + \hat{s}^2 LC}$$

b) As $\omega \to \infty$, $|\hat{H}|$ gets small; $\omega \to 0$, $|\hat{H}| \sim 1$ $\Rightarrow$ **low-pass**

c) No zeroes

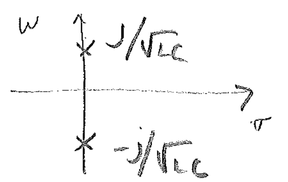
Poles for $1 + \hat{s}L/R + \hat{s}^2 LC = 0$

$$\Rightarrow \hat{s}^2 + \hat{s}/RC + 1/LC = 0$$

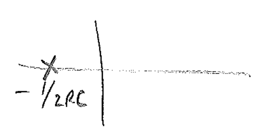
$$\boxed{\hat{s} = \frac{-1/RC \pm \sqrt{\left(\frac{1}{RC}\right)^2 - 4/LC}}{2}}$$

2 poles

d) For $R \to \infty$, poles $\to$ $\hat{s} = \pm \frac{1}{2} 2\sqrt{\frac{1}{LC}}\, j = \pm \frac{j}{\sqrt{LC}}$



e) For poles together on x-axis $\left(\frac{1}{RC}\right)^2 = \frac{4}{LC}$



$$\Rightarrow R = \sqrt{\frac{L}{4C}}, \quad \hat{s}_{1,2} = -\frac{1}{2RC}$$

f) Bode plot for $R \to \infty$

$$\hat{H} \sim \frac{1}{1 + \hat{s}^2 LC} = \frac{1}{1 - w^2 LC}$$

large for $w_c = \frac{1}{\sqrt{LC}}$