Content Questions

How do you do an integral with $d\vec{s} \times \hat{r}$ in it? What does it mean to have an integral of a cross-product?

I think $d\vec{s} \times \hat{r}$ is not really as scary as it looks. It’s just like any other vector cross-product, except that one of the vectors in it is an infinitesimal length element. This $d\vec{s}$ is just a little length vector along the wire pointing in the direction of the current. $\hat{r}$ is just the unit vector pointing from where $d\vec{s}$ is to the point where you’re evaluating the magnetic field. The resulting contribution to the integral is then a vector, and the sum you do when you integrate is a vector sum. This vector sum sounds complicated, but in many practical cases, it’s not so bad: often, each little vector piece in the integral $d\vec{s} \times \hat{r}$ has the same direction, and in this case the sum is just the sum of the magnitudes $|d\vec{s} \times \hat{r}|$, in the direction of any one of them.

The integrals you often get when applying the Biot-Savart formula over a finite length, however, sometimes can be a bit awkward. Often you have to change variables to be integrating over an angle, or something like that.

If the line is not straight, it’s really the same idea. Sometimes simple circular paths can make the integral relatively easy, but funny wiggly paths are hard and may need to be done numerically.

How do you write $ds$ in terms of $\theta$?

It will depend on the problem, but often you can use basic trig relations to relate the length to $\theta$. The idea is to first find a functional relation between the length and the angle. For example, in example 30.1 of your text (which is really just the same situation as both of the Biot-Savart homework problems, although different variable names), the length element $ds$ corresponds to $dx$. The variable $x$ is related to $\theta$ by $x = -a \tan \theta$. So we can relate $dx$ to $d\theta$ by taking the derivative, $dx = -\frac{a d\theta}{\cos^2 \theta}$. 
If I’m applying the formula $B = \frac{\mu_0 l}{4\pi a} (\sin \theta_1 - \sin \theta_2)$, how do I choose the angles?

The formula $B = \frac{\mu_0 l}{4\pi a} (\sin \theta_1 - \sin \theta_2)$ is for the $B$ field at a point $P$ a perpendicular distance $a$ away from a straight finite line segment, where the angles are defined as in Figure 30.3. You can use this formula for corresponding situations. The angle $\theta_1$ goes between the perpendicular dropped from $P$ to the wire, and the line between $P$ and one end of the wire; the angle $\theta_2$ goes between the perpendicular dropped from $P$ to the wire and the line between $P$ and the other end of the wire. Note that if the two angles are on opposite sides of the perpendicular, one will be negative. (If one end of the wire is at infinity, the corresponding angle will be $\pi/2$.)

For example, in homework problem 30-17, for the horizontal current segment, $\sin \theta_1 = \frac{a}{\sqrt{a^2 + d^2}}$ and $\sin \theta_2 = -\sin \theta_1$. For the vertical current segment, both angles are on the same side of the line between $P$ (point $O$) and the wire: $\sin \theta_1 = \frac{d}{\sqrt{a^2 + d^2}}$ and $\sin \theta_2 = \pi/2$.

How do you find the direction when using the Biot-Savart Law?

The direction of the contribution to the magnetic field $d\vec{B}$ is the direction of $d\vec{s} \times \hat{r}$. The quantity $d\vec{s}$ is a length element with direction along the current direction; $\hat{r}$ is a vector that goes from the current element to the place where you are evaluating the magnetic field. The cross-product direction is given by the RHR: put your fingers along $d\vec{s}$, curl them towards $\hat{r}$, and your thumb will point along $d\vec{B}$ (or you can use the other variants of the RHR if you prefer).

How do you apply Biot-Savart Law to non-straight lines of charge?

You can pick an current element $d\vec{s}$ that’s curved, and integrate over that in the line integral. Your text example 30-2 gives an example.

How do you choose an Ampèrian loop? How many different shapes can you choose?

Just as Gauss’s Law is true for any surface, Ampère’s Law is true for any loop. (Note that the loop must be closed for Ampère’s Law to hold.) Some loops will give you an easier time than others. As for Gauss’ Law, typically
you choose loops with the same symmetry as the current distribution. For instance, for a cylindrical wire, choose a concentric circular loop.

**In the Ampère examples in the homework, how come \( |\vec{B}| \) is constant over the loop?**

It’s constant by symmetry. The \( \vec{B} \) field’s magnitude is the same everywhere a constant distance from the wire (by Biot-Savart), so it’s constant on a circle. By the RHR, the direction of the magnetic field generated by the wire is in a circle around the wire (thumb in direction of current, magnetic field curls around with fingers). So at any point on this circular loop, \( \vec{B} \) and \( d\vec{s} \) are tangent to the circle, and so parallel to each other. So \( \oint \vec{B} \cdot d\vec{s} = \oint Bdl = B \oint dl \).

**Can you give an example of an Ampèrian loop where \( \vec{B} \perp d\vec{s} \), or \( \vec{B} = 0 \)?**

An example would be the solenoid. Here the Ampèrian loop chosen is a rectangle with one side inside the solenoid and one side outside the solenoid (where the field is zero for an ideal infinite solenoid— we could choose it infinitely far away where the field is zero for even a non-ideal solenoid). \( \vec{B} = 0 \) along the parallel side outside, so that part disappears from the integral. On the sides of the loop that are perpendicular to the solenoid’s length, and inside the solenoid, the \( \vec{B} \) field is \( \perp \) to the \( d\vec{s} \) of the loop, so \( \vec{B} \perp d\vec{s} = 0 \). So the only part that counts is the part of the loop parallel to the solenoid.

**How do you deal with a square Ampèrian loop like in the solenoid? (like in Fig. 30-18 of the text)**

The LHS of Ampère’s law is a line integral that can be treated as the sum of contributions from each straight side. We treat the solenoid as ideal, which means magnetic field is uniform inside and along the axis of the solenoid, and zero outside the solenoid. For the side of the square Ampérian loop that’s outside the solenoid, the contribution to \( \oint \vec{B} \cdot d\vec{s} = 0 \) because \( B = 0 \) everywhere along that side. For the side of the square Ampérian loop inside the solenoid, the \( d\vec{s} \) (which points along that side) is parallel to \( \vec{B} \), so \( \vec{B} \cdot d\vec{s} = Bds \) the LHS of Ampère’s Law is \( \oint \vec{B} \cdot d\vec{s} \), which is just \( Bl \), where \( l \) is the length of the side. For the sides of the square Ampèrian loop that
are perpendicular to the axis of the solenoid, either $B = 0$ or $B \perp d\vec{s}$, and either case there’s no contribution to $\oint \vec{B} \cdot d\vec{s}$. So the total LHS of Ampère’s Law is just $Bl$. Now the RHS is $\mu_0$ times the total current going through the loop, which is $I$ in the coil times the number of turns through the loop $N$. Putting together the RHS and LHS of Ampère’s Law, that gives $Bl = \mu_0 IN$, or $B = \mu_0 nI$, where $n = N/l$, the turns per unit length.

Note: I think the discussion in the text, p. 876, is not quite right. The text states “The contribution along side 3 is zero because the magnetic field lines are perpendicular to the path in this region.” Not so– there is a weak field outside, and it’s not perpendicular to the path. However, you can take the side outside to be at infinity where the field is zero everywhere along side 3 and so the line integral will be too. (Or, you can just take the approximation of the ideal infinite solenoid, for which $\vec{B}$ is zero outside).

Can you explain solenoids?

These are multiple loops of current in a spiral coil; you can find the field from Ampère’s Law (see above, also section 30-4 in text). In the approximation of an ideal infinite solenoid, the field inside is parallel to the axis of the solenoid, constant everywhere inside the solenoid, and goes to zero outside (not strictly true but in real life this approximation tends to be quite good, except near the ends of the solenoid). See also above couple of questions.

Why is $\vec{B}$ not zero inside a solenoid, since you can draw an Ampèrian loop inside with no enclosed current?

According to Ampère’s Law, the integral $\oint \vec{B} \cdot d\vec{l}$ is zero over a concentric loop inside the solenoid, no matter what $\vec{B}$ is. The magnetic field need not be zero along the loop– it’s the loop integral that must be. If you choose an Ampérian loop with plane perpendicular to the solenoid axis, each element $\vec{B} \cdot d\vec{s}$ in the integral is zero because $\vec{B} \perp d\vec{s}$ ($\vec{B}$ is along the axis of the solenoid). Fully-interior loops with planes parallel to the axis of the solenoid will have line integrals for opposing parallel sides that cancel. So an Ampèrian loop completely inside the solenoid isn’t going to help you find $B$. (To find $B$ you choose a loop partly inside and partly outside– see above questions)

You can conclude $B = 0$ for an Ampèrian loop enclosing zero current only when the symmetry of the system indicates that $B$ is constant (and non-zero) everywhere on the loop, such as for a loop concentric with a wire.
Can you explain the torque for the loop in the solenoid? How do the forces add up?

You can consider the field inside an ideal solenoid to be constant and parallel to the axis of the solenoid. Then the problem is just a loop of current in an external magnetic field, like the problems in the last chapter.

If the loop’s plane is perpendicular to the solenoid axis, using the RHR for current in an external field, \( F = I\vec{L} \times \vec{B} \); the forces on each side are all outward from the center. The net force is zero. The net torque on the loop, evaluated by just looking at the forces on each side, is zero. The forces on the sides are not going to make the loop rotate.

Another way of thinking about the torque on the loop is in terms of magnetic moment. The loop is also a source of magnetic field due to its current: it acts as a magnetic moment \( \vec{\mu} = I\vec{A} \). A magnetic moment in an external field is torqued in such a way as to line up with the field: the torque is \( \tau = \vec{\mu} \times \vec{B} \) (this is in section 29.5 of your text).

In problem 30-41, the loop is already oriented so that the \( \vec{\mu} \) due to its current is lined up with \( \vec{B} \). So there’s no torque on it. (If the loop’s plane were parallel to the axis of the solenoid, there would be a torque and the loop would try to line up perpendicular to the axis of the solenoid.)