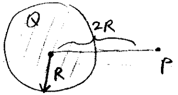


A solid spherical ball of uniform charge density has total charge $Q_0 = 3.0 \times 10^{-12}$ C and radius $R = 0.010$ m.

As always, we're free to decide the "zero point" of potential energy. Please follow the usual textbook convention of saying a particle has zero potential energy when it is infinitely far away. Mathematically speaking, $V = 0$ at $r = \infty$.

Solve everything symbolically *before* plugging in numbers.

(a) What is the potential a distance $2R$ from the ball's center? Derive any formulas you use.

(b) A tiny particle of charge $q_1 = 4.0\ \mu$C and mass $m_1 = 2.0 \times 10^{-6}$ kg is released from rest from the surface of the ball. What velocity does the particle have when it reaches a distance $2R$ from the ball's center?

(c) (*Very hard*) What is the potential inside the ball, at radius $r_2 = 0.0075$ m from the ball's center? **Remember to solve symbolically before cranking the numbers.**

(d) (*Very hard*) What is the potential at the ball's center?

a) Apply howto to find $V(r)$

Step 1 Find $\vec{E}(r)$: use Gauss' Law how-to

$r > R$ spherical symmetry

$LHS = \oint \vec{E} \cdot d\vec{A} = E \int dA = E\, 4\pi r^2$

$RHS = q_{in}/\varepsilon_0$

$E \cdot 4\pi r^2 = Q/\varepsilon_0 \Rightarrow \vec{E} = \dfrac{Q}{4\pi \varepsilon_0 r^2}\hat{r}$

Step 2 Choose $V = 0$ @ ∞

what point do you know V at? ∞

Step 3 $\Delta V = V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{s}$

where you want $V(r)$

$A \to \infty$

$B \to r$

choose path $\vec{E} \parallel d\vec{r} \Rightarrow \vec{E} \cdot d\vec{s} = E\, dr$ (radial path)

Step 4

$$V(r) - V(\infty) = -\int_{\infty}^{r} \frac{Q}{4\pi\varepsilon_0 r'^2}\,dr'$$ plug in $\vec{E}$

$$V(r) = \frac{-Q}{4\pi\varepsilon_0}\left[-\frac{1}{r'}\right]_{\infty}^{r}$$

$$= \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r} - \cancelto{0}{\frac{1}{\infty}}\right]$$

Step 5

Solve: $$V(r) = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r}\right]$$

At P, $r = 2R$ plug in

$$V_P = \frac{Q}{4\pi\varepsilon_0 (2R)} = \underline{\underline{1.35\,V}}$$

b) $q_1 = 4\mu C$, $m_1 = 2\times 10^{-6}$ kg

Does the charge "fall" with constant acceleration?

No, the force is changing

$\Rightarrow$ use energy considerations

$$\Delta U_{S\to P} = U_P - U_S = qV_P - qV_S$$

$$= q\left(\frac{Q}{4\pi\varepsilon_0 R} - \frac{Q}{4\pi\varepsilon_0 (2R)}\right)$$

$$= \frac{qQ}{4\pi\varepsilon_0}\left(\frac{1}{R} - \frac{1}{2R}\right)$$

$$= \frac{qQ}{4\pi\varepsilon_0}\left(\frac{1}{2R}\right)$$

$$\Delta U_{s\to p} = \Delta KE = \frac{1}{2}mv_p^2 - \frac{1}{2}mv_s^2$$

$$\Rightarrow \quad V_p = \sqrt{\frac{qQ}{4\pi\varepsilon_0 mR}} = \underline{\underline{2.3 \text{ m/s}}}$$

c) Apply the how-to now for $r < R$

<u>Step 1</u> Find $E(r)$ for $r < R$

G's Law $\quad LHS = E \cdot 4\pi r^2$

$\qquad\qquad RHS = q_{in}/\varepsilon_0$

how much charge is inside radius r?

$$\frac{q_{in}}{Q} = \frac{V_{in}}{V}$$

$$q_{in} = \frac{Q}{\frac{4}{3}\pi R^3}\,\frac{4}{3}\pi r^3 = Q\frac{r^3}{R^3}$$

$$\vec{E}(r) = \frac{1}{4\pi r^2}\,\frac{Qr^3}{\varepsilon_0 R^3} = \frac{Q}{4\pi\varepsilon_0}\,\frac{1}{R^3}\,r\,\hat{r}$$

Step 2 $\quad V = 0 \ @ \ \infty$

We know V at the surface from part a:

$$V_S = \frac{Q}{4\pi\varepsilon_0 R}$$

Step 3

$$\Delta V = \underbrace{V_B}_{V(r)} - \underbrace{V_A}_{\substack{\text{at surface,} \\ r=R}} = -\int_{\underbrace{A}_{\text{radial path}}}^{B} \underbrace{\vec{E}}_{\text{from step 1}} \cdot d\vec{s}$$

$$\Delta V = V(r) - V_S = -\int_R^r \vec{E} \cdot d\vec{s}$$

Step 4

Plug in $E(r)$

$$= -\int_R^r \frac{Q}{4\pi\varepsilon_0 R^3} \, r' \, dr'$$

$$= \frac{-Q}{4\pi\varepsilon_0 R^3} \left. \frac{r'^2}{2} \right|_R^r$$

$$V(r) - V_S = \frac{Q}{4\pi\varepsilon_0 R^3} \frac{1}{2}\left[R^2 - r^2\right]$$

Step 5

Solve for $V(r)$

$$V(r) = \frac{Q}{4\pi\varepsilon_0 R^3} \frac{1}{2}\left[R^2 - r^2\right] + \frac{Q}{4\pi\varepsilon_0 R}$$

$$V(r) = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{R} + \frac{1}{2}\frac{1}{R} - \frac{r^2}{R^3}\right]$$

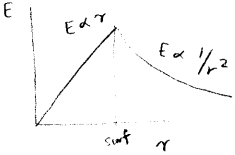
$$V(r) = \frac{Q}{4\pi\varepsilon_0}\left[\frac{3}{2R} - \frac{r^2}{R^3}\right]$$

At $r = r_a$, $\underline{\underline{V = 3.3\,V}}$

d) At $r = 0$, $V(r) = \frac{Q}{4\pi\varepsilon_0}\frac{3}{2R}$ (same formula)

$$\underline{\underline{V = 4\,V}}$$

Plot of E vs r



Plot of V vs r