

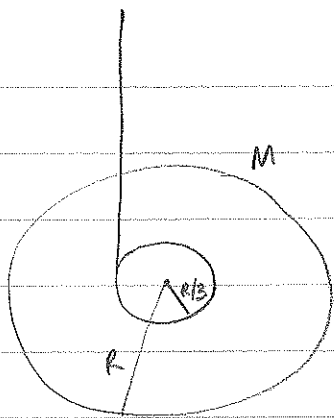
Consider a yo-yo (mass M , radius R) with a very light string wrapped around an "inner ridge" of radius $\frac{R}{3}$. The yo-yo is approximately a solid disk. Somebody holds the end of the string motionless and releases the yo-yo from rest. As a result, the yo-yo falls and rotates.

What is the tension in the string? Express your answer in terms of M , g , and R .

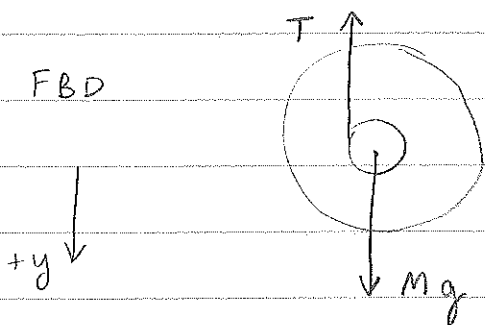
Hint: You can think of the yo-yo as "rolling" down the string.

Yoyo

Yo-yo "rolls without slipping"



What is T?



Note: not static
 $a \neq g$

We have (linear) $\begin{cases} Mg > T \\ \Sigma F_y = M a_{com} \text{ in } y\text{-dir} \\ Mg - T = M a_{com} \text{ CM linear motion} \end{cases}$

We also have rotational motion

RWOS $\Rightarrow v_{com} = r \omega$
 \downarrow this is $R/3$, the inner radius

$$a_{com} = \frac{R}{3} \alpha$$

We also have $\Sigma \tau = I \alpha$, pivot point at center of yoyo
 treating rotation separately

Mg creates no torque about center

$$\Sigma \tau = T \frac{R}{3} = \frac{1}{2} M R^2 \alpha$$

I of yo-yo (disk)

(2)

We now have 3 equations:

$$\left. \begin{array}{l} 1) \quad Mg - T = Ma_{\text{com}} \\ 2) \quad \frac{TR}{3} = \frac{1}{2} MR^2 \alpha \\ 3) \quad a_{\text{com}} = \frac{R}{3} \alpha \end{array} \right\}$$

3 unknowns:

T, α, a

↓

we can solve

Plug 3 into 2

$$\frac{TR}{3} = \frac{1}{2} MR^2 \cdot 3 \frac{a_{\text{com}}}{R}$$

$$T = \frac{9}{2} M a_{\text{com}}$$

Plug in 1 $T = \frac{9}{2} M \left[\frac{Mg - T}{M} \right]$

$$T = \frac{9}{2} Mg - \frac{9}{2} T$$

$$\frac{11}{2} T = \frac{9}{2} Mg$$

$$\Rightarrow \boxed{T = \frac{9}{11} Mg}$$

$$a_{\text{com}} = \frac{2}{9} \frac{T}{M} = \frac{9}{11} Mg \cdot \frac{2}{9} \frac{1}{M}$$

$$a_{\text{com}} = \frac{2}{11} g < \text{gravitational acceleration}$$