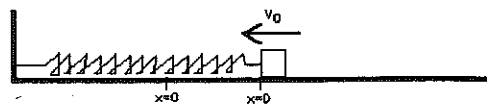
A block of mass m is attached to one end of a spring, the other end of which is attached to a wall. The floor is frictionless, and the spring has spring constant k. Let $x=0$ denote the block's position when the spring is at its equilibrium length.

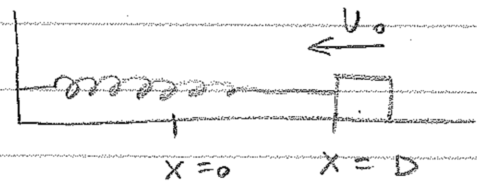
Suppose the block is displaced to $x=D$. Instead of releasing the block from rest, however, someone shoves the block towards $x=0$ with speed v_0, right at time $t=0$.

(a) What is the block's velocity at arbitrary later time t_1?



(b) During its oscillations, when the block is momentarily motionless, what is its acceleration?

(c) When the block passes through $x=0$, what is its speed?



a) SHM: $x(t) = x_m \cos(\omega t + \phi)$

We need to find x_m, ω, ϕ

Then: $v(t) = -x_m \omega \sin(\omega t + \phi)$

$\omega = \sqrt{\frac{k}{m}}$ for simple spring SHM

To find A: $E_0 = E_f$

At release: $(K_0 + U_0) = (K_f + U_f) \rightarrow$ at maximum

$\cancel{\tfrac{1}{2}} m v_0^2 + \cancel{\tfrac{1}{2}} k D^2 = 0 + \cancel{\tfrac{1}{2}} k x_m^2$ $\quad x = x_m,\ v = 0$

$\Rightarrow x_m = \sqrt{\frac{m v_0^2 + k D^2}{k}}$

To find the phase: At $t = 0$, $x = D$

$D = x_m \cos\phi$

$\Rightarrow \cos\phi = \frac{D}{x_m}$

$\Rightarrow \phi = \cos^{-1}\left[\frac{D}{x_m}\right]$

So the motion is described by

$x(t) = \sqrt{\frac{m v_0^2 + k D^2}{k}} \cos\left[\sqrt{\frac{k}{m}}\, t + \cos^{-1}\left(\frac{D}{A}\right)\right]$

plug in t_1 $\longrightarrow$ $\boxed{v(t) = -\sqrt{\frac{m v_0^2 + k D^2}{k}} \sqrt{\frac{k}{m}} \sin\left[\sqrt{\frac{k}{m}}\, t + \cos^{-1}\left(\frac{D}{A}\right)\right]}$

b) What is a when block is motionless?

$v = 0$ when block is at max (or min)

but $\underline{a \neq 0}$ there

$$a(t) = \pm X_m \omega^2 \cos[\omega t + \phi]$$

when $v = 0$, $\sin(\omega t + \phi) = 0 \Rightarrow \cos(\omega t + \phi) = 1$ (or -1), $X = X_m$

$$a = - X_m \omega^2$$

$$\boxed{a = -\frac{k}{m}\sqrt{\frac{m v_0^2 + k D^2}{k}}}$$ (or pos value)

c) At $x = 0$, $\cos(\omega t + \phi) = 0$

$\sin(\omega t + \phi) = 1$ (or -1)

$$\Rightarrow v = v_{max} = \pm X_m \omega = \underline{\pm\sqrt{\frac{k}{m}}\sqrt{\frac{m v_0^2 + k D^2}{k}}}$$

(note could also do this by conservation of energy)