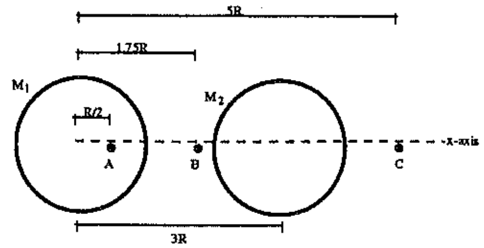
In outer space, two spherical shells are configured as drawn below. They both have radius R. Sphere 1, of mass M_1, is centered at $x = 0$. Sphere 2, of mass M_2, is centered at $x = 3R$.



Find the total gravitational force (magnitude and direction) on a particle of mass m sitting on

(a) Point A, located at $x = \frac{R}{2}$.

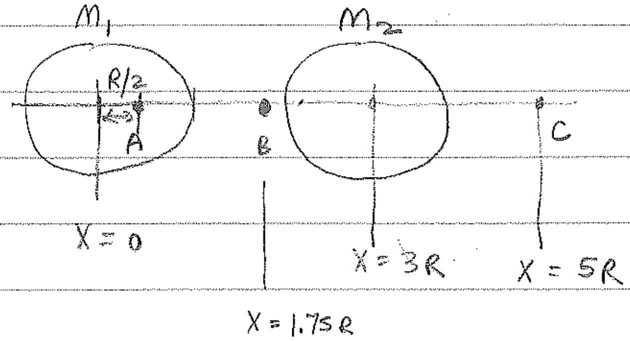
(b) Point B, located at $x = 1.75R$; and

(c) Point C, located at $x = 5R$.

Express your answers in terms of M, R, and any other constants you need.

If you are inside a shell ⇒ no force on you

If you are outside a shell ⇒ treat shell mass as a point at the center



a) Total force at A on mass m : · ignore force due to M_1

· feel attractive force due to M_2 as if M_2 is at $X = 3R$, a distance $3R - \frac{R}{2} = \frac{5}{2}R$ away

$$F_A = \frac{G m M_2}{(2.5R)^2}$$

(pos x direction)

b) Total force at B : treat M_1 as a point mass at $x = 0$, M_2 as a point mass at $x = 3R$

M_1 can be treated as $1.75R$ away, force in $-x$ dir

M_2 can be treated as $(3 - 1.75)R$ away, force in $+x$ dir

$$F_{tot} = -\frac{G m M_1}{(1.75R)^2} + \frac{G m M_2}{(1.25R)^2}$$

$$F_{tot_B} = \frac{G m}{R^2}\left(\frac{-M_1}{(1.75)^2} + \frac{M_2}{(1.25)^2}\right)$$

c) Total force at C : M_1 treated as point mass 5 R away

M_2 treated as point mass 2R away

both in $-x$ dir

$$F_{tot_C} = -\frac{Gm}{R^2}\left(\frac{M_1}{5^2} + \frac{M_2}{2^2}\right)$$