

Consider a soccer ball, which is approximately a hollow spherical shell of mass  $M$  and radius  $R$ . A player kicks the ball horizontally so as to give it initial speed  $v_0$ ; but the kick does not give the

ball any spin. So, the initial angular speed is 0. In other words, the ball initially slides. But the soccer field is not frictionless. In fact, the coefficient of kinetic friction between the field and the ball is  $\mu$ . Consequently, as the ball slides across the field, it gradually starts rotating. Eventually, it starts rolling without slipping.

How fast is the ball moving when it first starts to roll without slipping? Express your answer in terms of  $v_0$ ,  $M$ ,  $R$ ,  $g$ , and  $\mu$ .

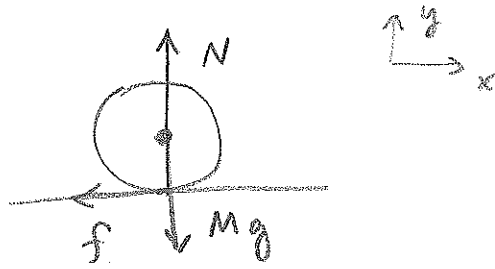
*Hint:* The "kick" force gives the ball speed  $v_0$ , but then "turns off." So, as the ball slides across the field, the kick force no longer acts.



①

Soccer ball initially slide  
and then starts RWOS

Extended  
FBD



$N = Mg$  from force  
balance in  $y$

$$f = \mu N = \mu Mg$$

Linear  
motion

$$\sum F_x = Ma_x$$

$$-\mu Mg = Ma$$

$$\Rightarrow a = -\mu g$$

$$v = v_0 + at$$

$$\textcircled{1} \quad v = v_0 - \mu g t$$

This is the velocity of  
the COM, which decreases  
with time.

But we still don't know  
when it starts RWOS.

Look at rotation & torque:

$$\sum \tau = I \alpha$$

Choose COM as pivot point

$$\tau = f R = \mu Mg R$$

For  
spherical  
shell

$$I_{\text{sphere}} = \frac{2}{3} MR^2$$

$$\tau = I \alpha \Rightarrow \mu Mg R = \frac{2}{3} MR^2 \alpha$$

$$\Rightarrow \alpha = \frac{3\mu g}{2R}$$

This gives us  $\omega = \omega_0 + \alpha t$  <sup>0 (starts not spinning)</sup> ②

So ②  $\omega = \frac{3\mu g}{2R} t$ , angular velocity as a function of time

Now to connect translation and rotation:

when RWOS happens

$$v = R\omega$$

Plug in ① & ② & solve for  $t$

$$v_0 - \mu g t = R \left( \frac{3\mu g}{2R} \right) t$$

$$\frac{5}{2} \mu g t = v_0$$

$$\Rightarrow t = \frac{2v_0}{5\mu g}$$

when RWOS happens

(Note:  $\propto v_0$ ,  
 $\propto 1/\mu$ )

Now find  $v$  from ①

$$v = v_0 - \mu g t$$

$$= v_0 - \mu g \left( \frac{2}{5} \frac{v_0}{\mu g} \right)$$

$$= v_0 - \frac{2}{5} v_0$$

$$v = \frac{3}{5} v_0$$