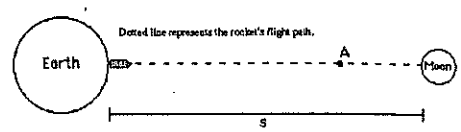
Consider a rocket, of mass m, that starts on the surface of the Earth, takes off with some initial velocity, and travels along a straight path to the Moon. After receiving its initial velocity, the rocket no longer uses its thrusters. (A realistic flight path would be curvy, but let us keep things simpler.)

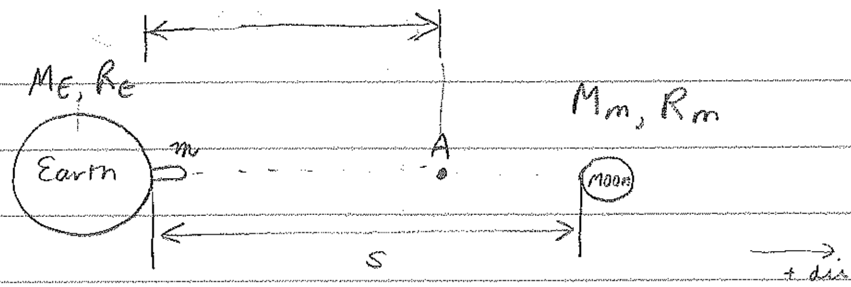


Let s denote the distance from the *surface* of the Earth to the *surface* of the Moon. Let M_E, M_m, R_E, and R_m denote the masses and radii of the Earth and Moon. Express your answers in terms of G and the constants listed above. Do not plug in numbers.

(a) Let point A refer to the spot in space where the net force on the rocket is zero. How far from the rocket's take-off point is point A? Set up, but do not solve, the relevant equation(s).

(b) What is the minimum initial speed the rocket must have in order to reach point A? Let x denote your answer to part (a), and leave your answer in terms of x.

(c) Suppose that when the rocket passes point A, it is barely moving. How fast will it be traveling right before crashing into the Moon? Again, leave your answer in terms of x.



a) At point A, the attractive forces in opposite directions cancel. X is unknown distance from surface of Earth to A

$$F_{\text{due to Earth on rocket}} = -\frac{GmM_E}{(X+R_E)^2}$$

neg x dir (attractive)

Note for a point outside a spherical mass distribution, treat as if entire mass at center - so distance is $X+R_E$

$$F_{\text{due to moon on rocket}} = \frac{GmM_m}{(S-X+R_m)^2} \quad \text{(pos dir)}$$

At A, the forces cancel

$$\frac{\cancel{G}\cancel{m}M_m}{(S-X+R_m)^2} = \frac{\cancel{G}\cancel{m}M_E}{(X+R_E)^2}$$

$$\frac{M_m}{(S-X+R_m)^2} = \frac{M_E}{(X+R_E)^2}$$

solve for x (not asked for)

b) Call the minimum initial speed v_0

Now consider energy. Mechanical energy is conserved.

At A, $K = 0$ (it just barely makes it). The system has potential energy due to interaction of the rocket and both Earth and Moon.

$$U_0 = \underset{+\text{Earth}\,0}{U_{\text{rocket}}} + \underset{+\text{Moon}\,0}{U_{\text{rocket}}}$$

$$E_0 = U_0 + K_0, \qquad K_0 = \frac{1}{2} m v_0^2$$

$$U_A = \underset{\substack{+\text{Earth}\\ \text{at } A}}{U_{\text{rocket}}} + \underset{\substack{+\text{Moon}\\ \text{at } A}}{U_{\text{rocket}}}$$

$$E_A = U_A + \cancel{K_A} \to 0$$

$$U_0 = -\frac{G m M_E}{R_E} - \frac{G m M_M}{(s + R_m)}$$

$$U_A = -\frac{G m M_E}{(x + R_E)} - \frac{G m M_m}{(s - x + R_m)}$$

Energy conservation equation

$$\frac{1}{2} \cancel{m} v_0^2 - \frac{G \cancel{m} M_E}{R_E} - \frac{G \cancel{m} M_M}{(s + R_M)} = -\frac{G \cancel{m} M_E}{(x + R_E)} - \frac{G \cancel{m} M_M}{(s - x + R_m)}$$

Solve for v_0:

$$v_0 = \left[2G \left[\frac{M_E}{R_E} + \frac{M_m}{s + R_M} - \frac{M_E}{x + R_E} - \frac{M_M}{s - x + R_M} \right] \right]^{1/2}$$

c) Suppose after reaching A, with $K \sim 0$, the rocket keeps moving. Energy is still conserved

$$U_f + K_f = U_A + \cancel{K_A}^{\,0}, \quad U_A \text{ is given above}$$

$$K_f = \frac{1}{2} m v_f^2 \qquad U_f = -\underbrace{\frac{G m M_E}{s + R_E}}_{\text{rocket + Earth}} - \frac{G m M_m}{R_m}$$

$$\frac{1}{2} \cancel{m} v_f^2 = U_A - U_f = -G \cancel{m} \left[\frac{M_E}{x + R_E} + \frac{M_m}{(s - x + R_m)} - \frac{M_E}{s + R_E} - \frac{M_m}{R_m} \right]$$

$$\Rightarrow \quad v_f = \left[2G \left[\frac{M_E}{s + R_E} + \frac{M_m}{R_m} - \frac{M_E}{x + R_E} - \frac{M_m}{s - x + R_m} \right] \right]^{1/2}$$