Water, which has density ρ, is pumped (at high pressure) at rate Q into the left end of a cylindrical pipe. Q is the mass flux, i.e., the mass per time pumped into the pipe. The radius of the left end of the pipe is r_1.

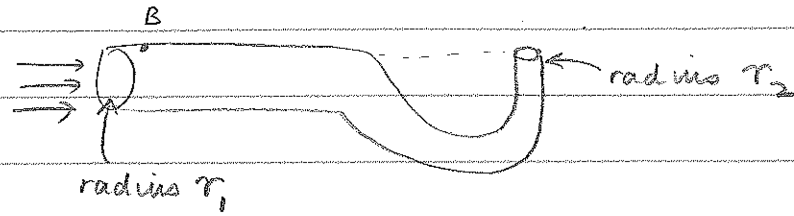
The pipe gradually narrows and curves as shown below. At the other end of the pipe (i.e., the "mouth"), the radius is r_2. The mouth of the pipe is at the same height as point B.

Express your answers symbolically in terms of the constants listed above, and g.

(a) How high into the air, above the mouth of the pipe, does the water shoot?

(b) If the air pressure is p_{air}, then what is the pressure exerted by the water on the wall of the pipe at point B?

Density ρ, pumped at rate Q $\left(\frac{\text{mass}}{\text{time}}\right)$



a) How high does the water shoot?

First find V_2 at the outlet

$$R_1 = R_2$$

$$A_1 V_1 = A_2 V_2$$

$$\Rightarrow V_2 = \frac{A_1 V_1}{A_2}$$

To find v_1: $Q = \rho A_1 V_1$ mass per time

$$V_1 = \frac{Q}{\rho A_1}$$

$$V_2 = \frac{\cancel{A_1} Q}{\rho \cancel{A_1}} \cdot \frac{1}{\pi r_2^2} = \frac{Q}{\rho \pi r_2^2}$$

Now the height reached is just for 1D kinem

$$V_f^2 = 0 = V_2^2 - 2gh$$

$$\Rightarrow h = \frac{V_2^2}{2g} = \frac{Q^2}{2g\rho^2\pi^2 r_2^4}$$

b) Pressure at point B: use Bernoulli

$$P_1 + \frac{1}{2}\rho V_1^2 + \cancel{\rho g y_1} = P_2 + \frac{1}{2}\rho V_2^2 + \cancel{\rho g y_2}$$

Same y, so those terms drop out

$y_1 = y_2$

$p_2 = p_{air}$, atmospheric pressure

$p_1 = p_B$

We know ρ, v_2, p_2

Need v_1, speed at point B

$$v_1 = \frac{Q}{\rho A_1} \quad \text{(see sol'n to part a)}$$

$$v_1 = \frac{Q}{\rho \pi r_1^2}$$

$$p_B = p_{air} + \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2$$

$$= p_{air} + \frac{1}{2}\rho\left[\left[\frac{Q}{\rho\pi r_2^2}\right]^2 - \left[\frac{Q}{\rho\pi r_1^2}\right]^2\right]$$

$$\boxed{p_B = p_{air} + \frac{1}{2}\frac{Q^2}{\pi^2}\frac{1}{\rho}\left[\frac{1}{r_2^4} - \frac{1}{r_1^4}\right]}$$