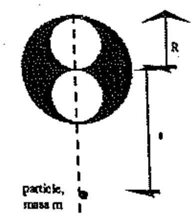
No difficult math is needed.

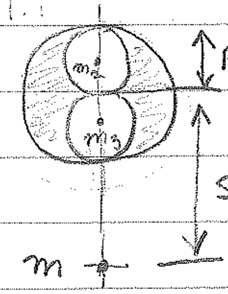
Two spherical holes (i.e., hollowed-out sections) are dug into a formerly uniform solid sphere of radius R. The holes are spheres of diameter R, as drawn below. Notice that one "edge" of each hole touches the center of the bigger sphere.

The mass of the big sphere, *before* it was hollowed out, was M.

With what gravitational force does the hollowed-out sphere attract a particle of mass m located a distance s from the center of the big sphere, as drawn below? (The particle lies on an imaginary straight line connecting the center of the big sphere to the centers of the holes.)

NOTE: The white areas are the dug-out holes. The grey areas are "solid matter."





For spherical symmetry, treat object as entire mass at center. By superposition, add contributions from components

Consider the sphere ~~before~~ hollowing: treat as M a distance S away from m
force on m is

$$F_{before} = \frac{GmM}{S^2} \quad \text{(in upward direction)}$$

Now consider the masses that would fill the holes as spherical objects
Call the top one m_2, bottom one m_3

These spheres exert force on m : $F_2 = \frac{Gmm_2}{(S+R/2)^2}$, $F_3 = \frac{Gmm_3}{(S-R/2)^2}$

Now the force on m due to the hollowed out sphere

$$\text{is } \underbrace{F_{before} - F_2 - F_3}_{\text{subtract the contributions}} = \frac{GmM}{S^2} - \frac{Gmm_2}{(S+R/2)^2} - \frac{Gmm_3}{(S-R/2)^2}$$

Now we need to find m_2; m_3 : assume uniform density

$$m_2 = V_2\rho = \cancel{\tfrac{4}{3}\pi}\left(\frac{R}{2}\right)^3 \cdot \frac{M}{\left(\cancel{\tfrac{4}{3}\pi} R^3\right)} \qquad \rho = \frac{M}{\frac{4}{3}\pi R^3}$$

$$m_2 = \frac{M}{8} \;; \quad \text{similarly } m_3 = \frac{M}{8}$$

$$\Rightarrow F_{hollowed} = GmM\left[\frac{1}{S^2} - \frac{(1/8)}{(S+R/2)^2} - \frac{(1/8)}{(S-R/2)^2}\right]$$

$$\boxed{F_{hollowed} = GmM\left[\frac{1}{S^2} - \frac{1}{8(S+R/2)^2} - \frac{1}{8(S-R/2)^2}\right]}$$