Physics 318 R.G. Palmer
Electromagnetism 4/28/08

Final Exam 2008 (24hr takehome)

Notes:

You are on your honor to abide by the rules in the handout *About the Final*. Violations will be treated extremely seriously: see `http://www.phy.duke.edu/graduate/resources/integrity.pdf`.

For full credit you must *justify what you are doing*. **Show all work.** Pulling an obscure formula out of a hat may lose you points; you must at least say where it came from (e.g., [Jackson page 813]), or probably you should write how to show that from standard ways.

Credit may be reduced if you do something a very awkward or long-winded way, even if you get the right result in the end.

I will readily give hints, but will record what I tell you and allow for that in grading.

1. In this problem we wish to find the asymptotic form of the potential for a point charge q placed along the axis of a very long (∞), ground cylinder. In particular, we want to determine the behavior of potential within the cylinder for distances z large compared with cylinder radius R. The charge position is taken as the origin.

   a. Construct a Dirichlet Green function for this problem, using cylindrical coordinates. You might want to follow a similar procedure to that used in lecture to derive the Green function for the charge between two infinity planes.

   b. Use the Green function from part (a) to find a general expression for the potential due to the point charge.

   c. Determine the leading order term for $\boldsymbol{x} = (\rho = 0, \phi = 0, z \gg R)$. Compare your result with the asymptotic potential behavior for a point charge placed between two ground planes.

2. Consider a thin square slab of ferromagnetic material with side L and thickness $t \ll L$. The slab is centered on the origin, oriented normal to the z direction, and carries uniform magnetization $\boldsymbol{M} = M\hat{\mathbf{k}}$.

   a. Find an expression for the magnitude and direction of the $\boldsymbol{B}$ and $\boldsymbol{H}$ fields, to leading order in the small parameter t/L, at a point directly above the center of the slab. Use the fields just outside the slab *and* just inside, *i.e.*, with coordinates $(0, 0, t/2^+)$ and $(0, 0, t/2^-)$.

   b. A hole of radius $t/4$ and centered at the origin is now carved out from the slab. Calculate the effect that this has on your answer to part (a). Is this a large effect?

3. Jackson 5.28.
   We have not talked about M_{ij}, the mutual inductances, in the classes, but all you need to use Eq. 5.155.

   Note 1: Jackson use

   $$K(k) = \int_0^{\pi/2} \frac{d\theta}{\sqrt{1 - k^2 \sin^2\theta}}.$$

   Some books use different notions, with $K(k^2)$ for the same RHS above.

   Note 2: There is a typo in Jackson — $(a + b^2)$ should be $(a + b)^2$.