# Electromagnetic Energy

*A child of five can understand this; send someone to fetch a child of five.*
*— Groucho Marx*

## Energy in the fields can move from place to place

We have discussed the energy in an electrostatic field, such as that stored in a capacitor. We found that this energy can be thought of as distributed in space, with an energy per unit volume (energy density) at each point in space. We obtained a formula for this quantity, $u_e = \frac{1}{2}\varepsilon_0 E^2$ (if there are no dielectric materials). This formula applies to any E-field. We also found that magnetic fields possess energy distributed in space, described by the magnetic energy density $u_m = B^2 / 2\mu_0$. If there are both electric and magnetic fields, the total electromagnetic energy density is the sum of $u_e$ and $u_m$. These specify how much electromagnetic field energy there is at any point in space.

But we have not yet considered how this energy moves from place to place. Consider the energy flow in a flashlight. We know that energy moves from the battery to the bulb, where it is converted into heat and light. It is tempting to assume that this energy flows through the conductors, like water in a pipe. But if we look carefully we find that the electromagnetic energy density in the conductors is much too small to account for the amount of energy in transit. Nearly all of the energy gets to the bulb by flowing through space near the conductors. In a sense they guide the energy but do not carry much of it. But how does this happen?

And of course the flow of energy in electromagnetic waves such as light is not guided by any conductors, let alone carried by them. Clearly we need to understand better the nature of this kind of energy flow.

## Intensity and the Poynting vector

In describing any flow of energy through space — such as that in a sound wave — it is useful to talk in terms of the power crossing unit area perpendicular to the flow. This is the **intensity**, usually denoted by *I*.

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| Intensity | Intensity is the amount of power crossing unit area perpendicular to the direction of the flow. |

Intensity is simply related to the energy density $u$ and the speed of flow $v$:

$$I = uv$$

This is a general relation for any kind of energy flow, not specific to electromagnetic energy.

For electromagnetic fields, the energy density (in empty space) is

$$u_{\text{tot}} = u_e + u_m = \tfrac{1}{2}\left[\varepsilon_0 E^2 + \frac{B^2}{\mu_0}\right],$$

and the speed of flow (in a vacuum) is $c$, the speed of light.

Around 1900 an important insight about electromagnetic energy flow was gained from a theorem due to Poynting. To describe this flow he introduced a new quantity, now called the **Poynting vector**:

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| Poynting vector | $\mathbf{S} = \dfrac{\mathbf{E}\times\mathbf{B}}{\mu_0}$ |

To understand the important role played by this vector, consider a region of space surrounded by a closed surface. Within this surface there may be anything — charges, circuits, batteries, resistors, generators, motors, etc. The total electromagnetic field energy in this region is

$$U_{\text{e-m}} = \int u_{\text{tot}}\, dV$$

where the integral covers the volume of the region. Now let $P_Q$ represent the rate at which energy is being transformed *from* other forms *into* e-m field energy within the region. Poynting's theorem, which is derived from Maxwell's equations, states that

$$\frac{\partial U_{\text{e-m}}}{\partial t} + \oint \mathbf{S}\cdot d\mathbf{A} = P_Q\,.$$

The first term on the left is the rate of *increase* in total field energy within the region. If there were no flow of field energy in or out through the surface, this would be equal to $P_Q$ by conservation of energy. The second term on the left must therefore represent the net rate of flow of field energy *out* through the surface. It is the net flux of **S** through the surface, so we conclude that:

The Poynting vector **S** represents the flow of energy. Its direction is that of the flow, and its magnitude is the intensity**.**

The total electromagnetic power crossing a given surface $A$ is obtained by integrating

$$P(\text{across } A) = \int_A \mathbf{S}\cdot d\mathbf{A}\,.$$

Because it is the the scalar product of **S** and $d\mathbf{A}$, the power is proportional to the cosine of the angle between the directions of those vectors.

This dependence on the angle with which the flow passes the surface explains (among many other things) why the intensity of sunlight received at the earth’s surface varies with latitude, and explains the seasonal changes in that intensity.

## Maxwell predicts electromagnetic waves: light, among others

Probably the most important case of energy flow by means of electric and magnetic fields is electromagnetic (e-m) waves, which we discuss now.

In describing waves in a medium (waves in a string, sound waves, etc.) we found that the disturbance passing through the medium was described mathematically by a function obeying a wave equation.

The reference notes on mechanical waves give a review of the mathematics of wave motion.

If the disturbance moves in the $+x$-direction, this function has the general form

$$f(x,t) = f(x - vt),$$

where $v$ is the speed of the waves.

If the disturbance moves in the –x-direction the form of the function is $f(x) = f(x + vt)$.

Maxwell showed mathematically that there are situations where the E-field and the B-field in empty space are functions having this form, so an electromagnetic "disturbance" can propagate in the form of waves. We will omit the argument and just quote some of the results of Maxwell's analysis.

Not all configurations of fields give rise to waves, of course. What Maxwell showed is that there exist configurations that do constitute waves.

Among his most remarkable finding was that the speed of the waves should be $1/\sqrt{\varepsilon_0 \mu_0}$. Numerically this is $2.998 \times 10^8$ m/s, which is the observed speed of light.

This gave a strong theoretical reason to believe that light is an electromagnetic wave.

In 1887 Hertz (after Maxwell's death) succeeded in producing and detecting electromagnetic radiation using devices in his laboratory. He also showed that this radiation obeys the same laws of reflection, refraction, etc., as light.

The waves that Hertz produced and detected have wavelengths much longer than light waves and are usually called "radio" waves. About a decade later Marconi patented a method of using these waves for "wireless" communication.

Besides traveling at the speed of light, e-m waves have these general properties:

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| Properties of electromagnetic radiation | 1. Both $E$ and $B$ decrease with distance from the source as $1/r$.<br>2. **E** and **B** are mutually perpendicular.<br>3. Energy flows in the direction of $\mathbf{E} \times \mathbf{B}$.<br>4. The magnitudes are related by $E = cB$.<br>5. The intensity is $I = \varepsilon_0 c E^2$. |

The formula for the intensity is the magnitude of the Poynting vector, expressed in terms of $E$ by using item #4.

In any discussion of waves a special role is played by harmonic waves, which have a definite wavelength $\lambda$ and frequency *f*. Since the fields which constitute e-m waves are vector fields we must also specify the directions of those fields. One particular case is described by these functions:

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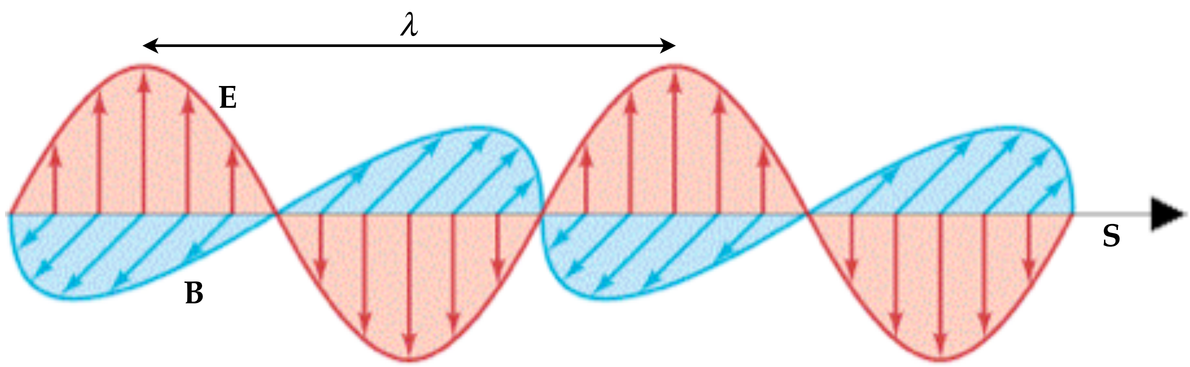
| Harmonic electromagnetic wave (traveling in +*x* direction, with **E** along *y*-axis) | $E_y(x,t) = E_0 \cos(kx - \omega t)$<br>$B_z(x,t) = (E_0 / c)\cos(kx - \omega t)$ |

Here, as usual, $\omega = 2\pi f$, $k = 2\pi / \lambda$, and $\omega / k = f\lambda = c$.




Shown above are the fields of an e-m wave, moving to the right. The fields oscillate exactly in phase with each other, and the Poynting vector representing energy flow is in the direction of $\mathbf{E} \times \mathbf{B}$.

For a harmonic wave the intensity $I = c\varepsilon_0 E^2$ oscillates between zero and its maximum twice per cycle. It is common to use the average over a cycle. Since the average of $\cos^2 \omega t$ is 1/2, we have

$$I_{av} = \tfrac{1}{2} c\varepsilon_0 E_0{}^2 .$$

This formula will be used in our discussion of light.

## Sources of e-m waves: classical radiation by a dipole antenna

We have not yet discussed an important question: How do the fields that constitute e-m waves get created? The general answer (in classical physics) is easy to state:

Accelerated charges radiate away part of their energy as e-m waves.

For a single point charge this process is very complicated to describe in detail. (It is an important case in practice, however: X-rays are produced by the rapid stopping of electrons in a beam.) Much easier to describe (and also of great practical importance) is the case of an oscillating electric dipole, a system in which charges move back and forth over a certain distance, creating a dipole moment that varies periodically with time.

We assume for simplicity that the motion of the charges is simple harmonic. Then the dipole moment varies sinusoidally with time:

$$\mathbf{p}(t) = \mathbf{p}_0 \cos \omega t .$$

Near the dipole the E-field of the charges and the B-field of the current produced by their motion give the dominant contributions to the fields. These are the "near" fields. However, during half of the cycle the energy in these fields moves out from the dipole, and during the other half it moves back; the net outward flow per cycle is zero. These fields each also fall off with distance at least as fast as $1/r^2$, so the Poynting vector falls off with distance at least as fast as $1/r^4$. This gives no net flow to great distances. These fields do *not* by themselves constitute the radiation.

But the oscillation in time of the "near" fields also produces *induced* fields, weak near the dipole but falling off only as $1/r$ with distance. The Poynting vector of these fields is always directed outward, giving a net flow of energy. This is the radiation.

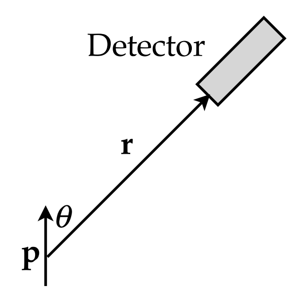
These induced E- and B-fields (called "radiation" or "far" fields) are each proportional to the *acceleration* of the charges, and therefore to the second time derivative of the dipole moment:

$$\frac{d^2\mathbf{p}}{dt^2} = -\omega^2\mathbf{p}.$$

The Poynting vector, being a product of **E** and **B**, is thus proportional to $\omega^4$, which means that the amount of power radiated increases rapidly with the frequency.

This is why energy loss by radiation is negligible in transmission of 60 Hz household power by ordinary wires, but becomes a significant factor at the frequencies of FM or TV power, 100 MHz or higher. At those frequencies coaxial cables, which (as we have seen in the assignments) produce negligible B-fields outside the cable and therefore little radiation, are useful.

The radiated intensity varies with direction in a fairly simple way. Shown is a dipole, and a detector at a large distance which absorbs some of the radiation along the direction of the vector **r** indicated. The angle between the dipole moment and **r** is $\theta$. Detailed analysis shows that the average intensity at the detector is given by




$$I_{av} = \frac{\omega^4 p_0^{\,2}}{32\pi^2 c^3 \varepsilon_0} \frac{\sin^2\theta}{r^2}.$$

The important things in this formula are that the intensity:

- Is proportional to $\omega^4$.
- Varies with direction as $\sin^2\theta$.
- Varies with distance as $1/r^2$.

We will make use of these facts later in our discussion of light. They are important also in design of transmitting and receiving antennas for radio, FM and TV waves.

## Field momentum and radiation pressure

The flow of energy in the e-m field is much like that of mass in a perfect fluid. As in the case of fluid flow, there is also momentum and pressure associated with the energy flow in the electromagnetic fields.

There is a general relation between the Poynting vector and the momentum per unit volume associated with the flow of energy:

$$\mathbf{S} = c^2 \mathbf{g}$$

Here **g** denotes the momentum per unit volume (*not* the gravitational field). For e-m radiation the magnitude of **S** gives the intensity, which is equal to the speed of flow (c) times the energy density, so we see that the magnitude of the momentum density and energy density in e-m radiation are simply related:

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| Energy and momentum of electromagnetic radiation | $\text{Energy density} = c \cdot \lvert\text{Momentum density}\rvert$ |

When radiation falls on a surface, there is a flow of momentum onto the surface. (It is like the momentum carried by a stream of water striking a wall.) The amount of this momentum delivered to the surface per unit time determines the force exerted on the surface by the radiation. The force per unit area constitutes a pressure exerted on the surface, called the **radiation pressure**. If the incident momentum is *completely absorbed* by the surface, the pressure is given by

$$P_{\text{rad}} = S / c \,.$$

If the radiation is *reflected* straight back, carrying equal momentum backwards, then the force and the radiation pressure will be twice this amount. Radiation impinging perpendicular to a perfectly reflecting surface thus exerts twice as much force as it does on a perfectly absorbing surface.