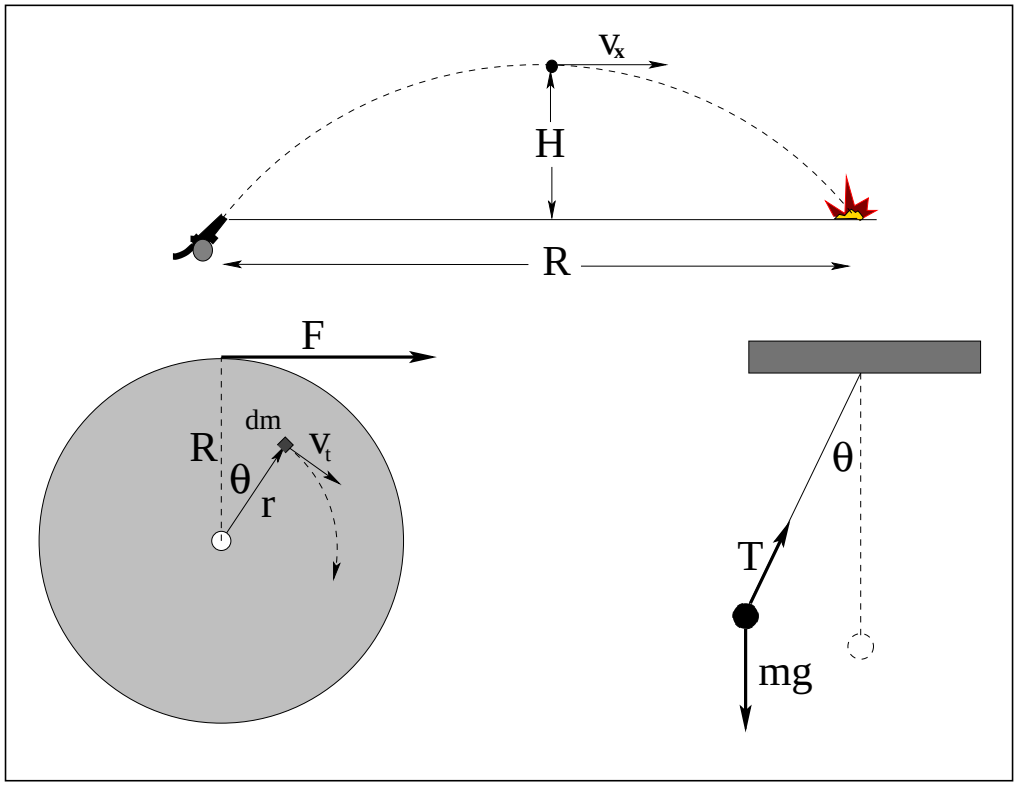
Introductory Physics 141/151/161

Self-Guided Learning Problems

Robert G. Brown



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About These Problems

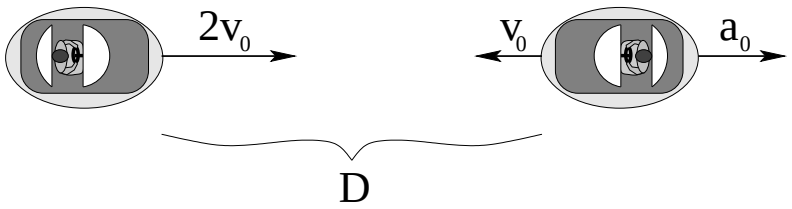
This is an experiment. Beamer allows me to make slides that will successively reveal lines of math-heavy text. This gives me a unique opportunity to build a collection of self-guided learning problems for physics that do what I've fantasized about doing for years now – present a problem, then provide a hint, then another hint, then another (or reveal a step, and then another step) until finally, the entire souution is presented, annotated.

Hopefully these problems will help students everywhere as they struggle to learn physics problems solving techniques and learn to "think like a physicist" as they do so.

To use this resource, pick a problem or topic from the table of contents and go directly to it, or work your way through all the problems systematically. Work on a separate sheet of paper, and when you get stuck, page down through the frames to see (hopefully) where you went wrong.

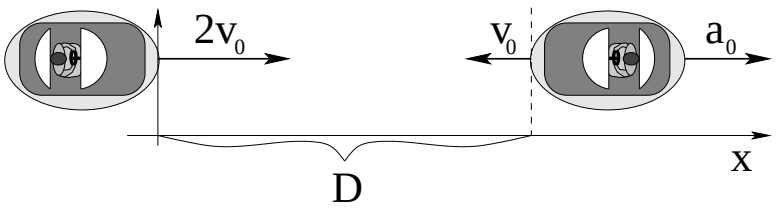
Remember, the point is to **master these problems**, not just to get through them. Make sure that before you are done, you can do every problem **without looking, without hints, and without remembering the exact solution** but rather, understanding *how* to find it!

Kinematics: Two Bumper Cars



- Two bumper cars are headed straight at one another, one travelling at $2v_0$ to the right, the other at speed v_0 to the left. When they are separated by a distance D, the car on the right slows down with a constant acceleration a_0. Does the right hand car manage to stop before being hit by the left hand car?

Kinematics: Two Bumper Cars



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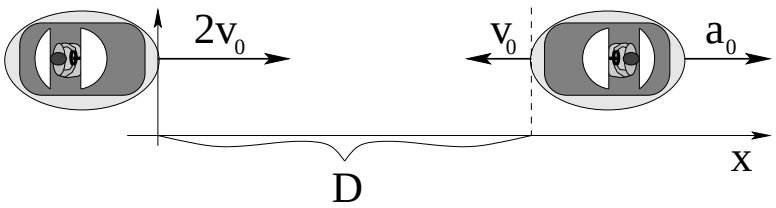

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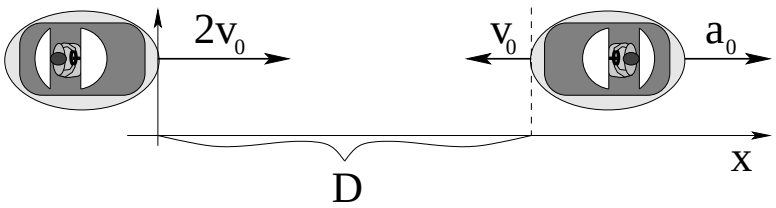

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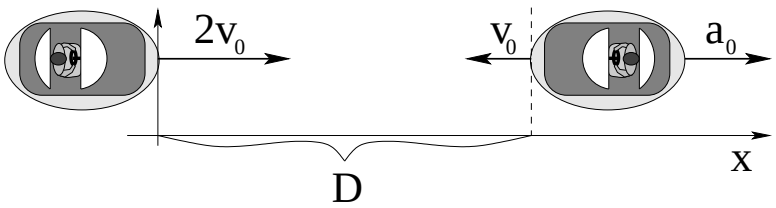

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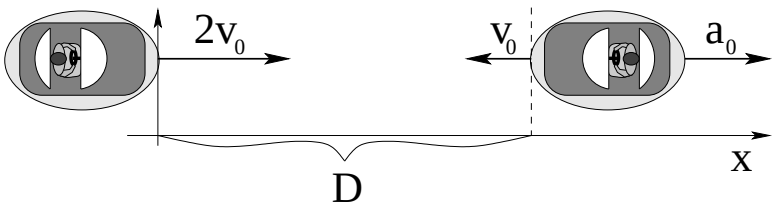

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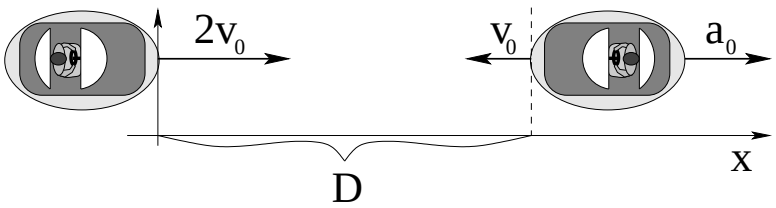

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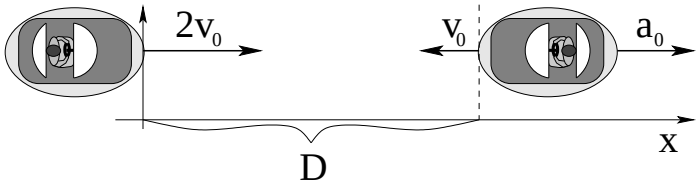
* - The solution is on the next page. Don't advance until you are ready!

Kinematics: Two Bumper Cars-Solution



Solution: $x_l(t) = 2v_0 t, \quad v_l(t) = 2v_0$

$$x_r(t) = D - v_0 t + \frac{1}{2} a_0 t^2, \quad v_r(t) = -v_0 + a_0 t$$

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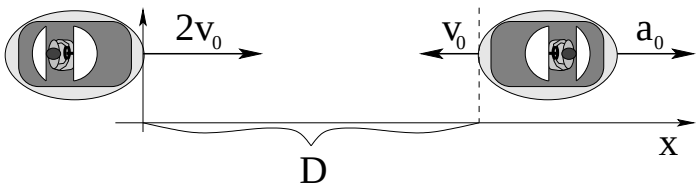

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Find the time: $t_r = \dfrac{v_0}{a_0}$.

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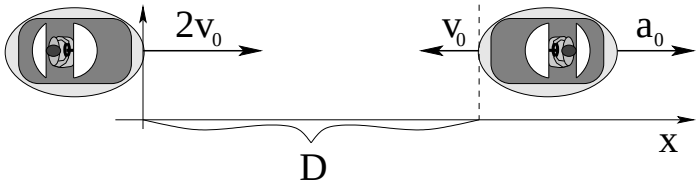

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Find the time: $t_r = \frac{v_0}{a_0}$. The left hand car is then at $x_l(t_r) = \frac{2v_0^2}{a_0}$ and the right hand car is at $x_r(t_r) = D - \frac{v_0^2}{a_0} + \frac{v_0^2}{2a_0} = D - \frac{v_0^2}{2a_0}$.

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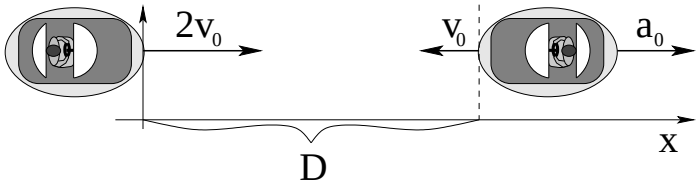

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$$D < \frac{5v_0^2}{2a_0}$$

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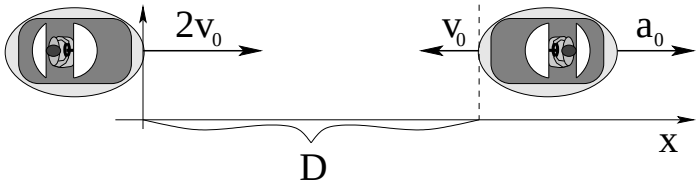

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It makes sense – larger D makes it *less* likely to collide, larger a_0 makes it *less* likely they will collide, larger v_0 makes it *more* likely.

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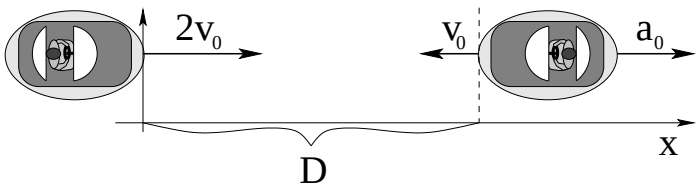

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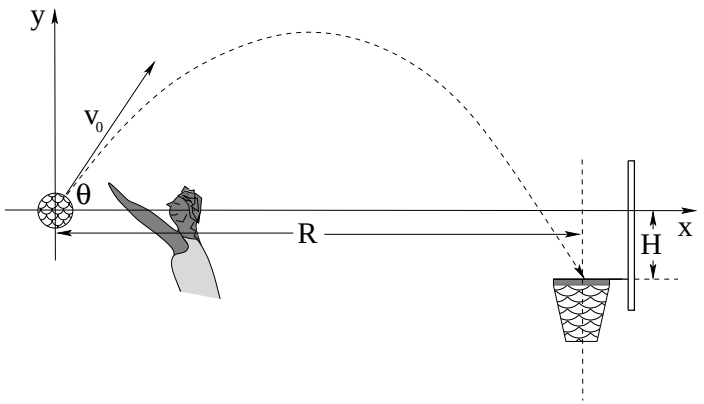
$$D < \frac{5v_0^2}{2a_0}$$

It makes sense – larger D makes it *less* likely to collide, larger a_0 makes it *less* likely they will collide, larger v_0 makes it *more* likely. Knowing they collide, if we write:

$$x_l(t_c) = 2v_0 t_c = D - v_0 t_c + \frac{1}{2} a_0 t_c^2 = x_r(t_c)$$

would let us find the time of collision and answer other questions about e.g. their relative velocity at that time. This is a simple quadratic equation for t_c.

Kinematics: 2D Basketball Trajectory



- A basketball player shoots a jump hook at a (horizontal) distance R from the basket, releasing the ball at a height H above the rim as shown. To shoot over his opponent's outstretched arm, he releases the basketball at an angle θ with respect to the horizontal.

Find v_0, the ***speed*** he must release the basketball with (in terms of H, R, g and θ) for the ball to go through the hoop "perfectly" as shown. Assume that his release is on line and undeflected, at initial speed v_0 and that the acceleration of the basketball is $\vec{\boldsymbol{a}} = -g\hat{\boldsymbol{j}}$, ignoring drag.

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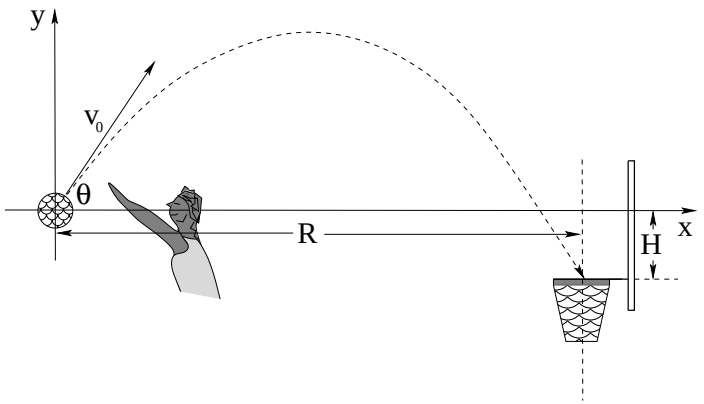

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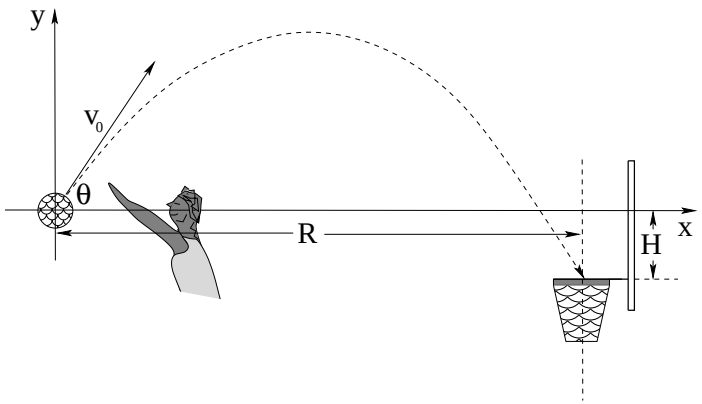

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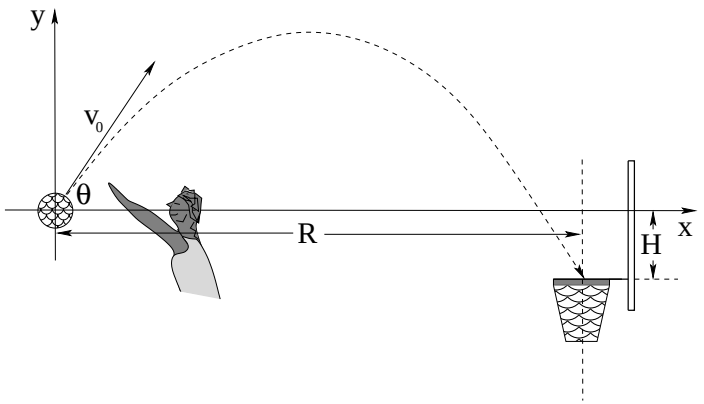

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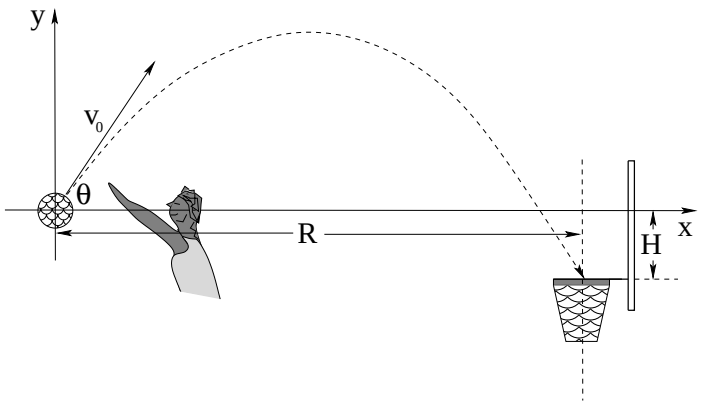

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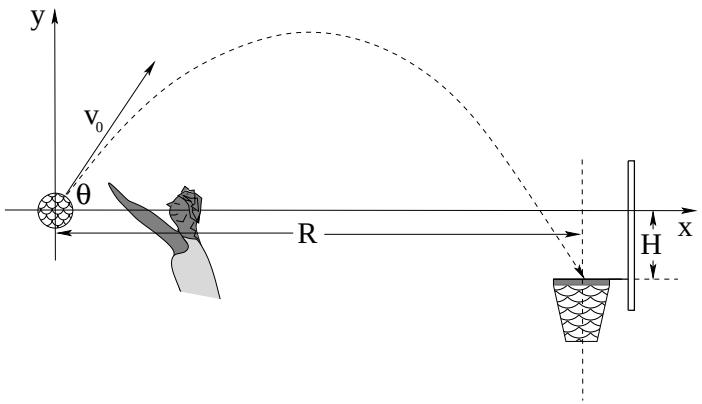

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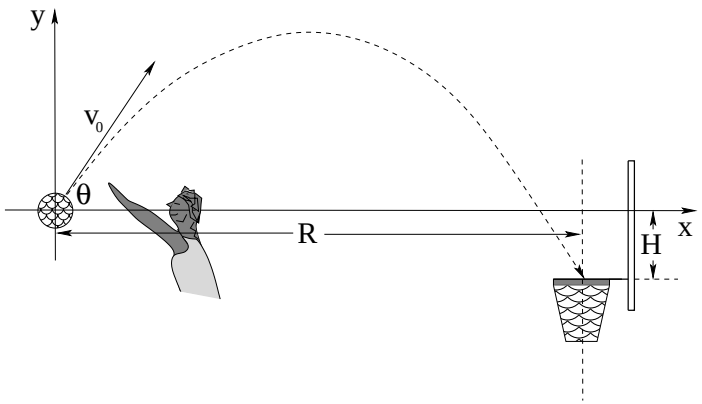

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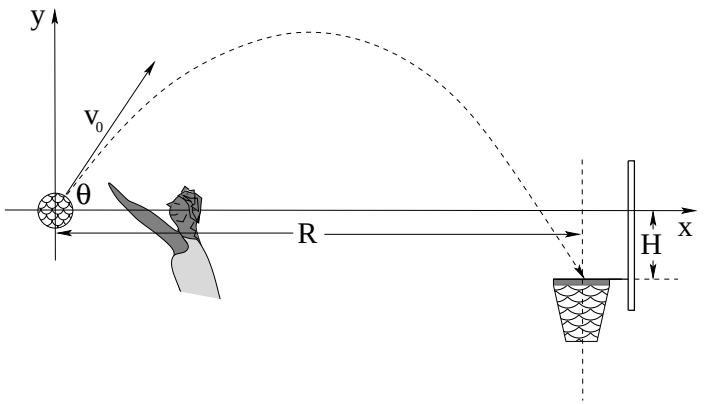

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Find v_0, the ***speed*** he must release the basketball with (in terms of H, R, g and θ) for the ball to go through the hoop "perfectly" as shown. Assume that his release is on line and undeflected, at initial speed v_0 and that the acceleration of the basketball is $\vec{\boldsymbol{a}} = -g\hat{\boldsymbol{j}}$, ignoring drag.

Hints: First write down $a_x = 0$, $a_y = -g$ and solve for $x(t)$, $v_x(t)$, $y(t)$ and $v_y(t)$. Note that there are really two unknowns – v_0 and t_b, the time the ball reaches the basket. In order to go through the basket, $x(t_b) = R$ at the same time that $y(t_b) = -H$. Find t_b in terms of v_0 and the givens from the x equation. Substitute it into the $y(t_b)$ equation. Solve for v_0. There is some algebra involved!

* - The solution is on the next page. Don't advance until you are ready!

Kinematics: 2D Basketball Trajectory-Solution

Initial Conditions:

$$a_x = 0, v_{0x} = v_0 \cos\theta, x_0 = 0 \text{ and } a_y = -g, v_{0y} = v_0 \sin\theta, y_0 = 0$$

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Integrate:

$$x(t) = v_0 \cos\theta t \quad v_x(t) = v_0 \cos\theta \quad y(t) = -\frac{1}{2}gt^2 + v_0 \sin\theta t \quad v_y(t) = v_0 \sin\theta - gt$$

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Find the ***time*** t_b that the basketball reaches the horizontal position of the hoop:

$$R = v_0 \cos\theta t_b \Rightarrow t_b = R/(v_0 \cos\theta)$$

This must also be the time that the ball has exactly the height of the hoop:

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$$\boxed{v_0 = \sqrt{\frac{gR^2}{2(R\sin\theta\cos\theta + H\cos^2\theta)}}}$$

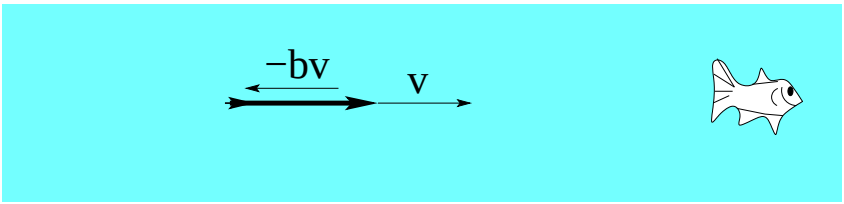
After doing the algebra, check the dimensions. Are they OK?

Firing a Speargun

- An underwater fisherman fires her speargun at a distant fish. The neutral-buoyancy spear leaves the gun at initial speed v_0 and experiences a *linear* drag force $F_d = -bv$ opposite to its velocity. Find $v(t)$ and R, the maximum range of the spear.

Hints:

Firing a Speargun



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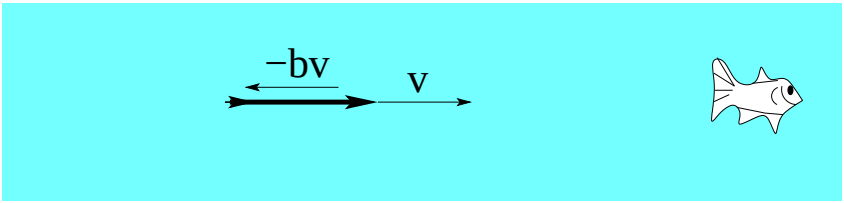

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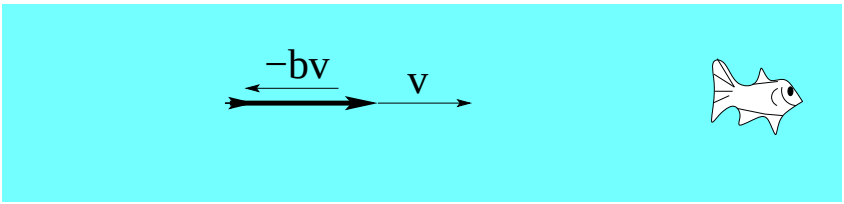

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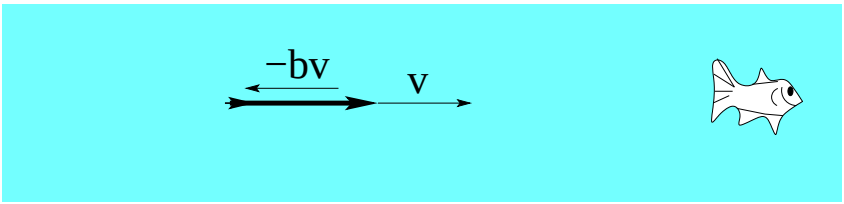

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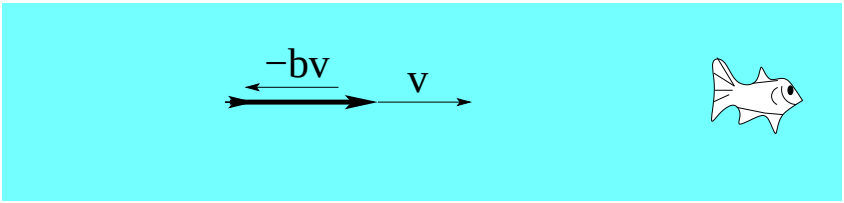

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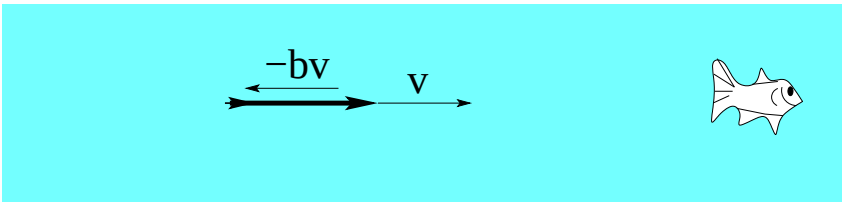

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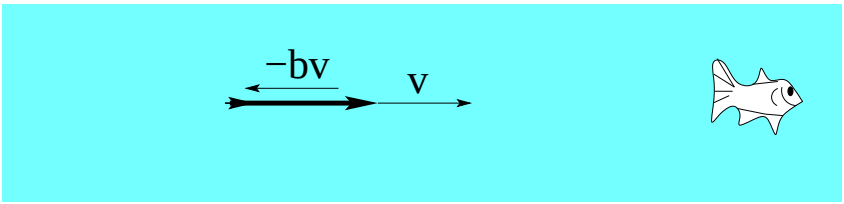

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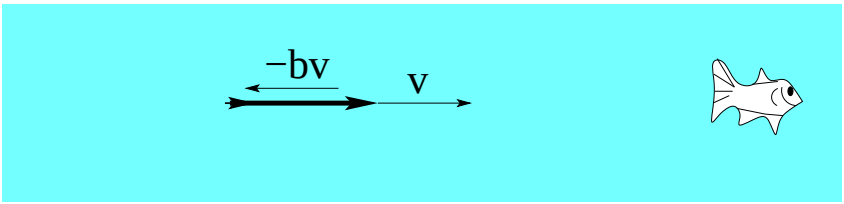

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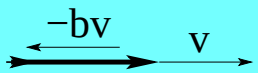

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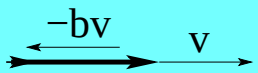

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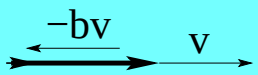

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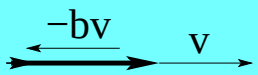

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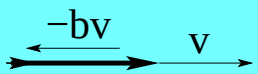

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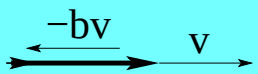

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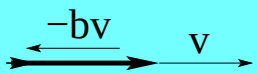

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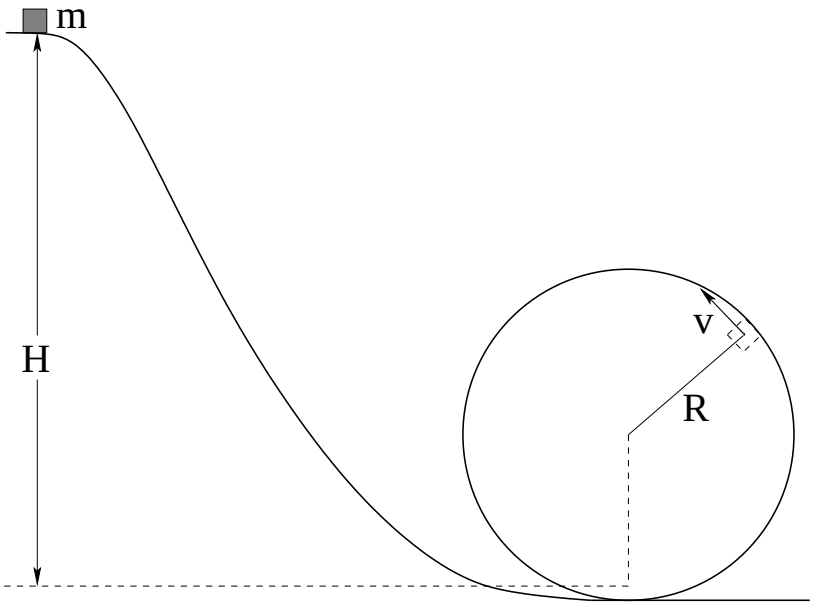
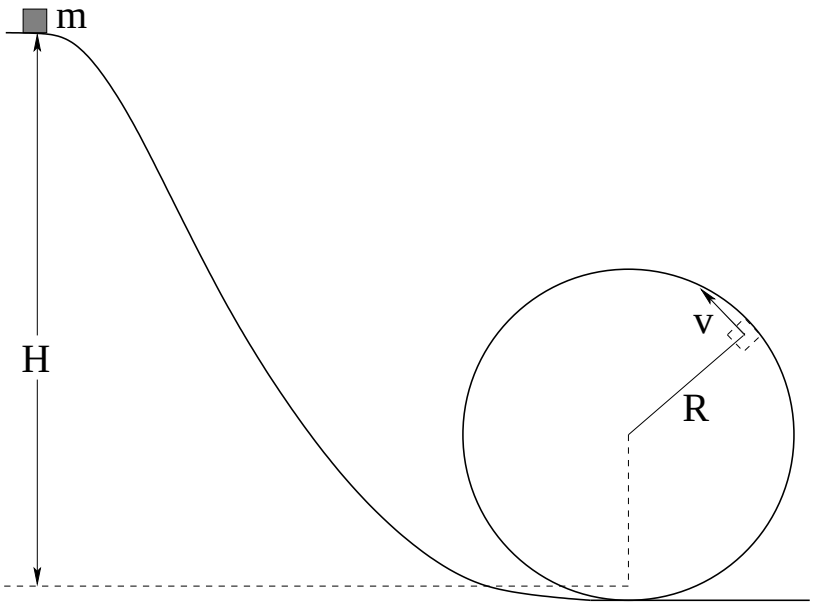
Loop the Loop



- A block of mass M sits at the top of a frictionless hill of height H leading to a circular loop-the-loop of radius R. Find the minimum height $H_{\rm min}$ for which the block *barely* gess around the loop staying on the track at the top. If the block is started at this position, what is the normal force exerted by the track at the *bottom* of the loop, where it is greatest?

Solution Map:

Loop the Loop



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Solution Map: First, draw a force diagram of the mass at the top of the loop (include gravity and the normal force).

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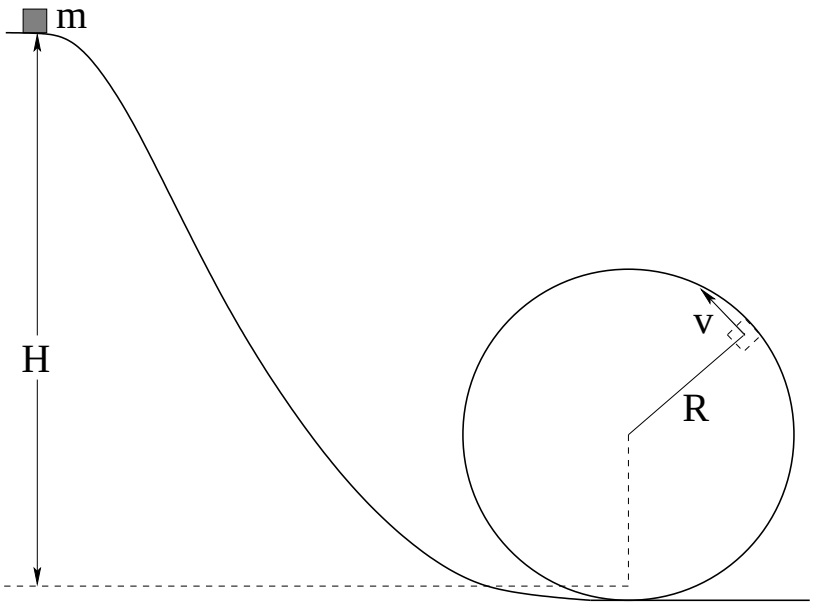

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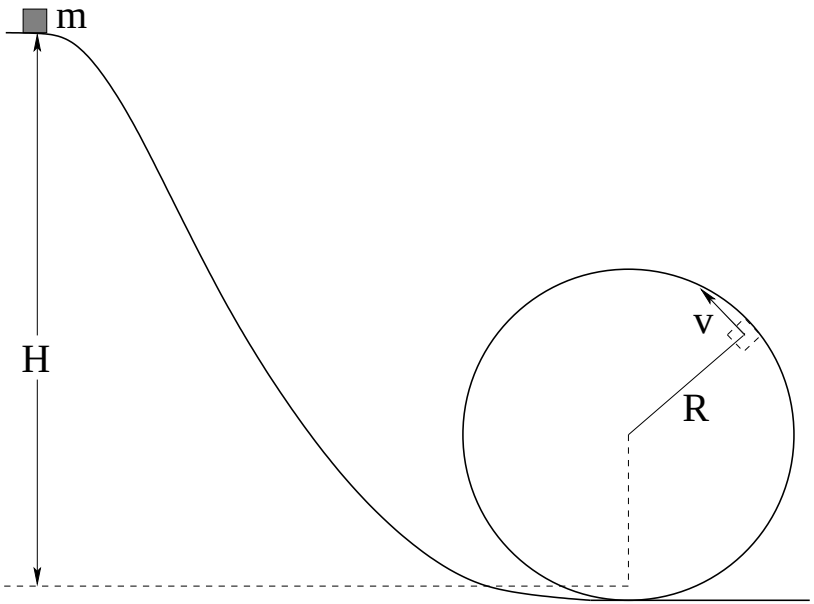
Loop the Loop



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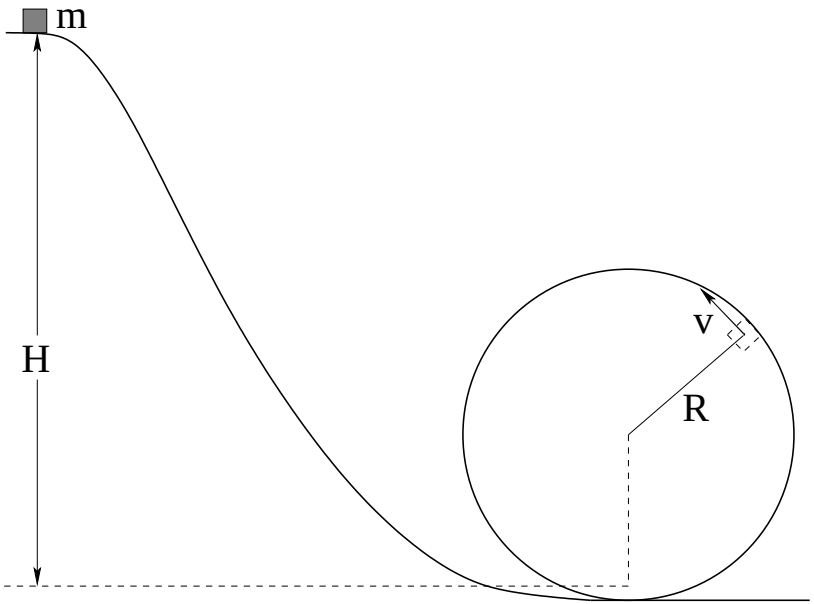

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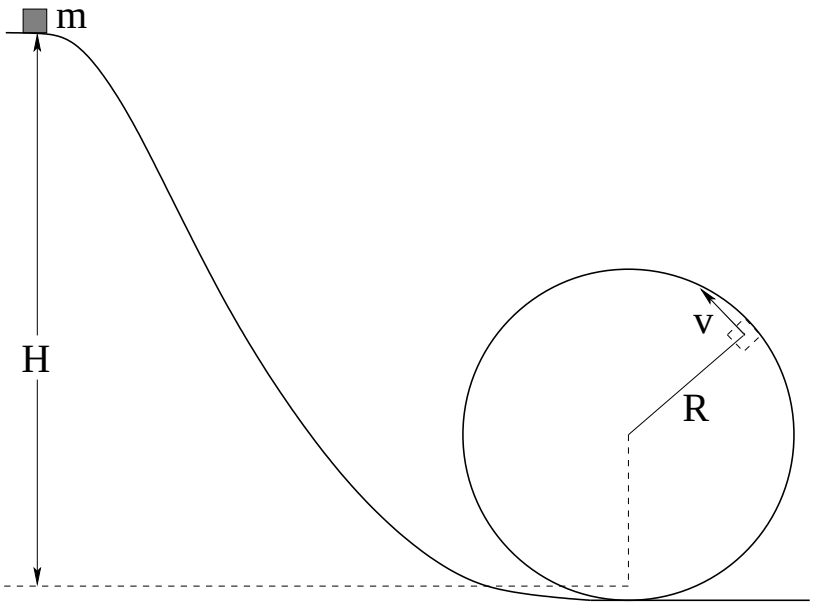

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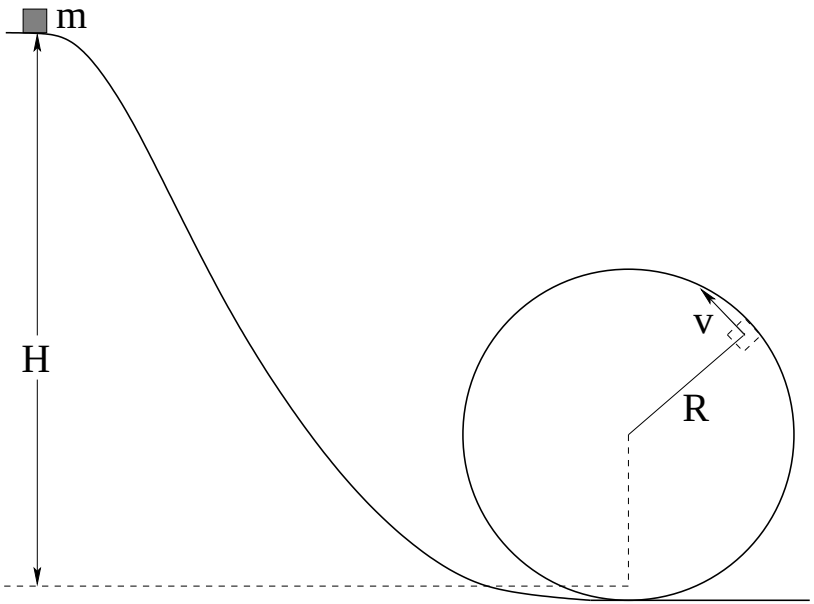

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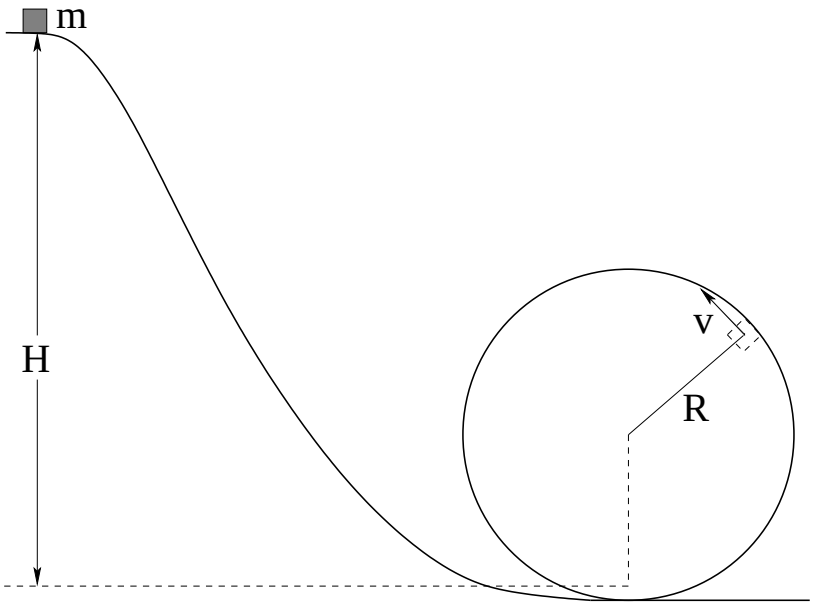

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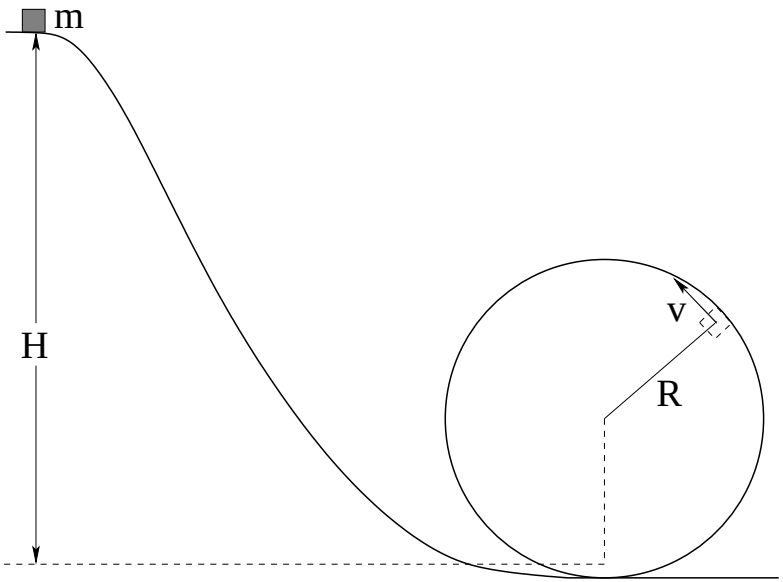
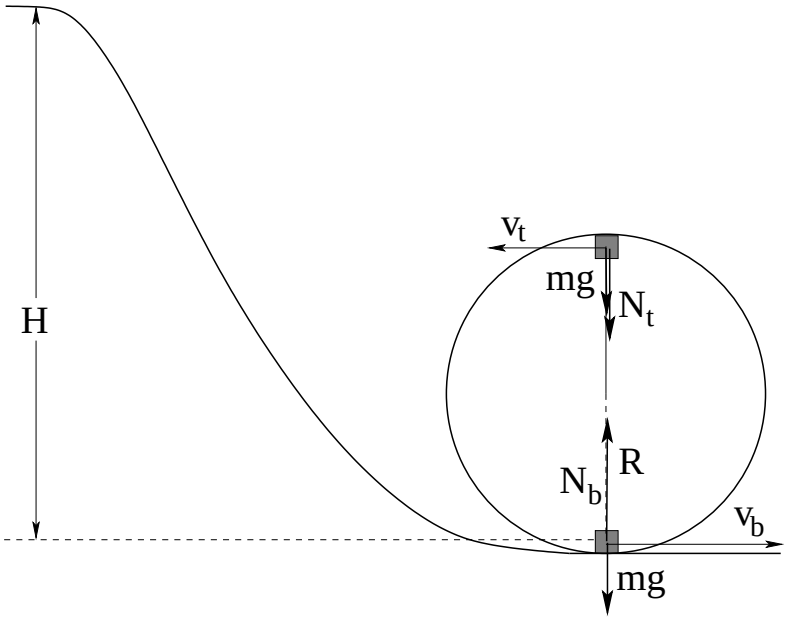
Loop the Loop-Solution



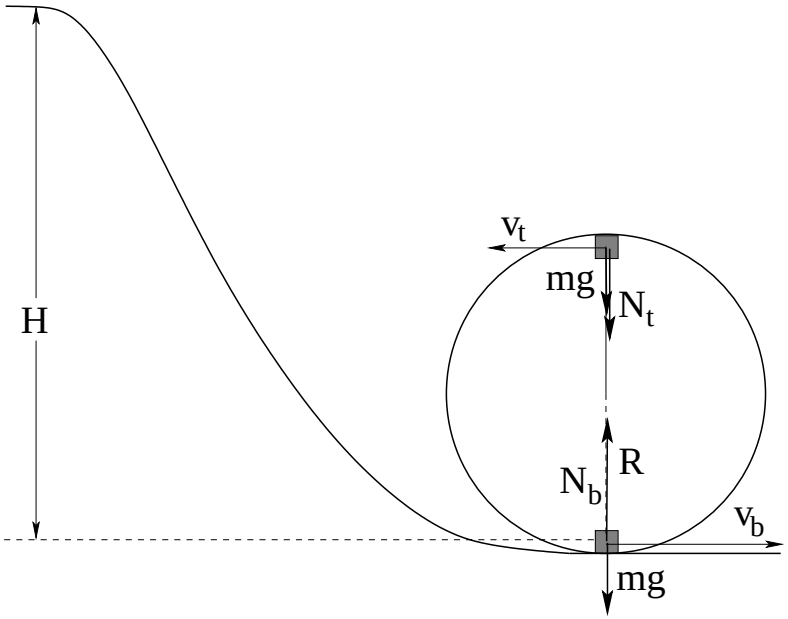
At the top:

Loop the Loop-Solution



At the top: $mgH_{\min} + 0 = mg2R + \frac{1}{2}mv_t^2$

Loop the Loop-Solution



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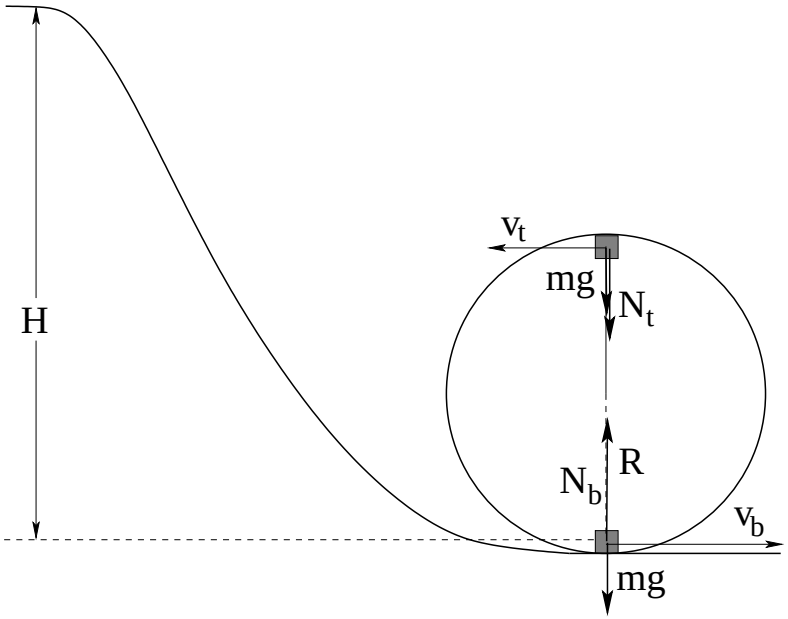
$$mg + \cancel{N_t} = \frac{mv_t^2}{R} \Rightarrow \frac{1}{2}mv_t^2 = \frac{1}{2}mgR$$

Loop the Loop-Solution



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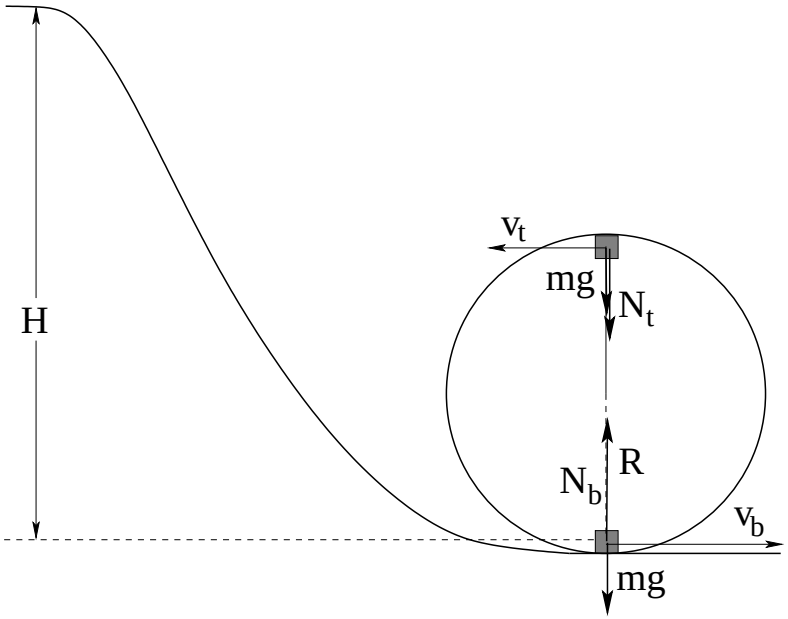

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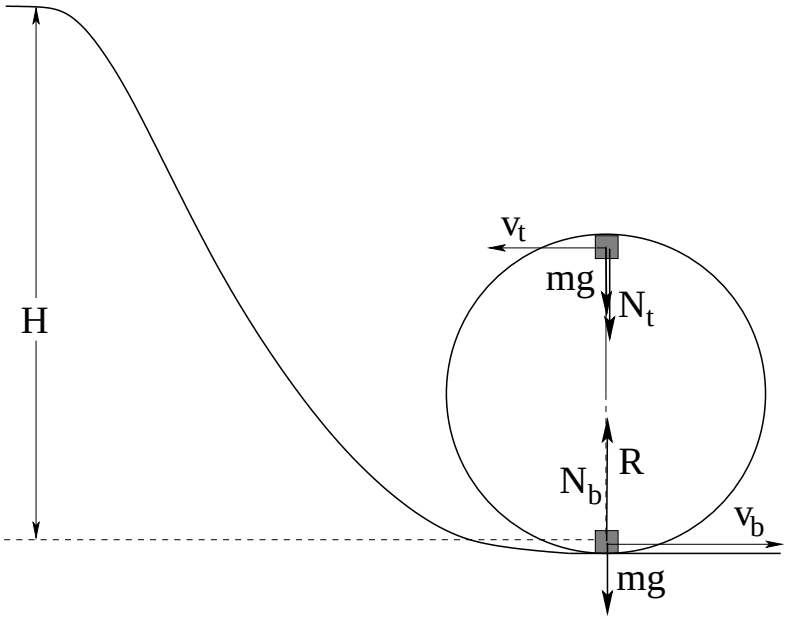

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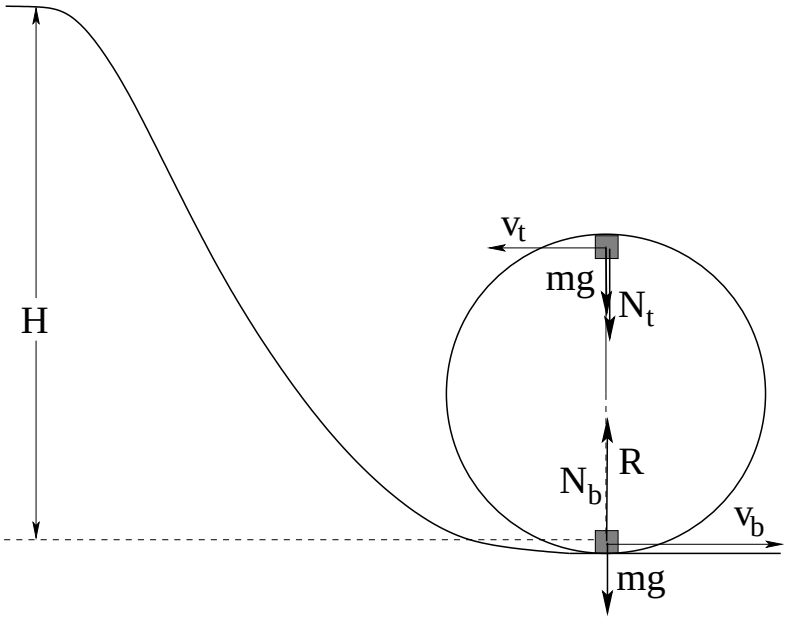

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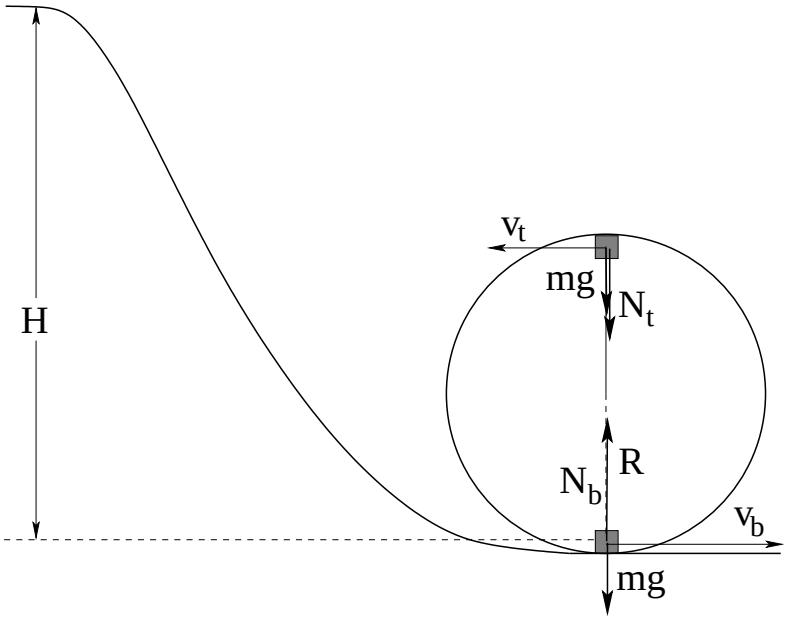
$$mgH_{\min} + 0 = \frac{5}{2}mgR \quad = \frac{1}{2}mv_b^2 \text{ and}$$

Loop the Loop-Solution



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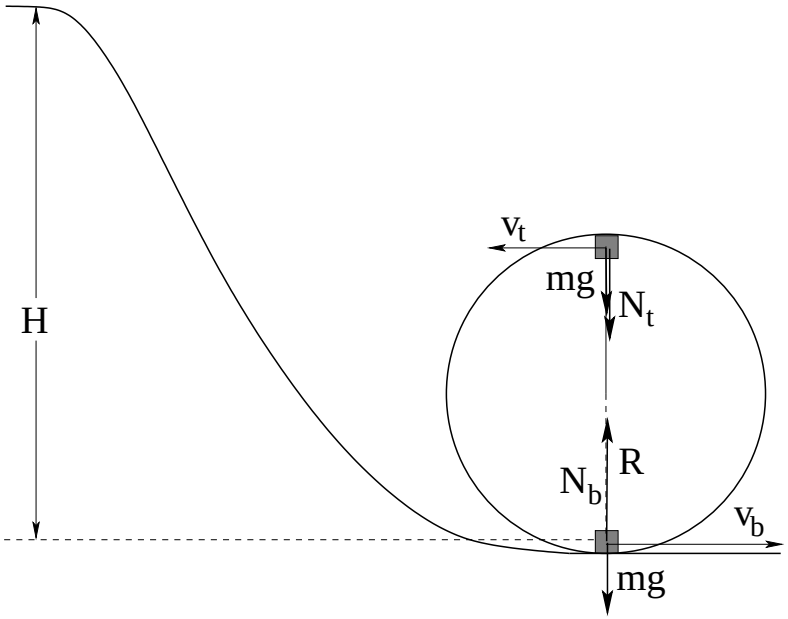
At the bottom:

$$mgH_{\min} + 0 = \frac{5}{2}mgR \quad = \frac{1}{2}mv_b^2 \text{ and } \quad N - mg = \frac{mv_b^2}{R} \Rightarrow$$

Loop the Loop-Solution



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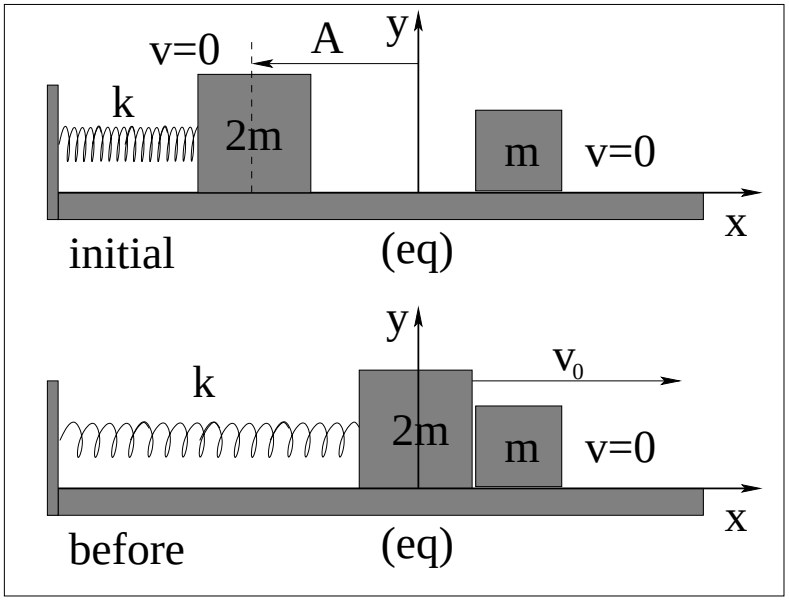
At the bottom:

$$mgH_{\min} + 0 = \frac{5}{2}mgR \quad = \frac{1}{2}mv_b^2 \text{ and } \quad N - mg = \frac{mv_b^2}{R} \Rightarrow N = mg + \frac{mv_b^2}{R}$$

or:

$$mv_b^2 = 5mg \Rightarrow \boxed{N = mg + 5mg = 6mg}$$

Spring-Driven Collision



In the figure above, a mass $2m$ connected to a spring k is released from rest when the spring is compressed to the position $x_0 = -A$ and collides **elastically** with a mass m just as it reaches its equilibrium position moving with velocity $v_0\hat{\boldsymbol{x}}$ as shown. Find: A in terms of v_0, m, and k; v_{2m} and v_m immediately after the collision; the maximum amplitude A' of the big $(2m)$ block's oscillation *after* the collision. Ignore friction, and m starts at rest.

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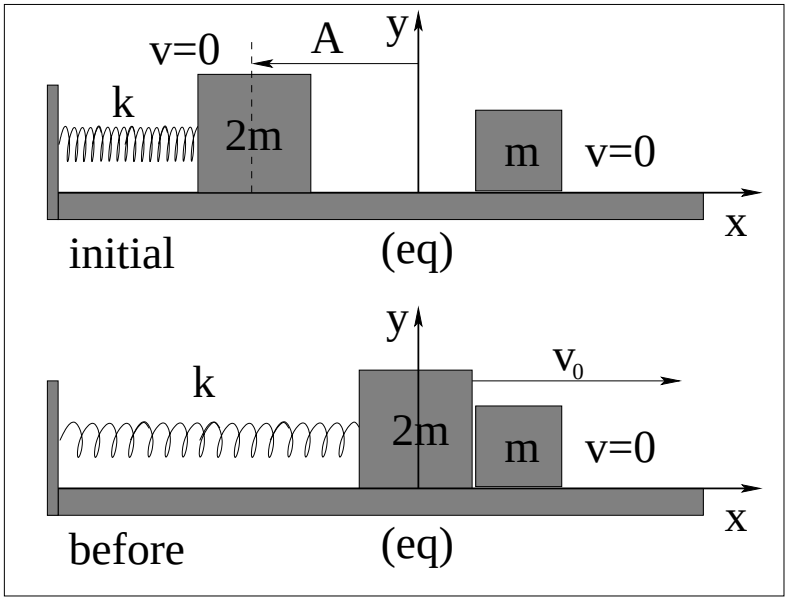

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Solution Map:

Use energy conservation to find A.

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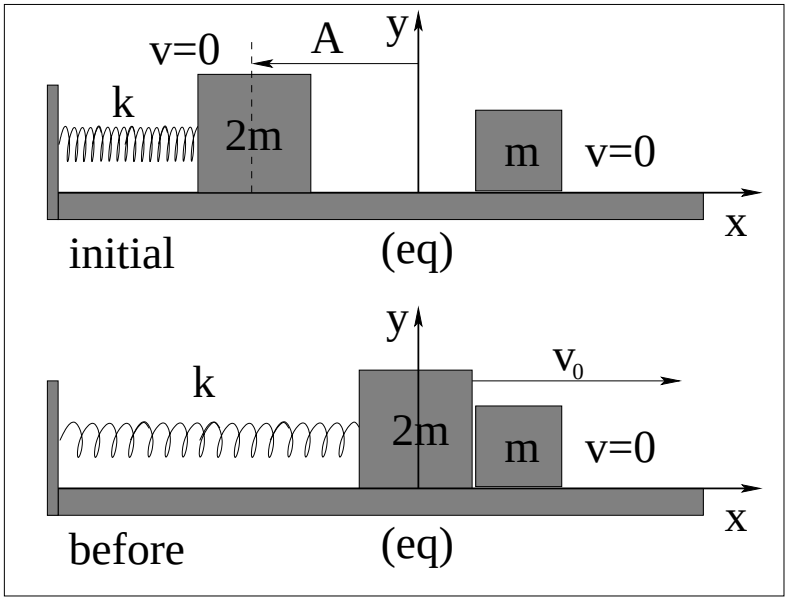

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Solution Map:

Use energy conservation to find A. Solve the 1D elastic collision for v_{2m} and v_m.

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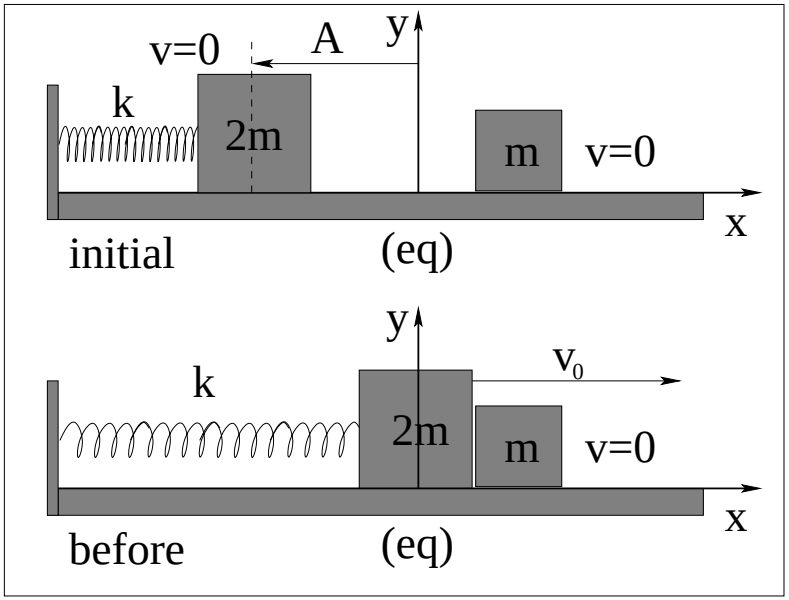

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Solution Map:
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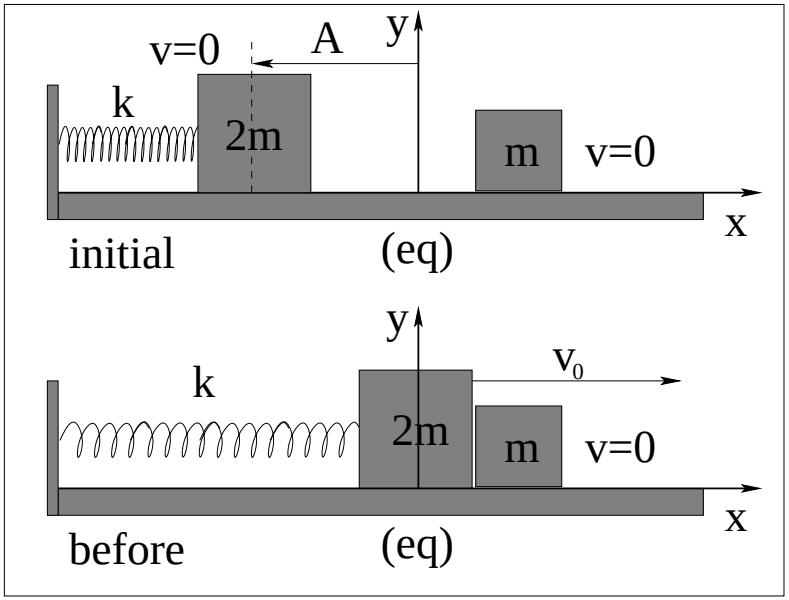
Use energy conservation again to find A'.

Spring-Driven Collision



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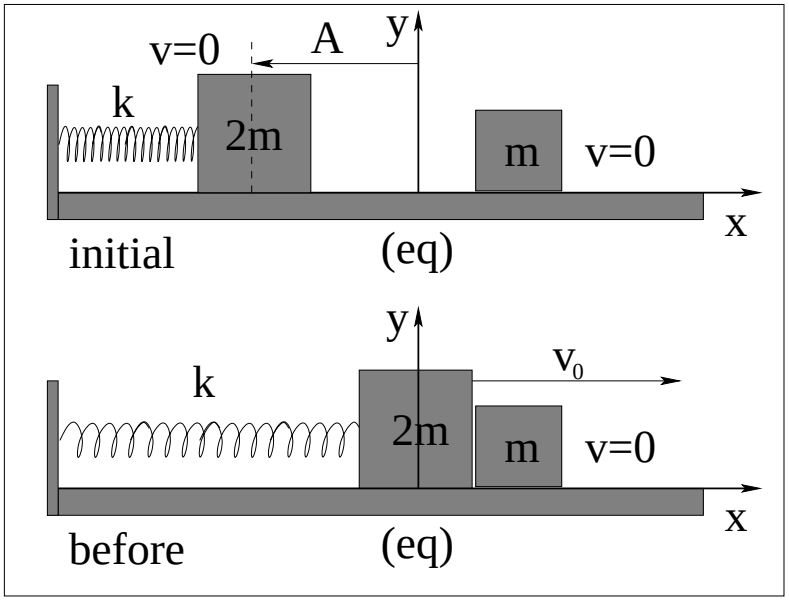

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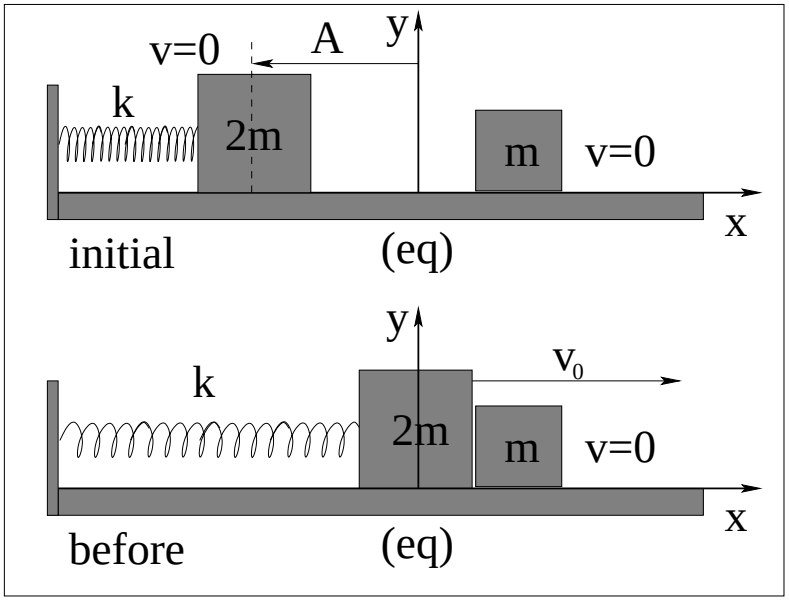
1D Elastic Collision Formula: $v_f = -v_i + 2v_{\rm cm}$ with $v_{\rm cm} = \dfrac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2}$

Spring-Driven Collision



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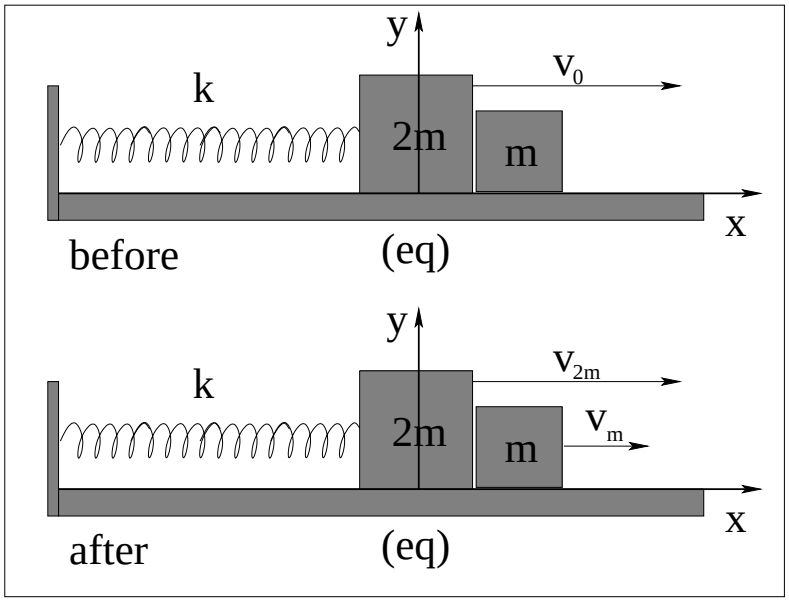
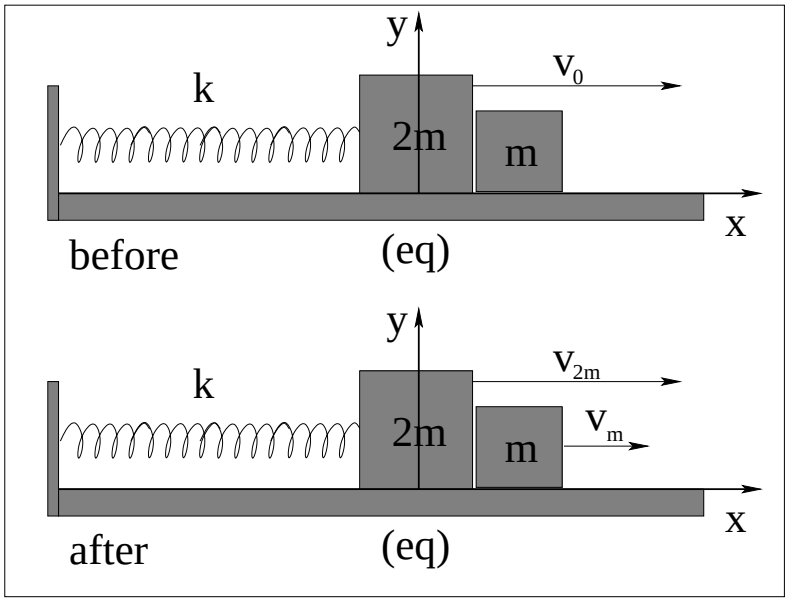
Spring-Driven Collision-Solution



Use mechanical energy conservation before the collision to find A:

$$\frac{1}{2}kA^2 = \frac{1}{2}(2m)v_0^2 \Rightarrow \boxed{A = \sqrt{\frac{2m}{k}}v_0}$$

Spring-Driven Collision-Solution



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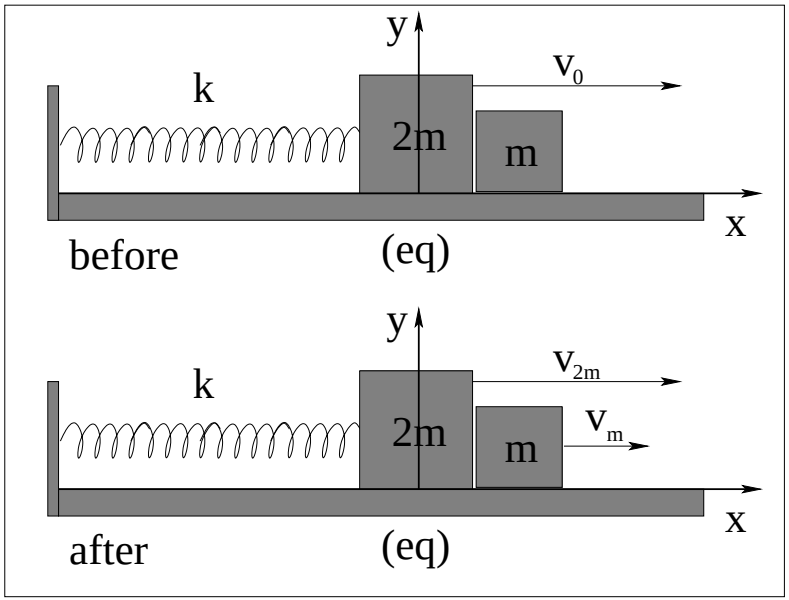

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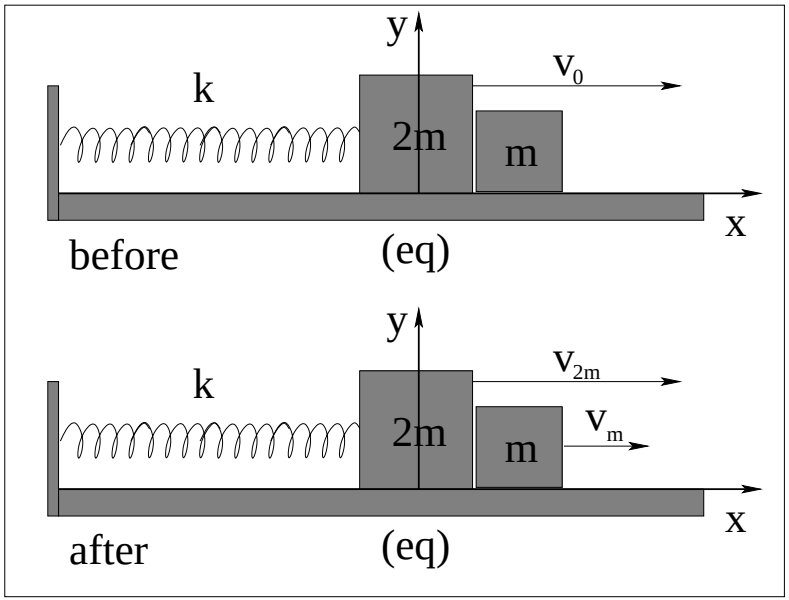
Solve the elastic collision:

$$v_{\rm cm} = \frac{2mv_0}{2m+m} = \frac{2v_0}{3} \Rightarrow \boxed{v_{2m} = \frac{v_0}{3} \qquad v_m = \frac{4v_0}{3}}$$

Spring-Driven Collision-Solution



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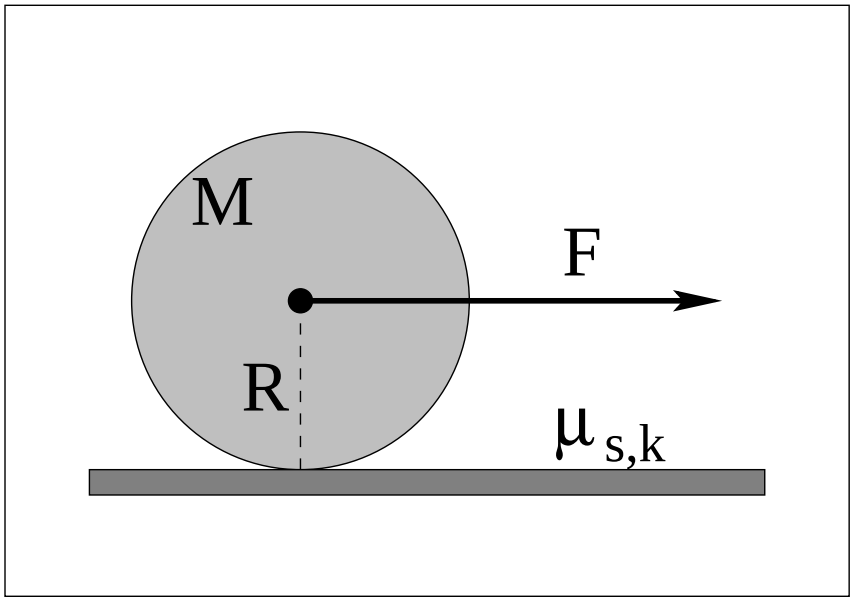
Use mechanical energy conservation again to find A':

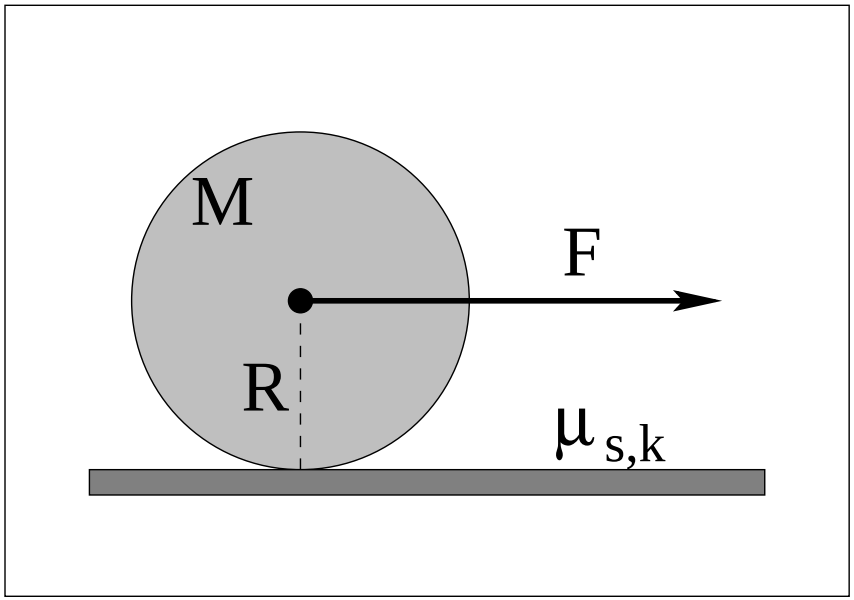
$$\frac{1}{2}(2m)v_{2m}^2 = \frac{1}{9}mv_0^2 = \frac{1}{2}kA'^2 \Rightarrow \boxed{A' = \sqrt{\frac{2m}{9k}}v_0}$$

Rolling with Slipping



- A disk with mass M, radius R, is sitting on a rough floor with coefficients of friction $\mu_{k,s}$ respectively. It is pulled by a force $F(t) = At$ that **increases linearly with time** (A is a constant with units N/sec). Write an expression for $f_s(t)$, the **magnitude** of the force of static friction as a function of time. Find the time t_s that the disk starts to slip instead of roll without slipping.

Rolling with Slipping



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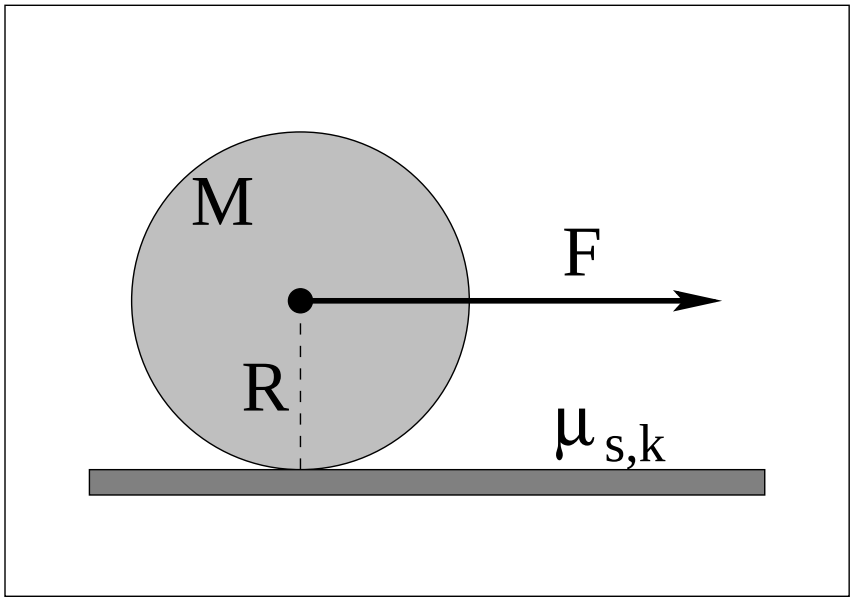

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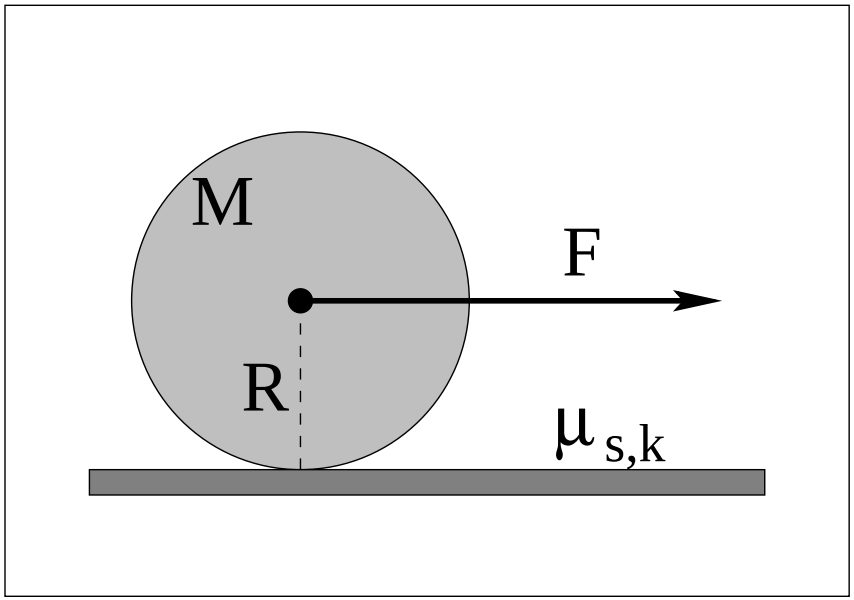

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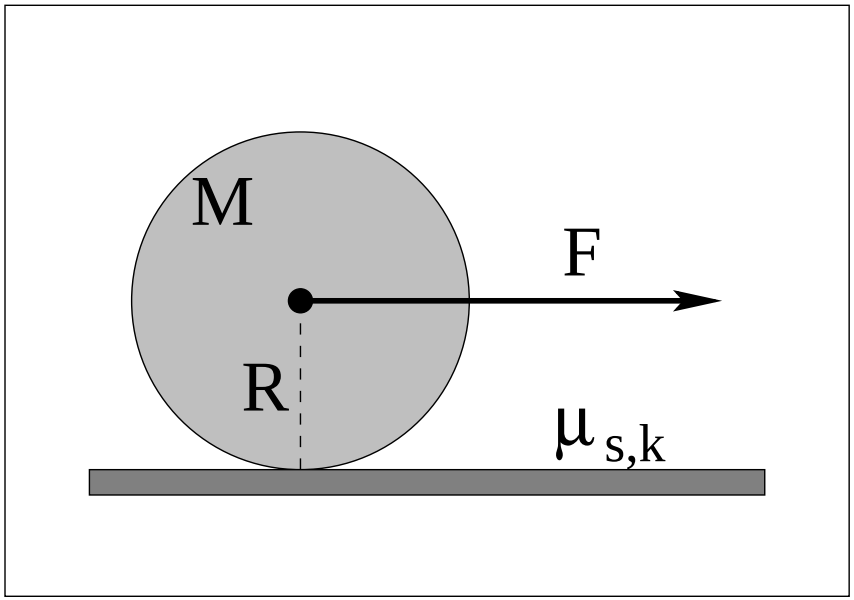

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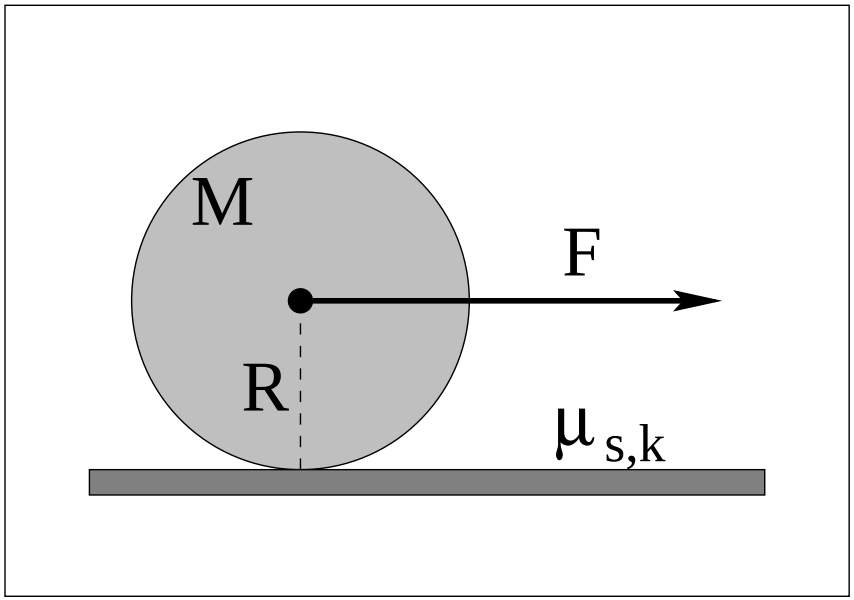

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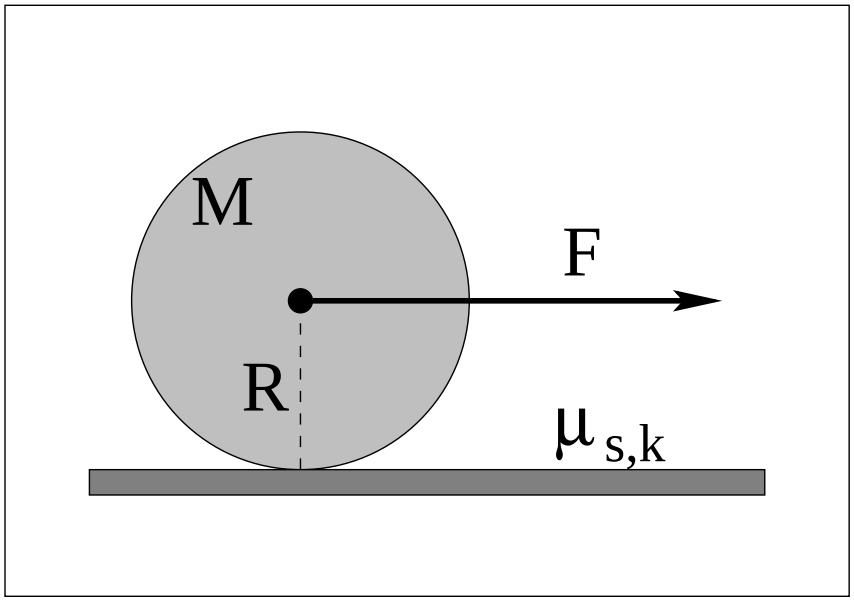

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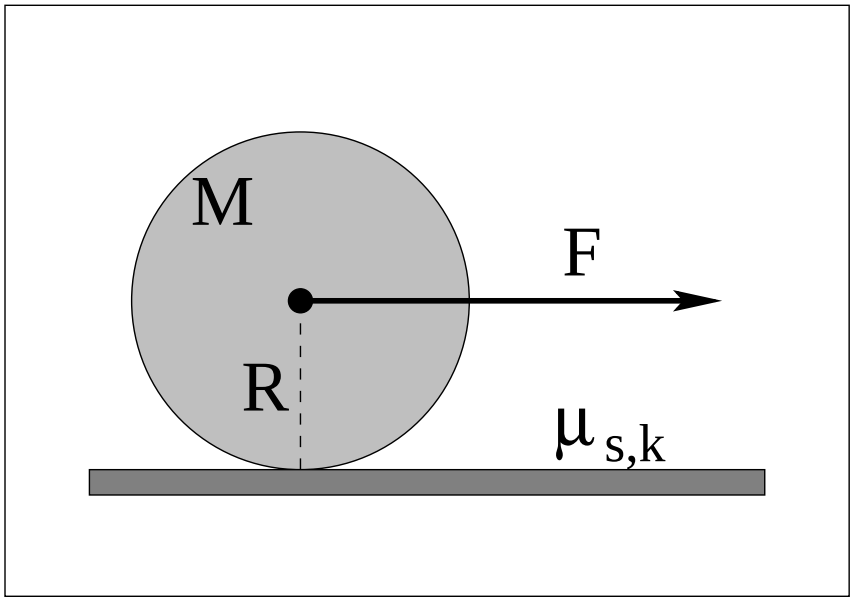

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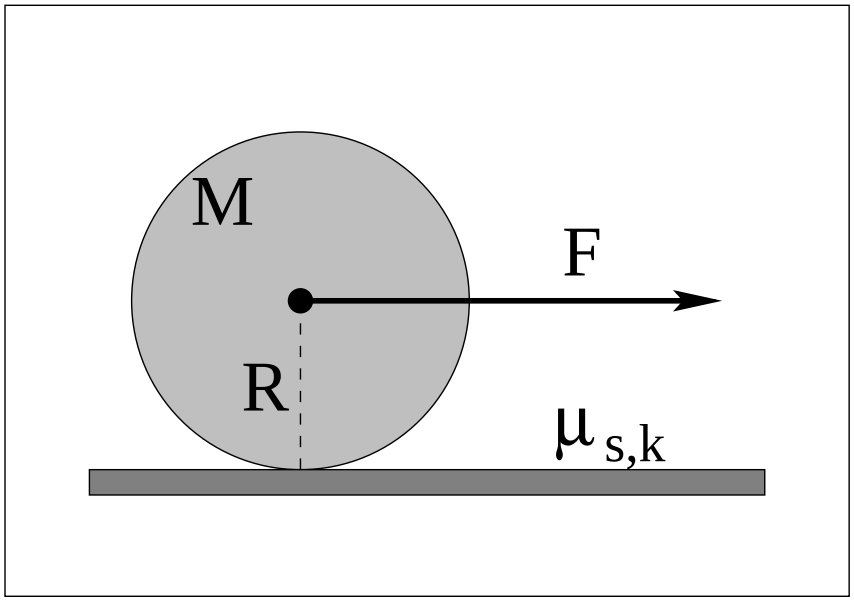

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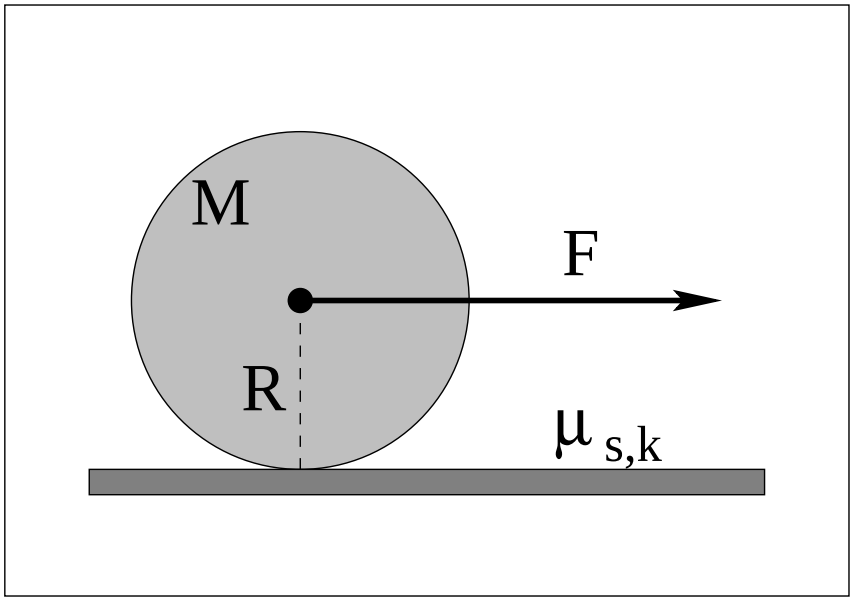
* - The solution is on the next page. Don't advance until you are ready!

Rolling with Slipping-Solution



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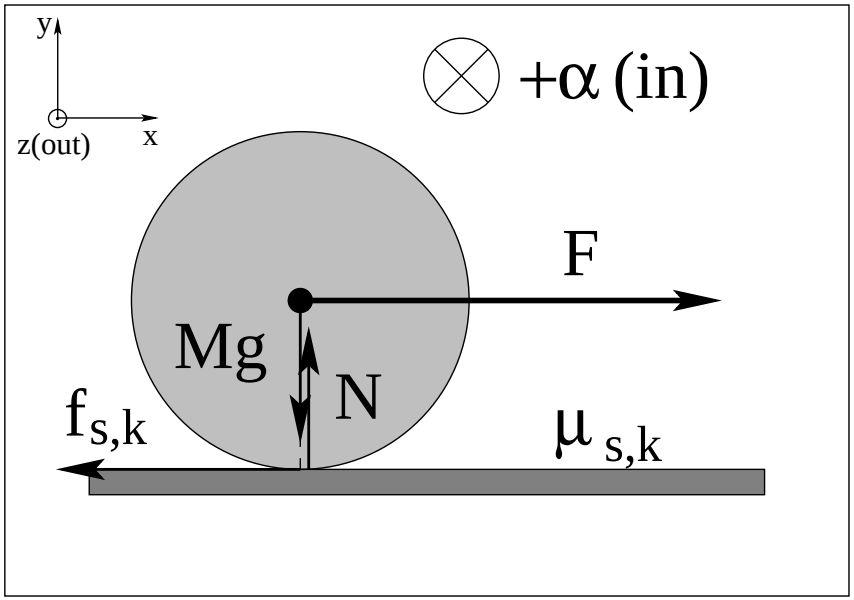

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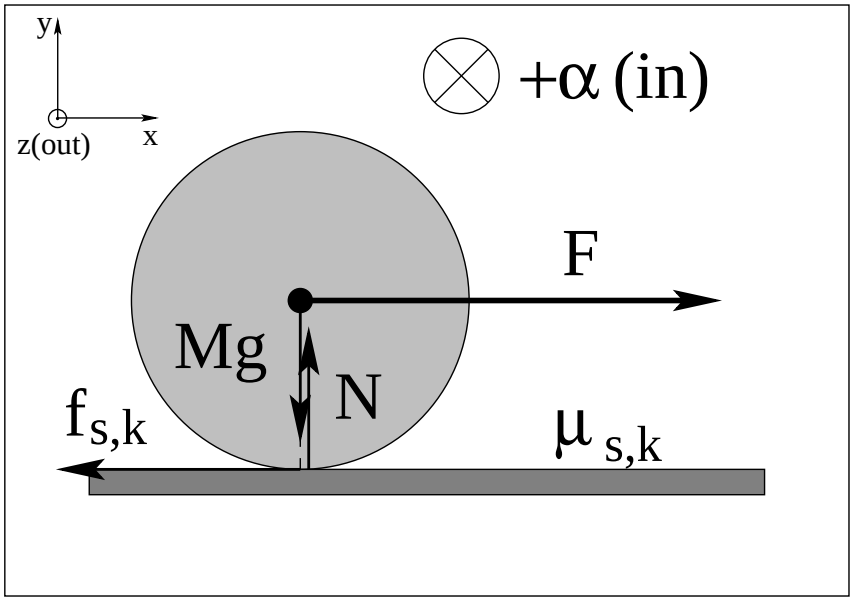

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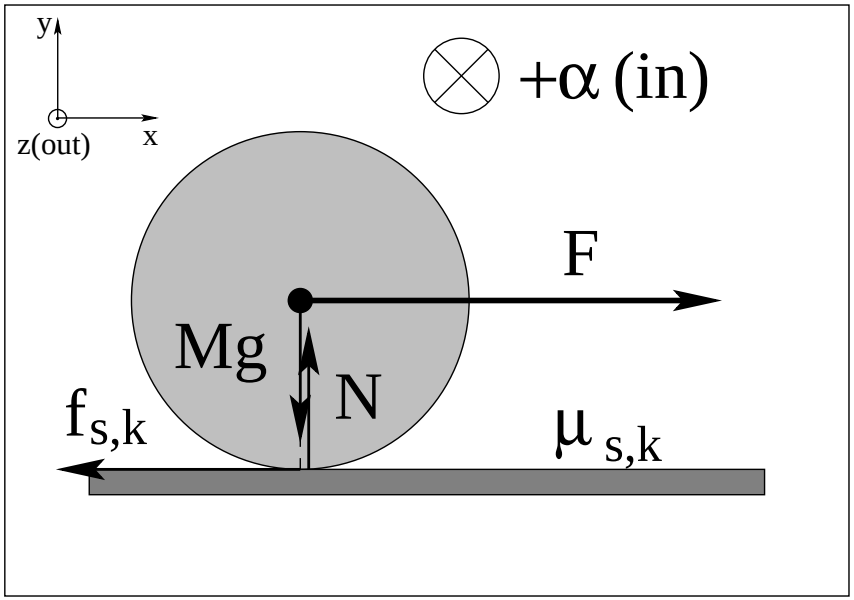

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$$f_s = \frac{1}{3} At < \mu_s N = \mu_s Mg \Rightarrow \frac{1}{3} At_s = \mu_s Mg \Rightarrow \boxed{t_s = \frac{3\mu_s Mg}{A}}$$

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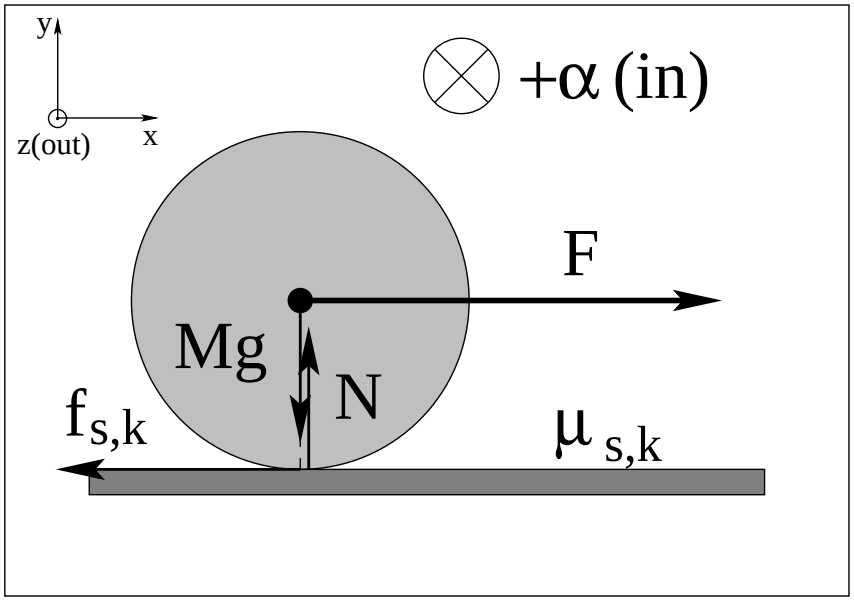

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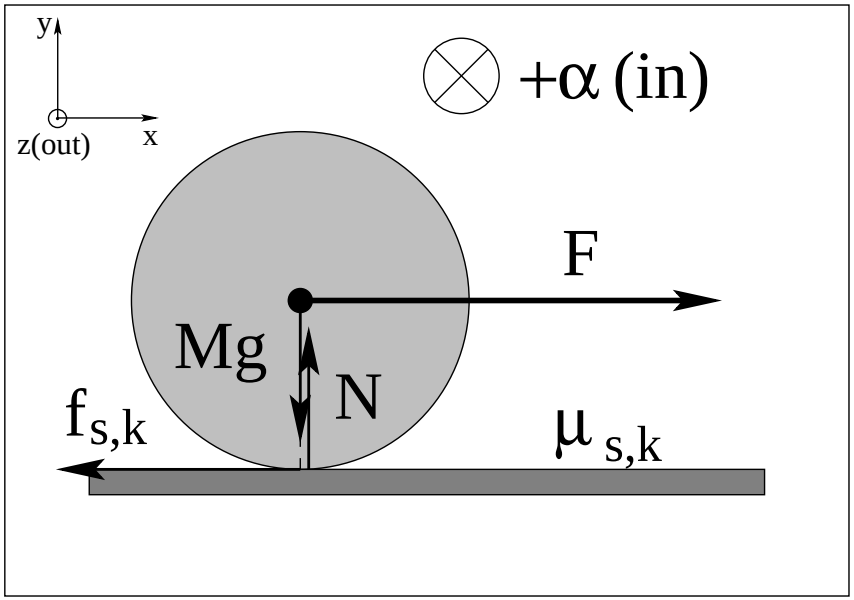

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$$F - \mu_k Mg = Ma' \Rightarrow \boxed{a' = \frac{At}{M} - \mu_k g} \qquad Rf_k = R\mu_k Mg = \frac{1}{2}MR^2\alpha' \Rightarrow \boxed{\alpha' = \frac{\mu_k g}{R}}$$

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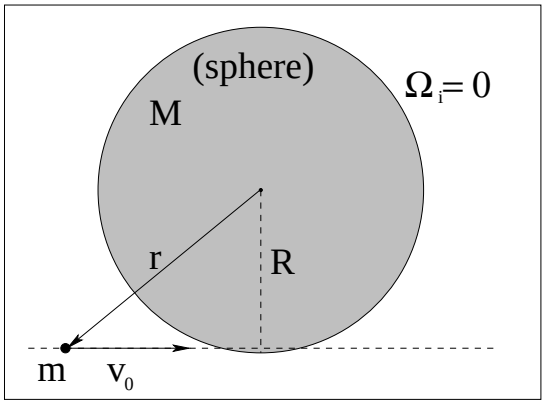
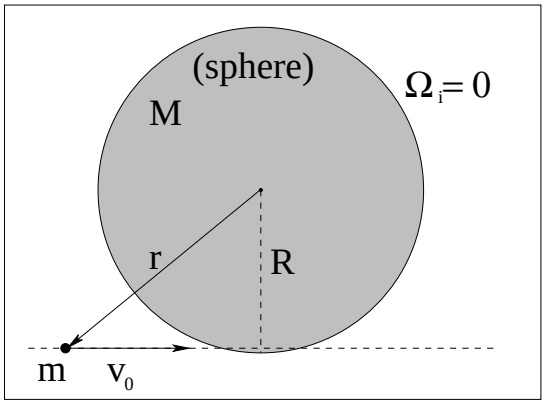
- Note Well! We can solve for everything using just F for the applied force and substitute $F = At$ only at the end when we need it!
- And *note above all* that $\boldsymbol{f_s = \frac{1}{3} At \neq \mu_s N}$. We *only* use $\mu_s N$ **to find the (critical) slipping point, never to find f_s itself!**

A Bullet Grazes a Sphere



A small bullet of mass m fired at an initial speed of v_0 grazes a free (unpivoted!) sphere sphere of mass M and radius R ($I = \frac{2}{5}MR^2$) grazes the surface of the sphere at its equator and emerges travelling along the same line at the reduced speed $v_1 = v_0/2$.

A Bullet Grazes a Sphere



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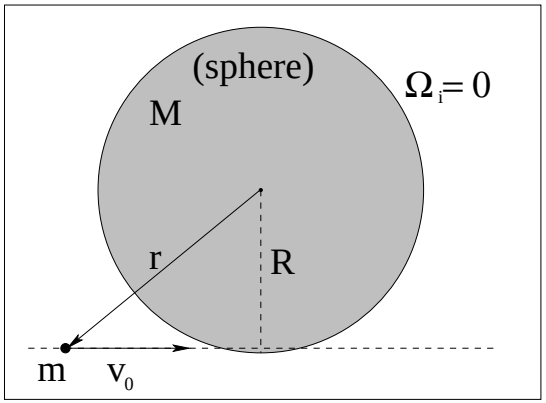

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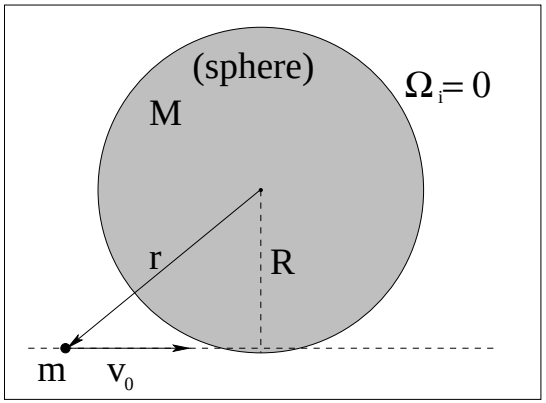

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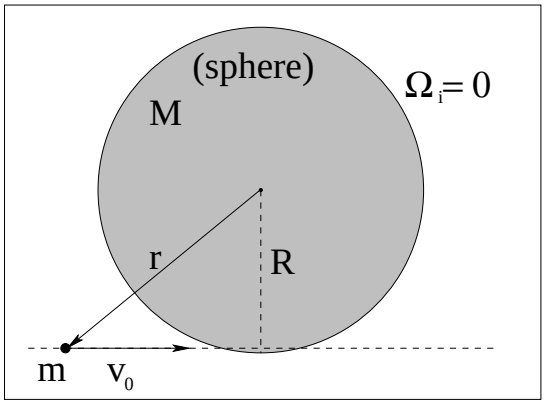

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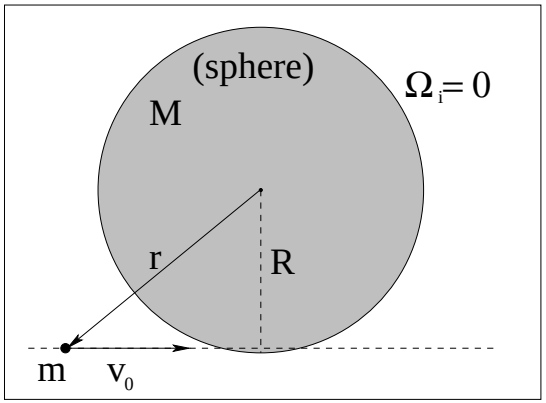

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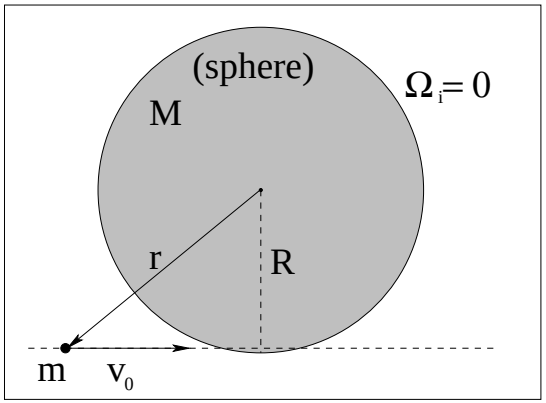

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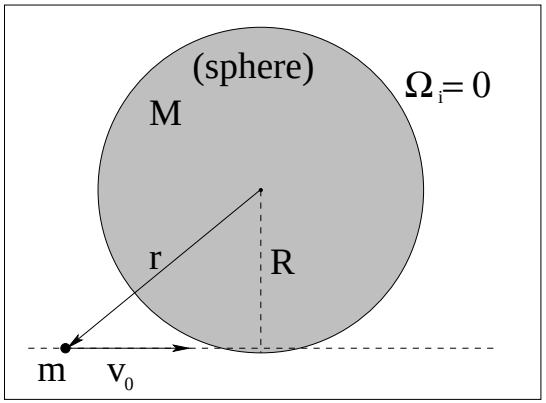

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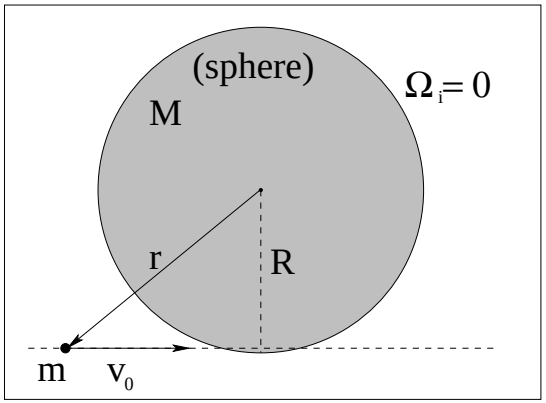
A Bullet Grazes a Sphere



A small bullet of mass m fired at an initial speed of v_0 grazes a free (unpivoted!) sphere sphere of mass M and radius R ($I = \frac{2}{5}MR^2$) grazes the surface of the sphere at its equator and emerges travelling along the same line at the reduced speed $v_1 = v_0/2$.

Find: Ω_f the angular velocity of the sphere after the collision; the velocity v_f of the center of mass of the sphere after the collision; the energy lost during the collision. Is there any value of m (relative to M) for which this collision is elastic?

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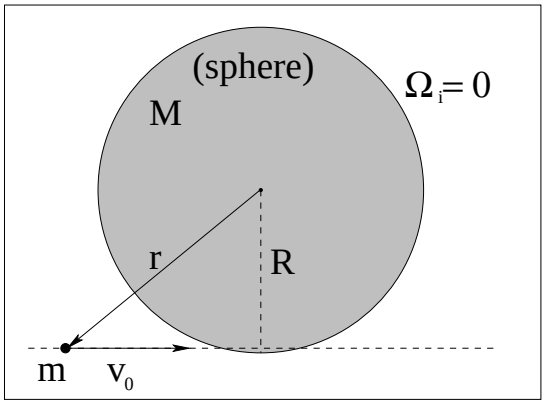

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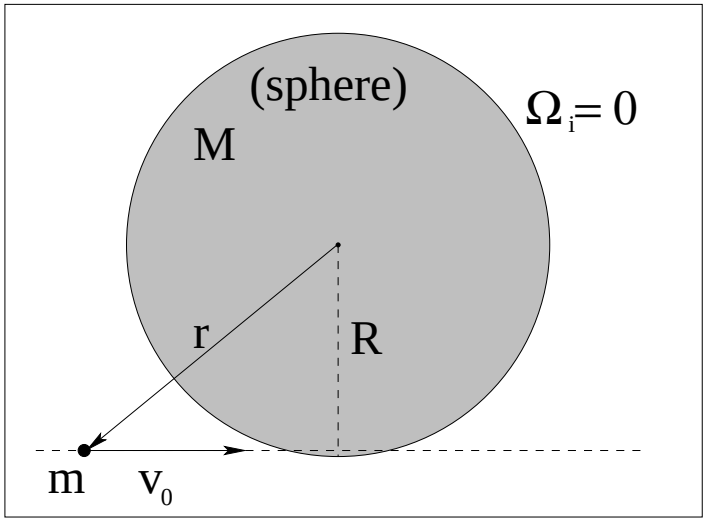
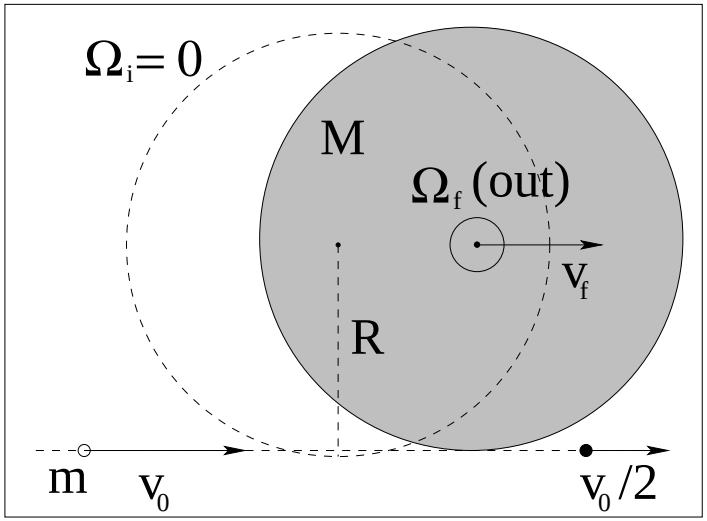


Start by drawing/visualizing the final state of the rotating sphere.

A Bullet Grazes a Sphere-Solution



Start by drawing/visualizing the final state of the rotating sphere. ($v_1 = v_0/2$ and x-directed motion only). Then:

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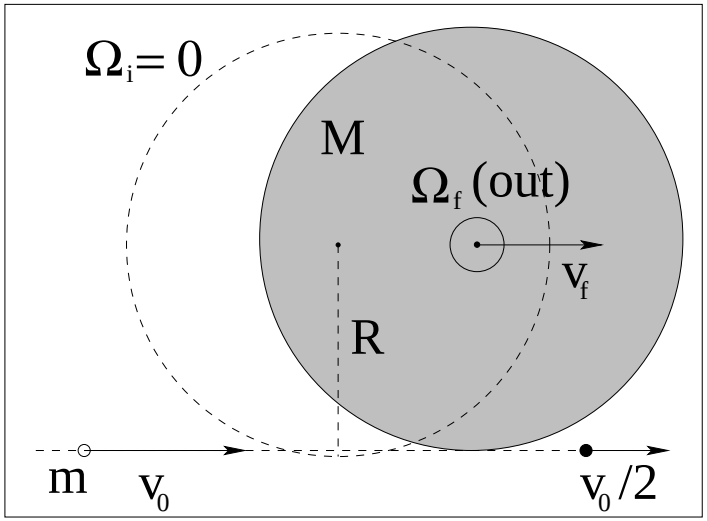

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$$p_i = mv_0 = mv_1 + Mv_f = p_f \Rightarrow \boxed{v_f = \frac{m}{2M}v_0}$$

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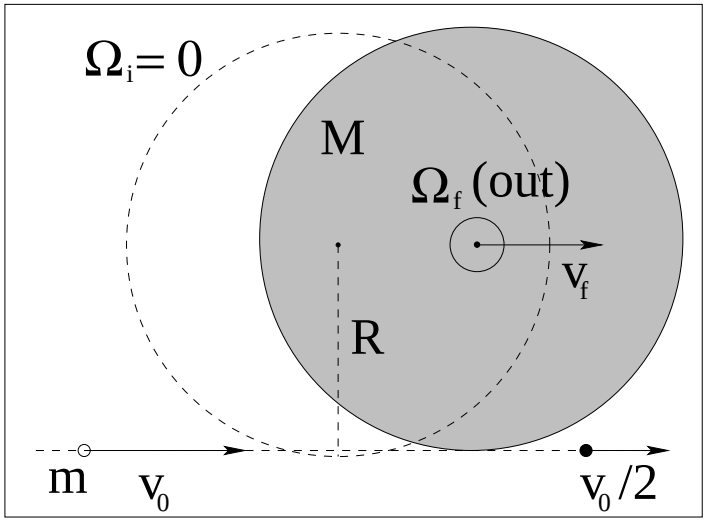

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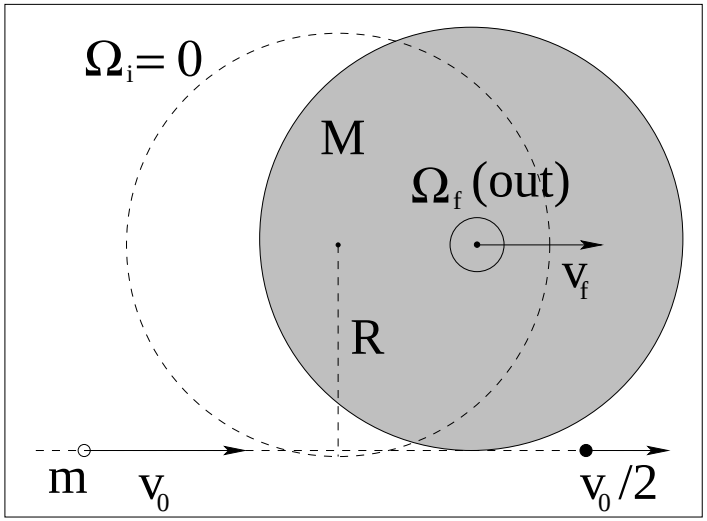

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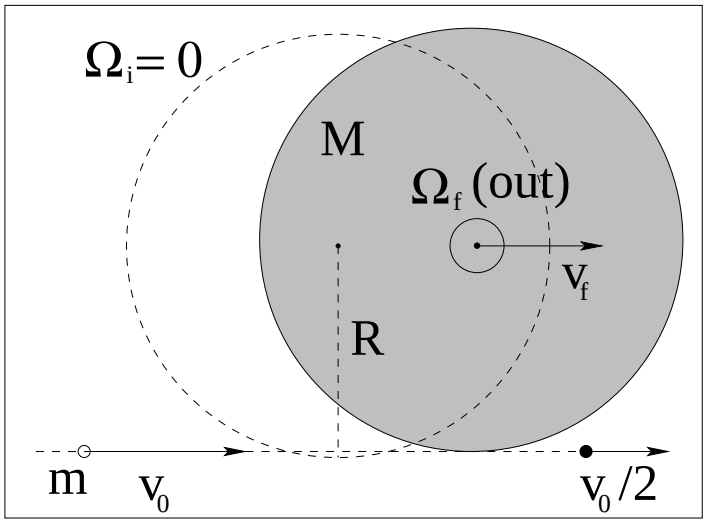

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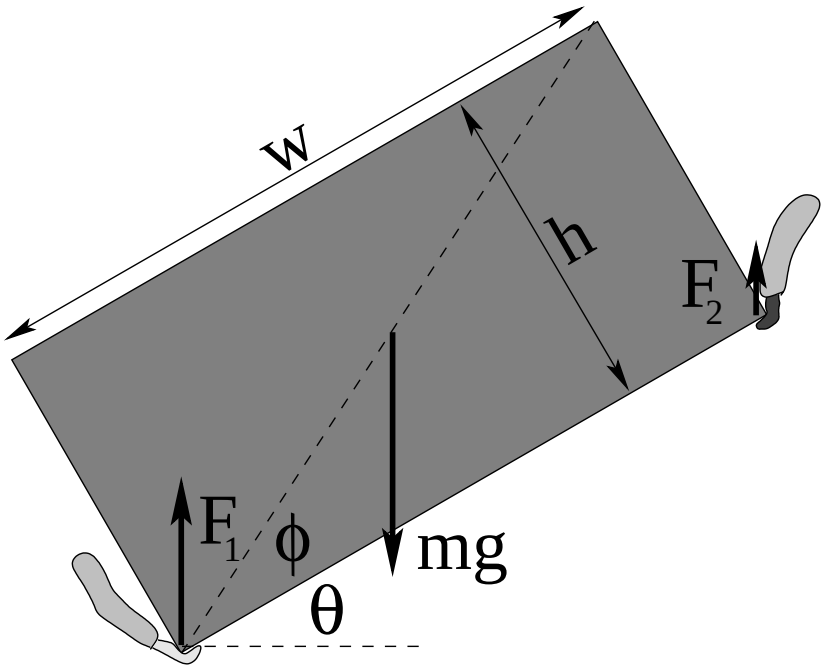
$$\boxed{\Delta E = \left(\left\{\frac{1}{4} + \frac{7m}{8M}\right\} - 1\right)\frac{1}{2}mv_0^2} \text{ and } \Delta E = 0 \text{ if } \boxed{m = \frac{6}{7}M}$$

Static Equilibrium: Carrying A Box Up the Stairs



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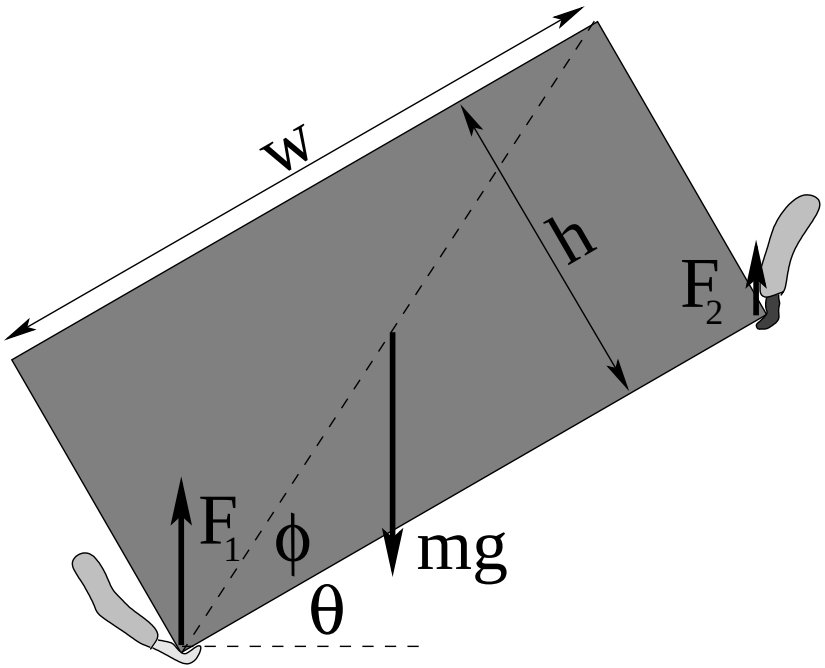

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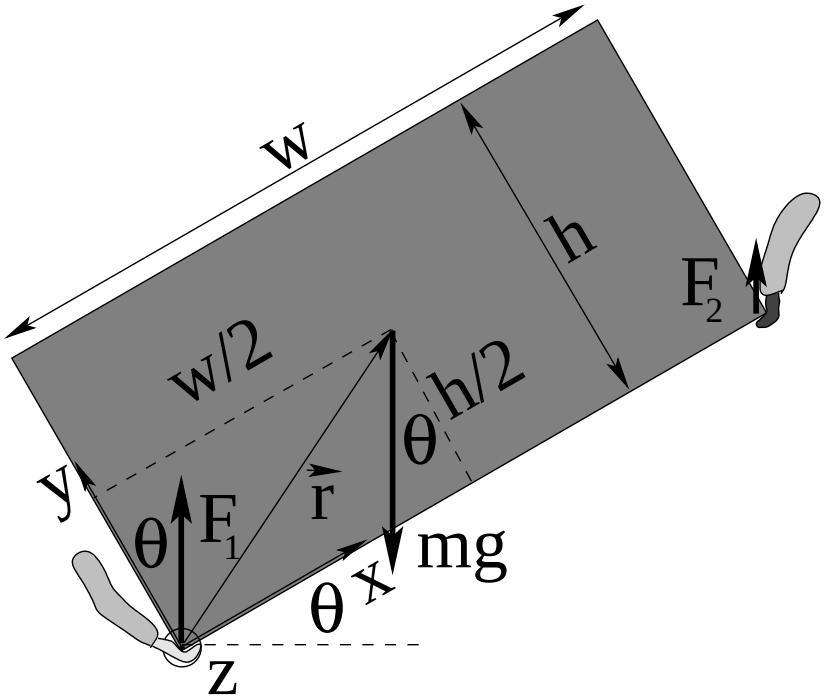

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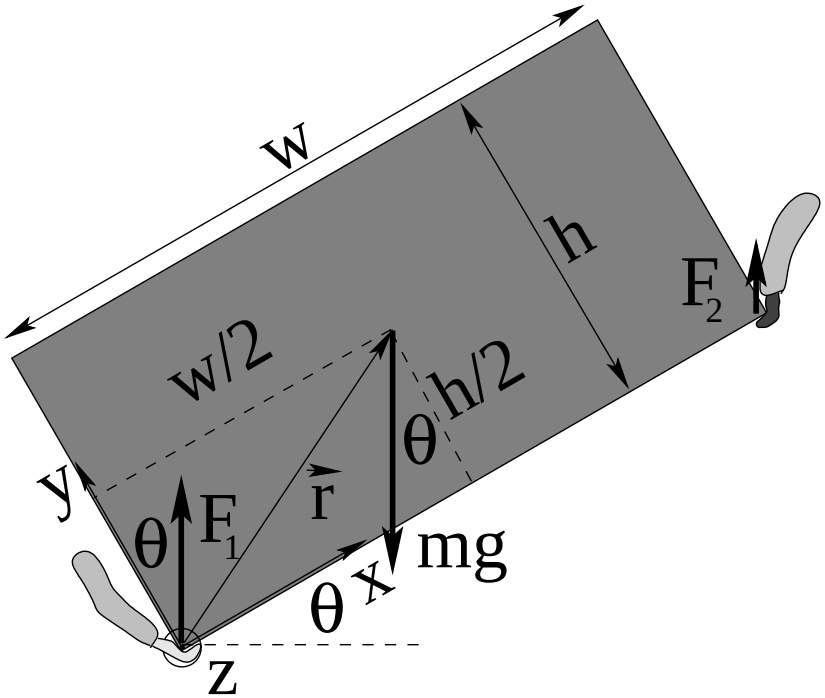

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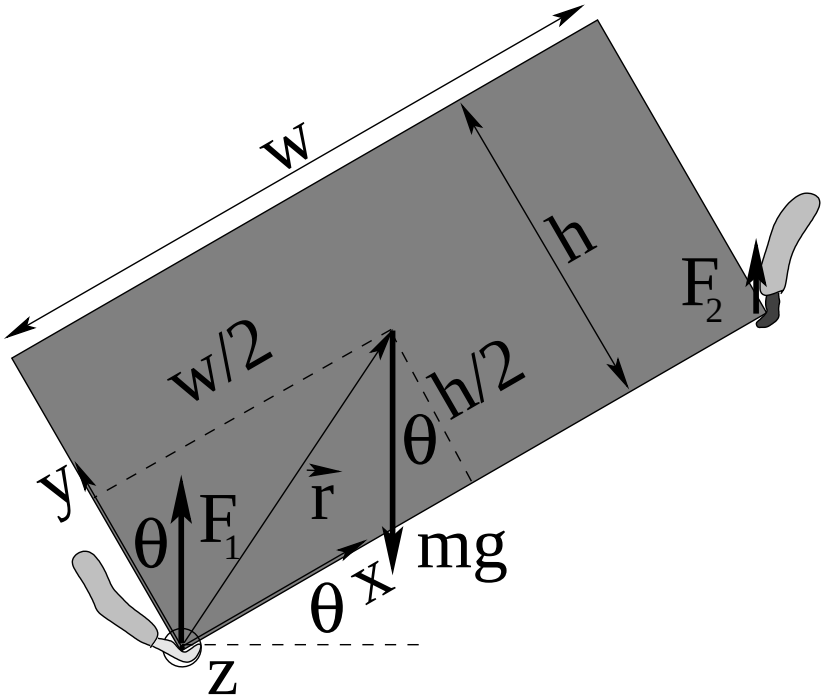

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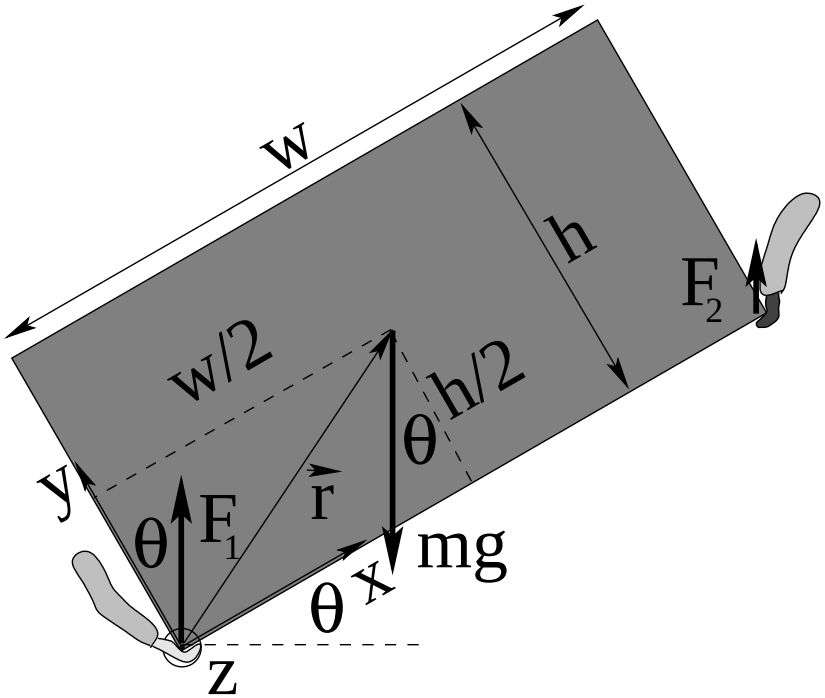

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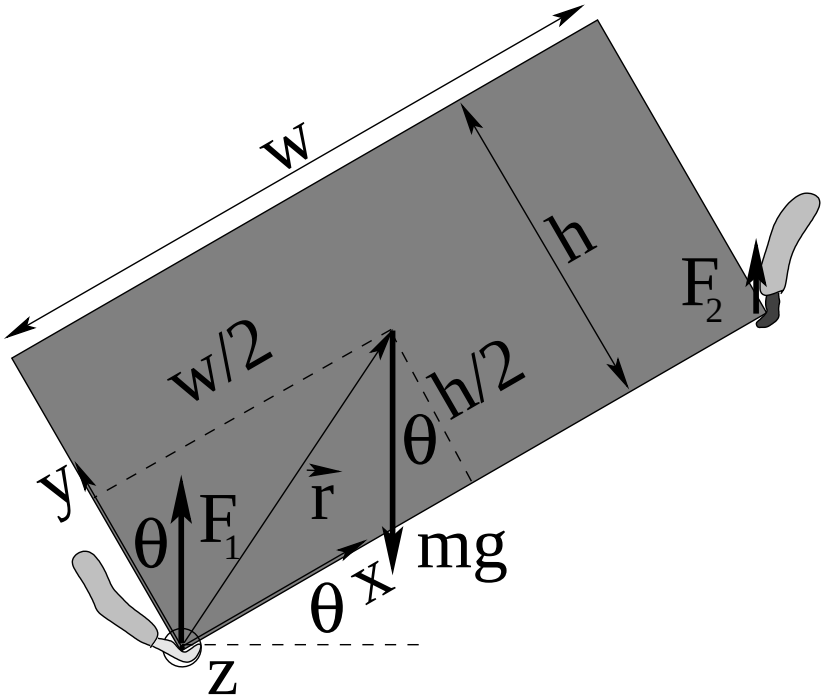

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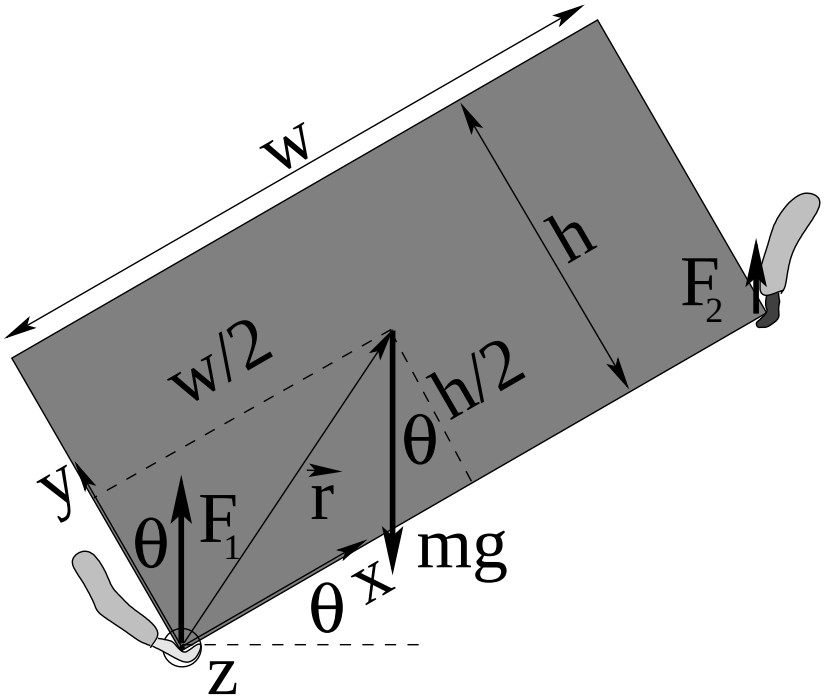

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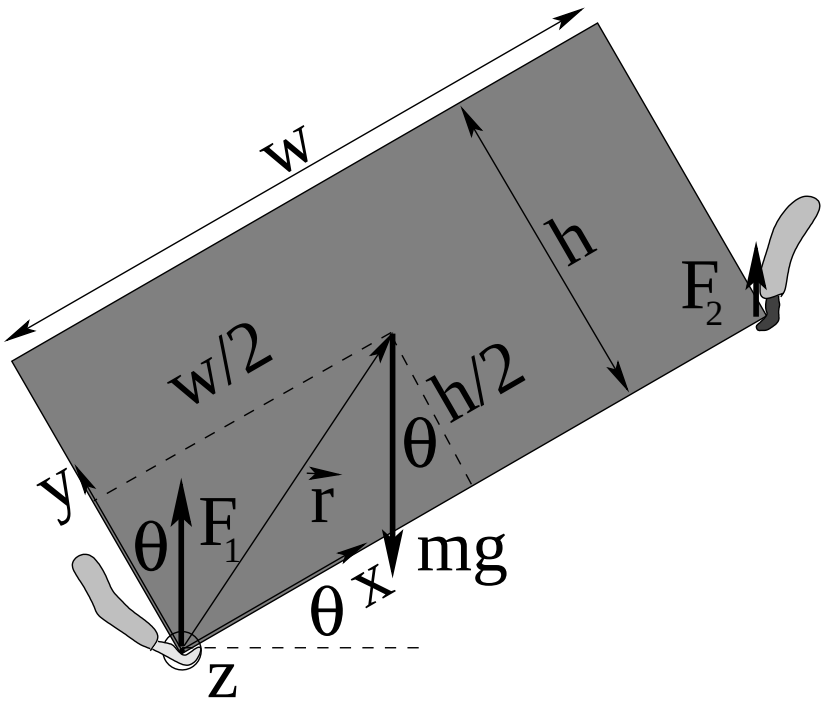

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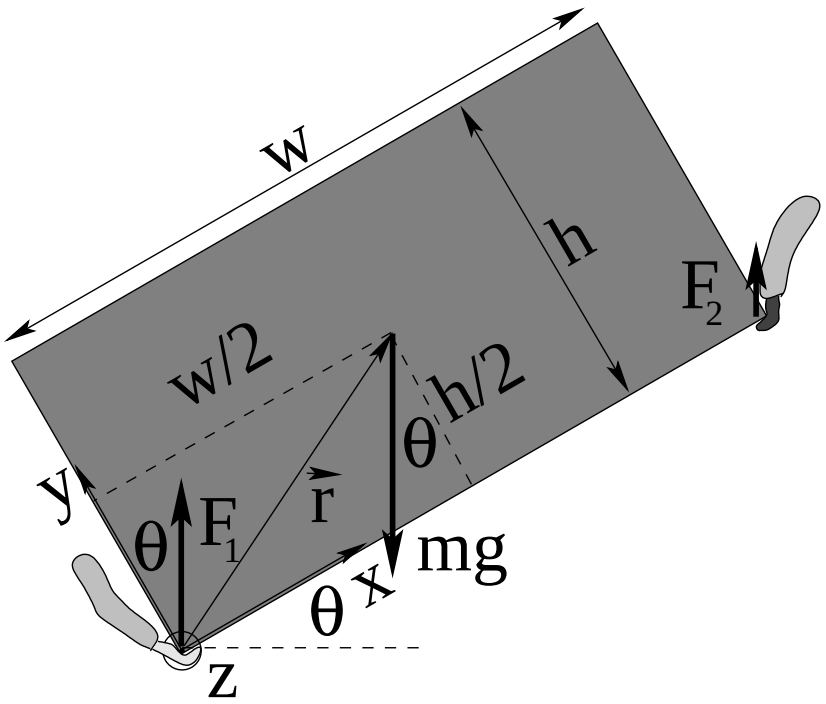

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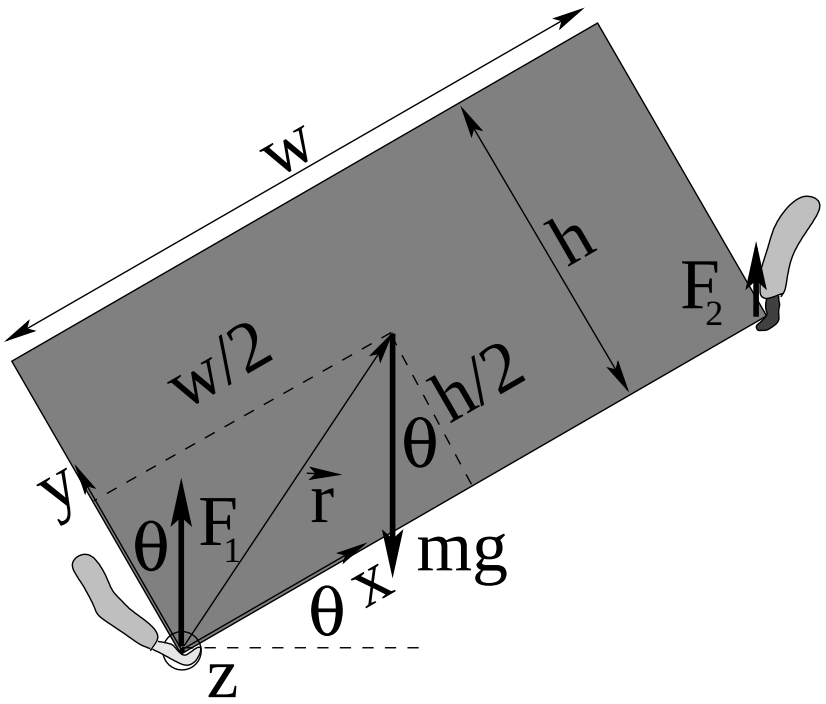

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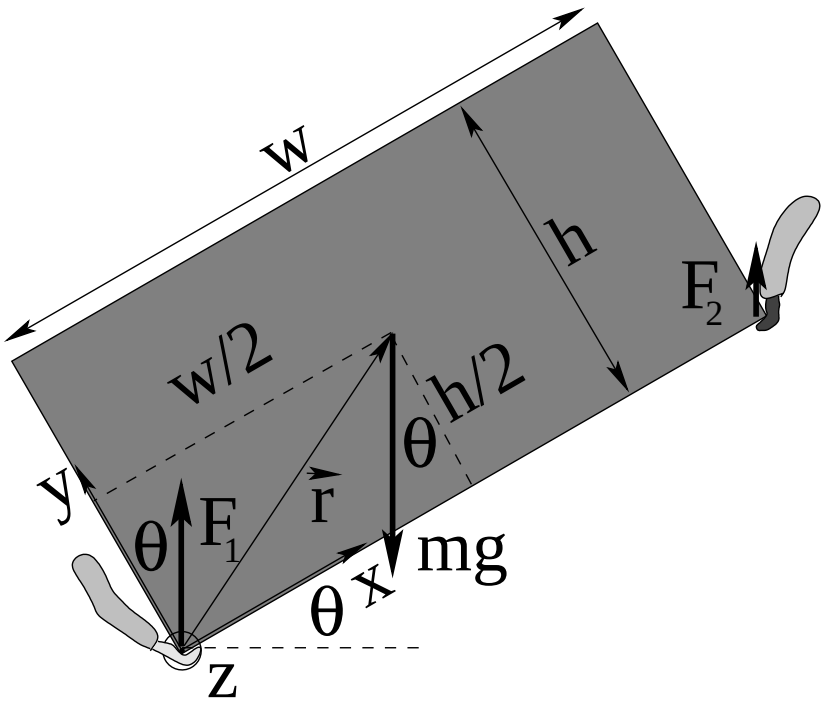

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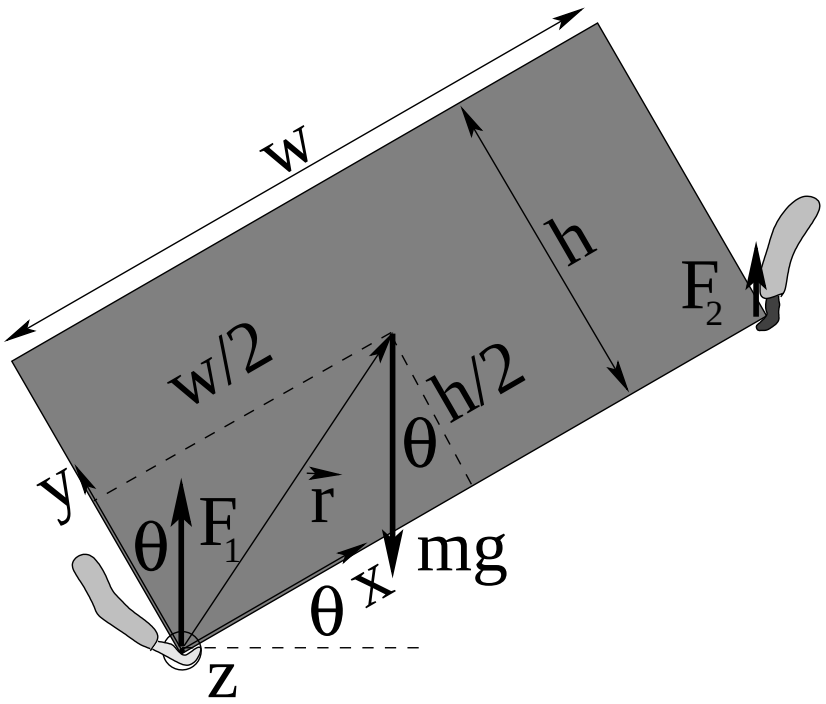

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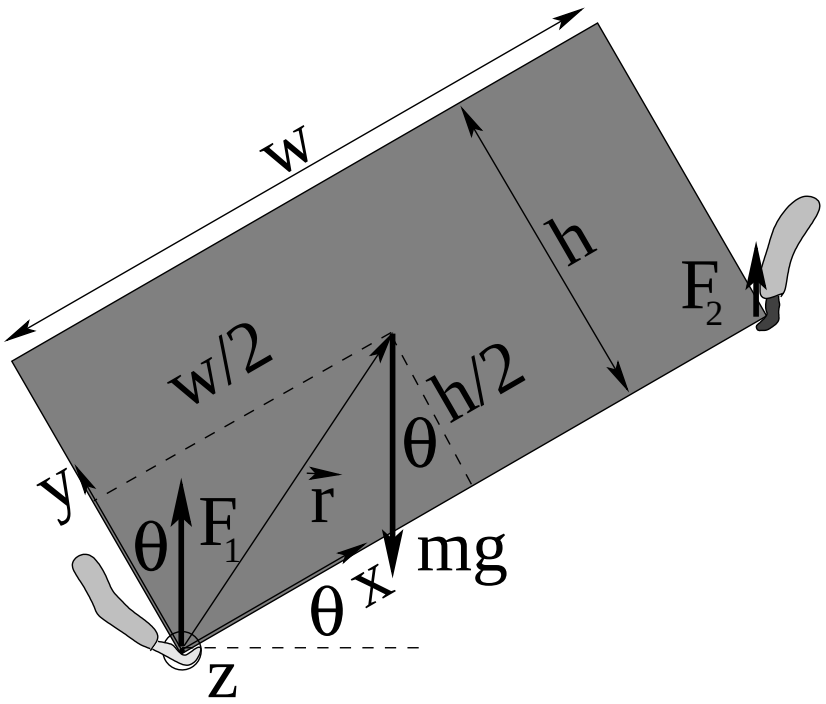

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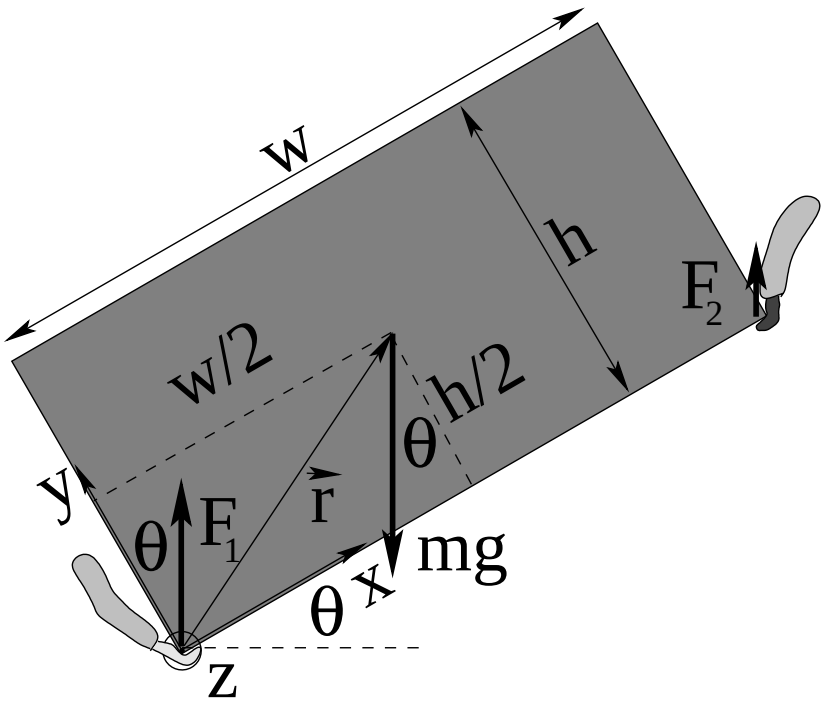

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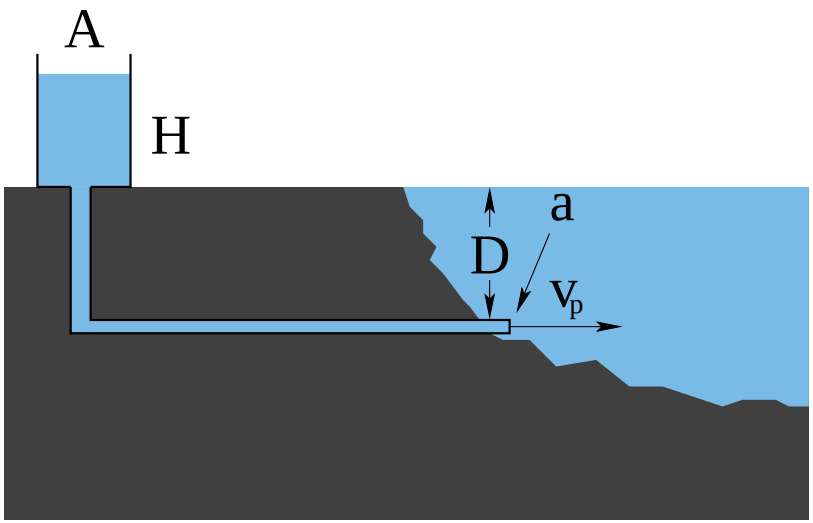
Note: $F_1 > F_2$ for all $\theta \in (0, \pi/2)$. It's better to be the student at the top! Note: $F_1 = mg$ when $\theta = \tan^{-1}\frac{w}{h}$. At this point $F_2 = 0$ and the student at the top is just helping balance the load!

Draining into a Lake



- The figure shows a rain-barrel that drains into a nearby lake through an underground pipe. The cross sectional area of the pipe is a, the top of the barrel has area $A \gg a$, the water has a height H in the barrel, and the pipe enters the lake at a depth D. Find v_p, the speed with which the water drains into the lake through the pipe, and the rate of flow through the pipe. Lake and barrel are obviously open to air pressure.

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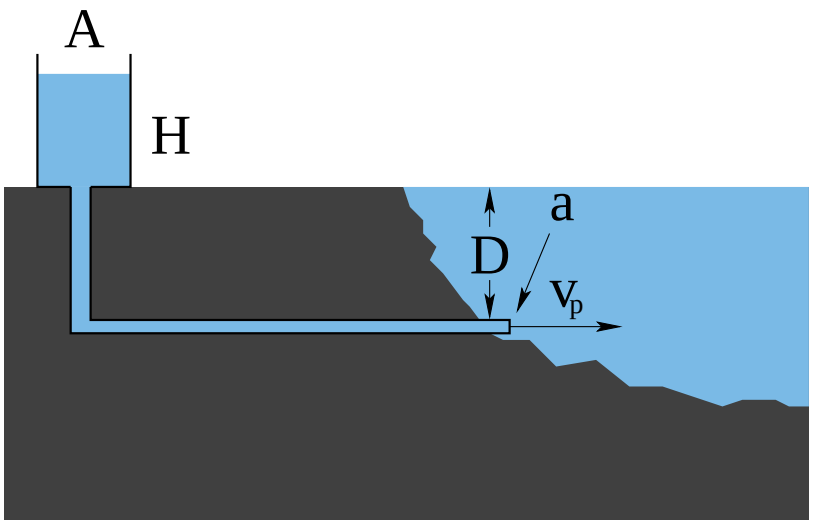

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Pick good coordinates – the y origin at the height of the pipe opening, for example.

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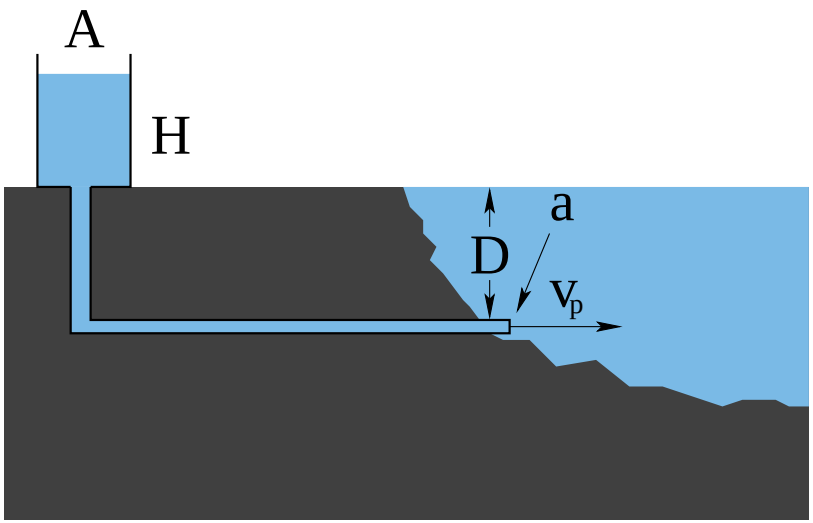

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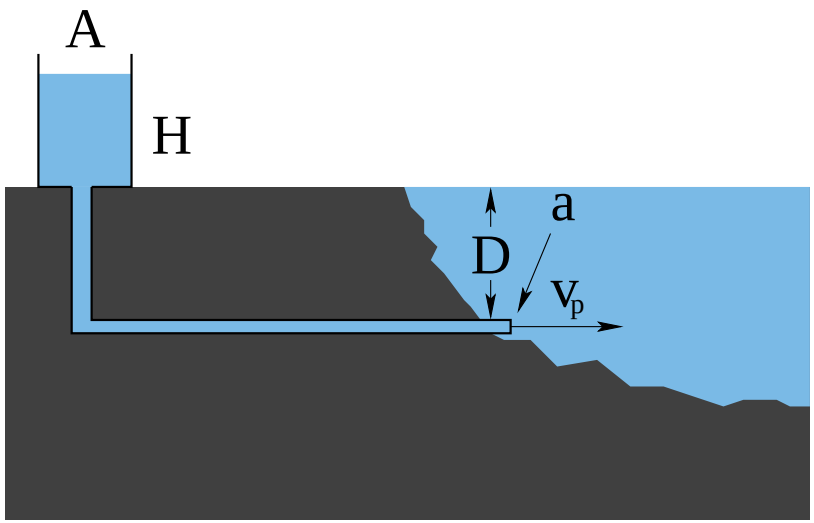
Draining into a Lake



- The figure shows a rain-barrel that drains into a nearby lake through an underground pipe. The cross sectional area of the pipe is a, the top of the barrel has area $A \gg a$, the water has a height H in the barrel, and the pipe enters the lake at a depth D. Find v_p, the speed with which the water drains into the lake through the pipe, and the rate of flow through the pipe. Lake and barrel are obviously open to air pressure.

Pick good coordinates – the y origin at the height of the pipe opening, for example. This is a Bernoulli's Formula problem, so start by writing it down. You'll also need conservation of flow at the top of the barrel and the opening of the pipe.

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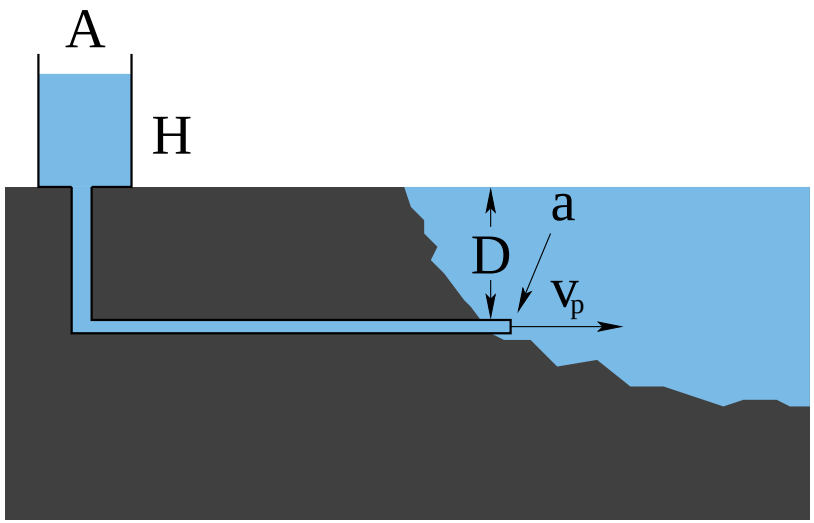

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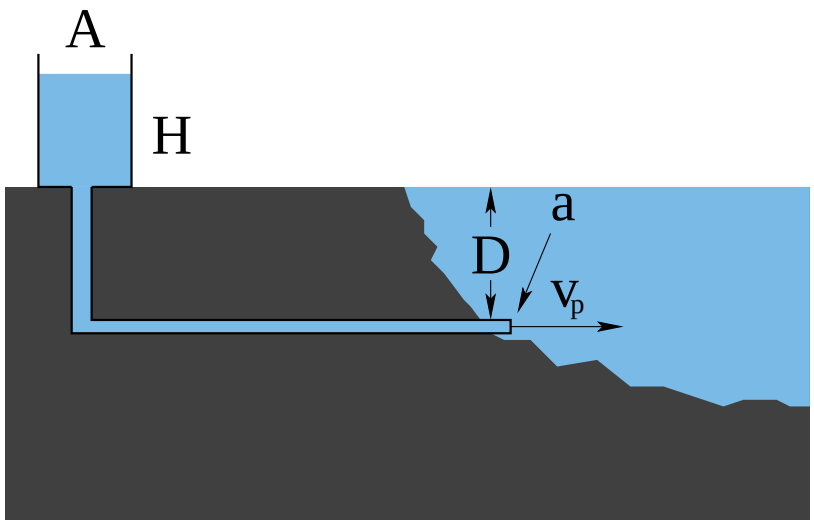

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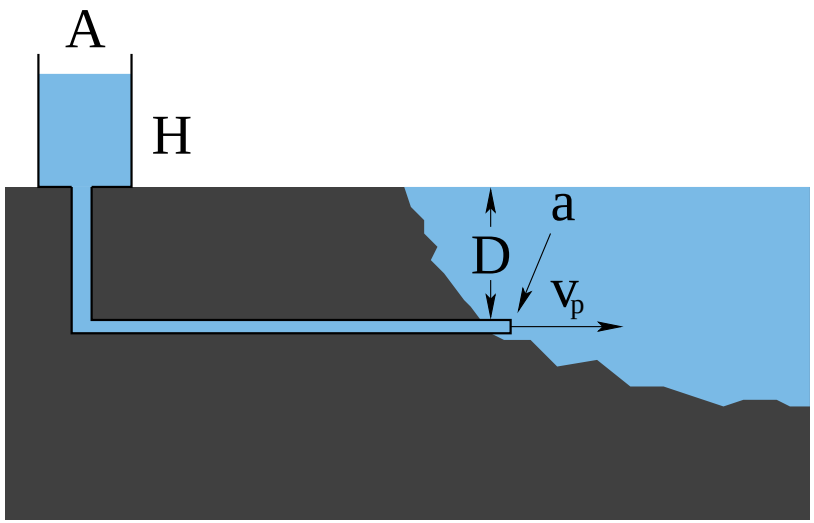

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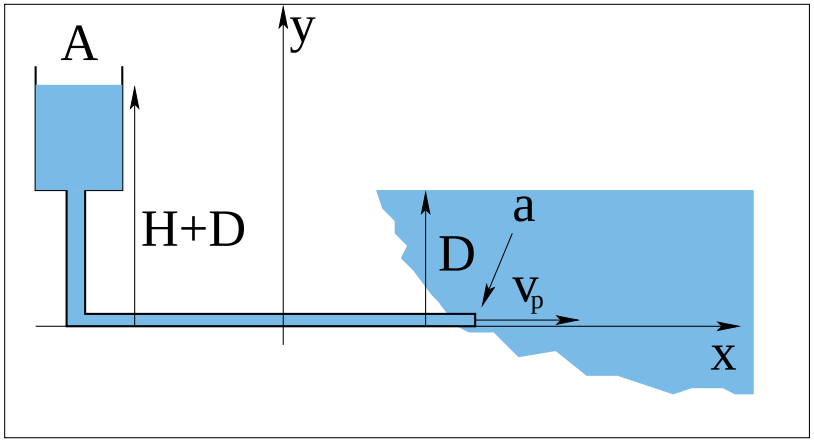

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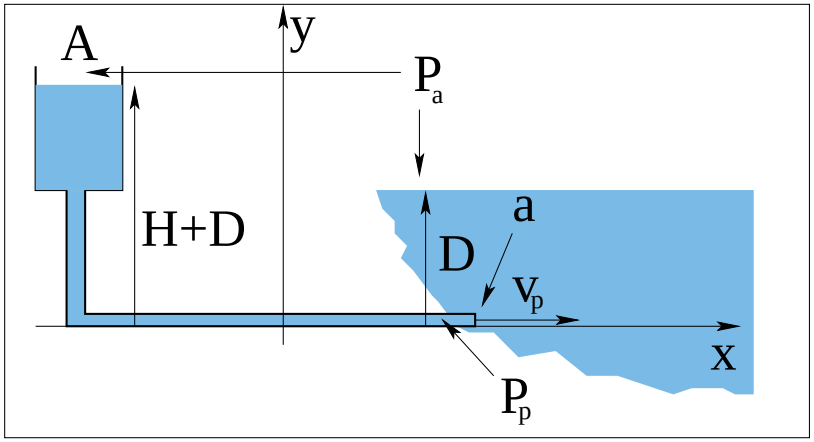
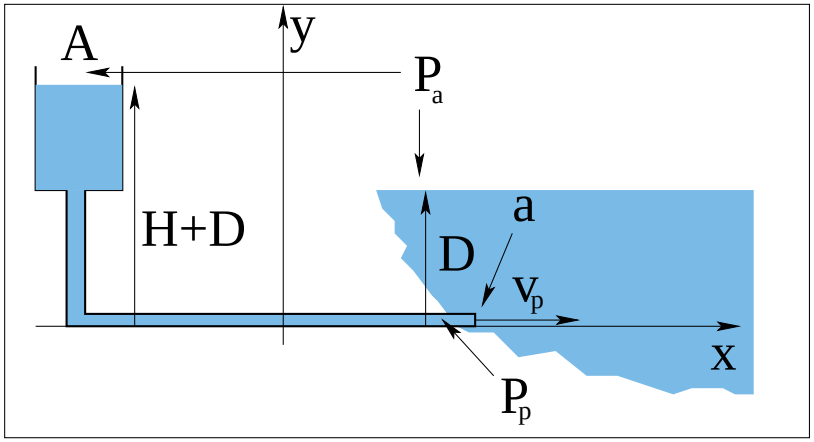


Choose coordinates like those shown.
Evaluate $P_p = P_a + \rho g D$.

Draining into a Lake-Solution



Choose coordinates like those shown. Evaluate $P_p = P_a + \rho g D$. Apply Bernoulli Formula:

$$P_a + \rho g(H + D) + \frac{1}{2}\rho v_b^2 = P_p + \frac{1}{2}\rho v_p^2$$

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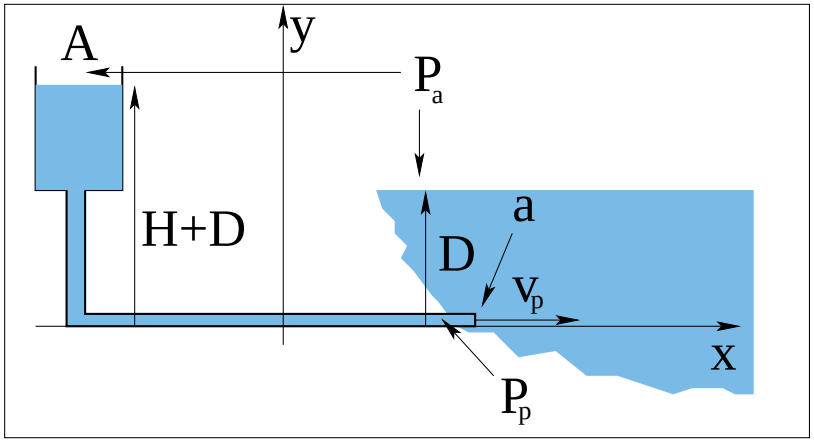

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or (neglecting $\frac{1}{2}\rho v_b^2$ because $v_b \ll v_p$ because $A \gg a$ – Torricelli assumption):

$$\frac{1}{2}\rho v_p^2 = P_a + \rho g H + \rho g D - P_a - \rho g D = \rho g H \Rightarrow \boxed{v_p = \sqrt{2gH}}$$

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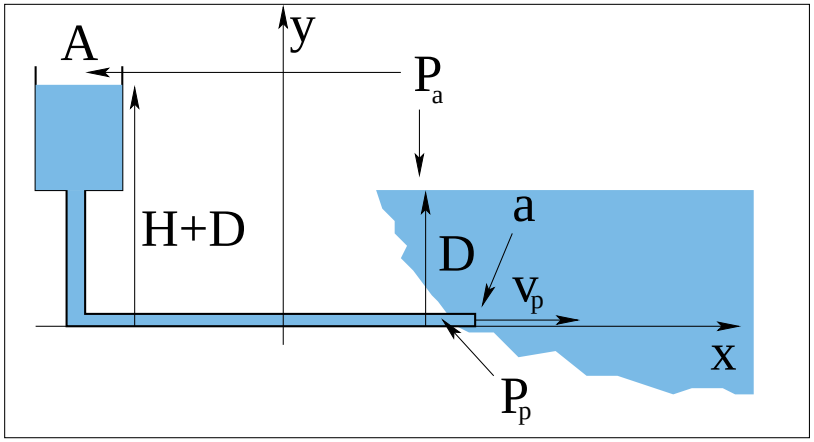

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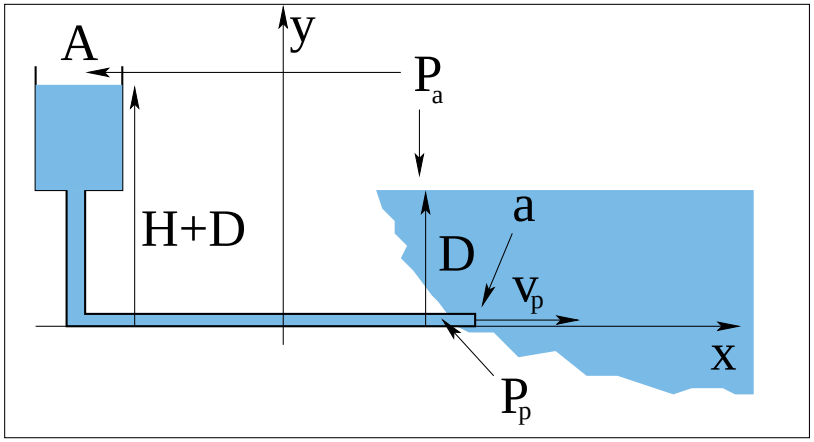

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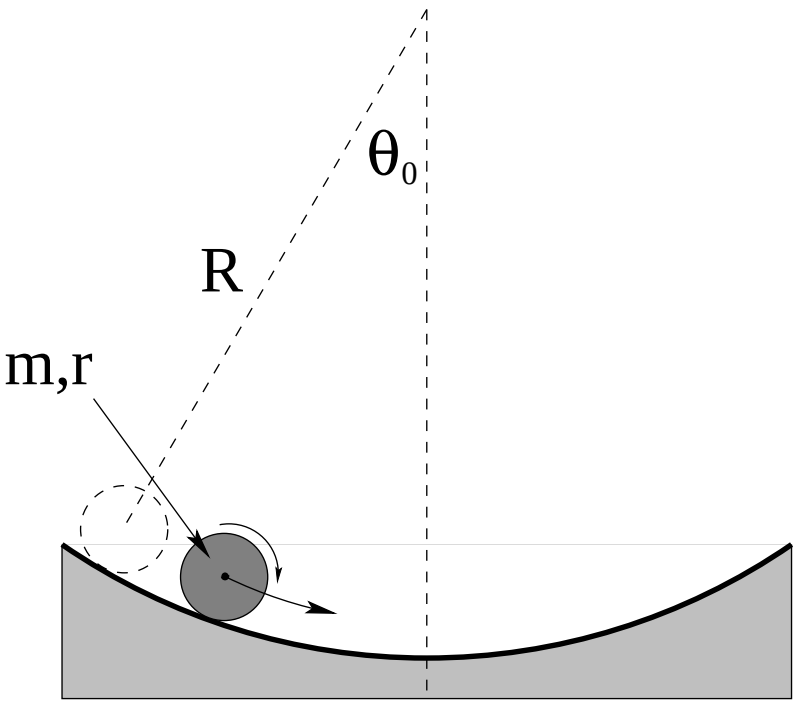
The flow is then easy:

$$\boxed{I_p = v_p a = a\sqrt{2gH}}$$

where the speed in the pipe is much larger than the speed of water in *either* the rainbarrel *or* the body of the lake.

Oscillations: A Rolling Pendulum



- A disk of mass m and radius r is gently set on a rough circular floor so that it makes an angle θ_0 relative to a vertical through the center of curvature of the floor, with its center of mass a distance R from the center of curvature as shown, and is released from rest at $t = 0$ so that it ***rolls without slipping and oscillates***.
- Find: a) ω of the oscillation; b) $\theta(t)$; c) $f_s(t)$ (the force of static friction *as a function of time!*).

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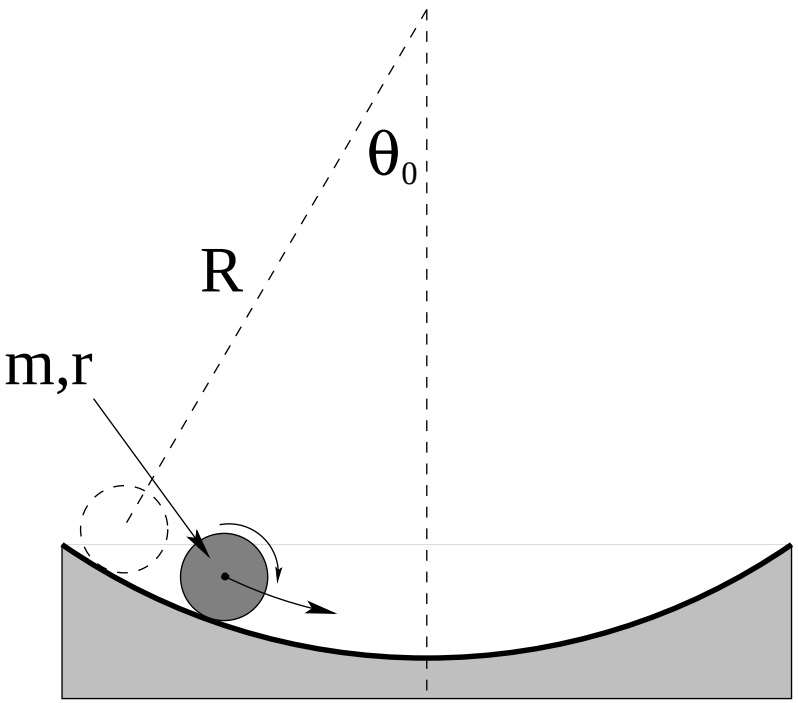

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Draw a force diagram on the rolling mass at a general angle θ.

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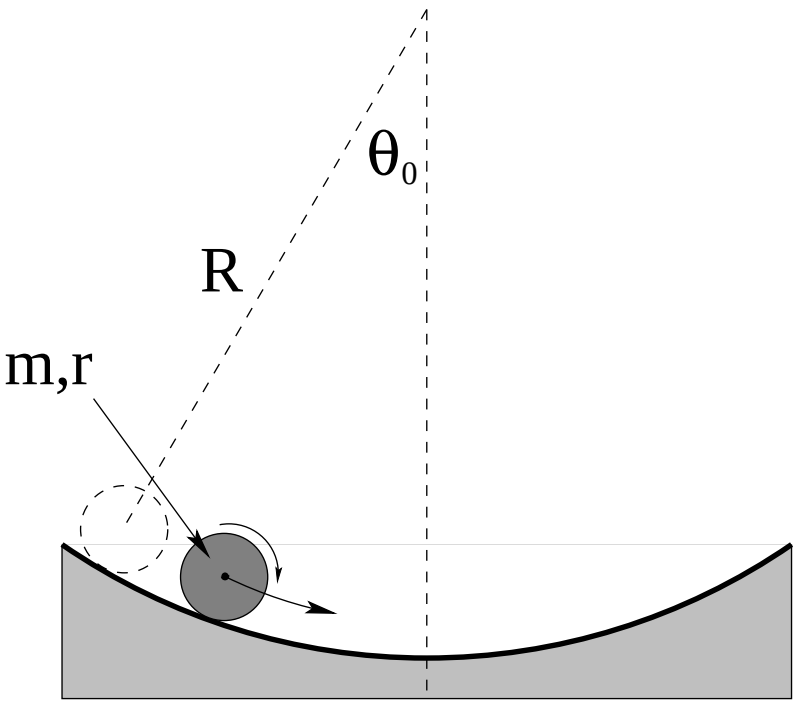

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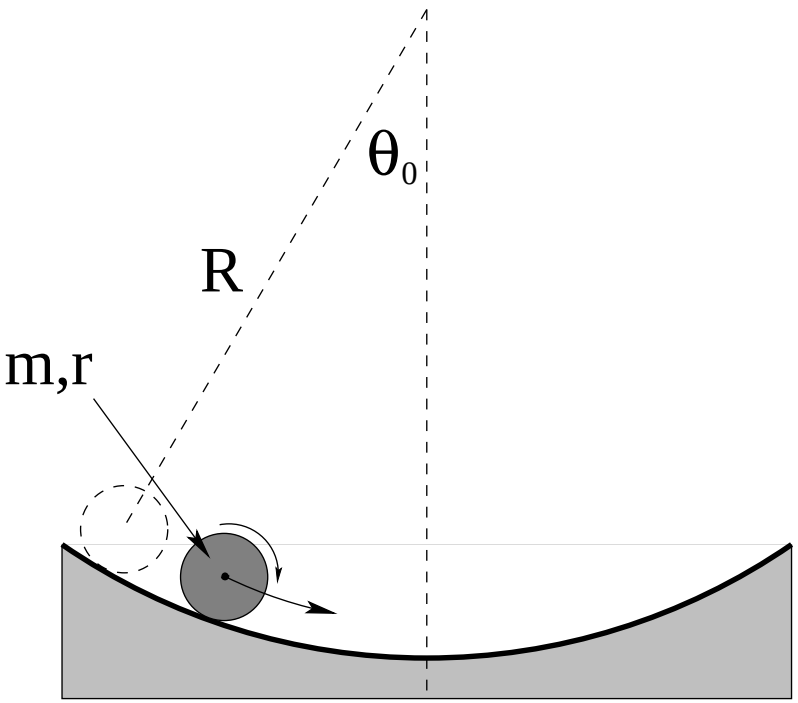

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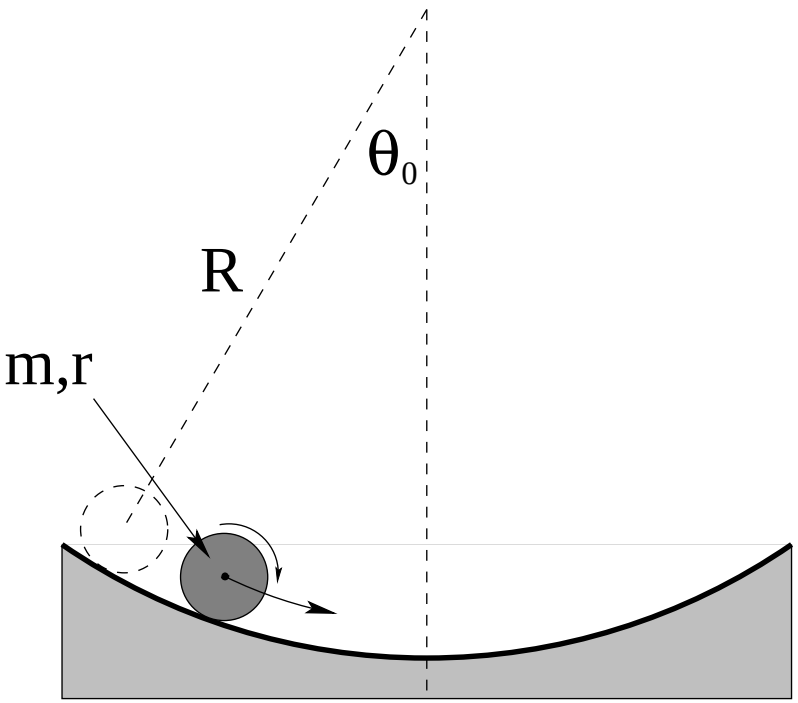

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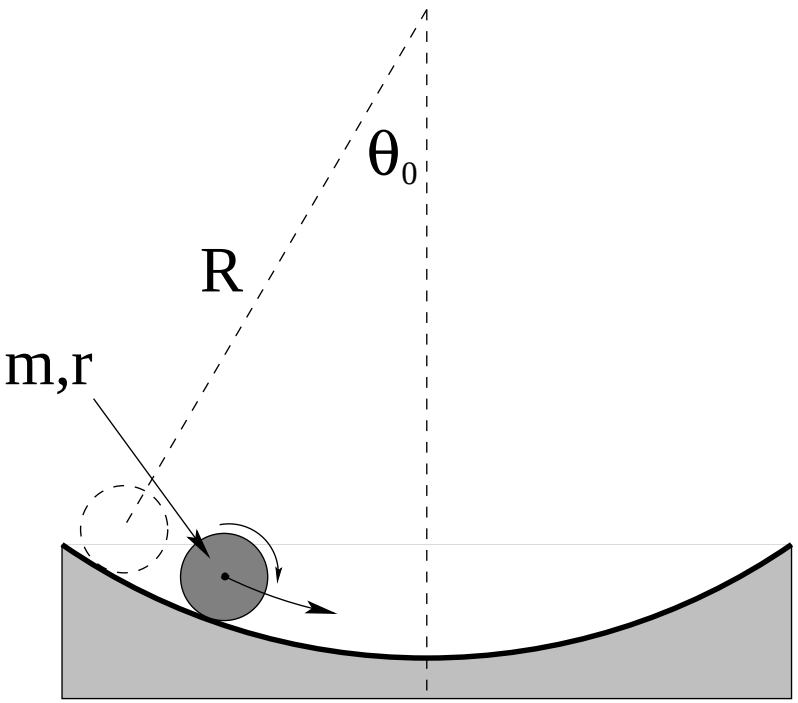

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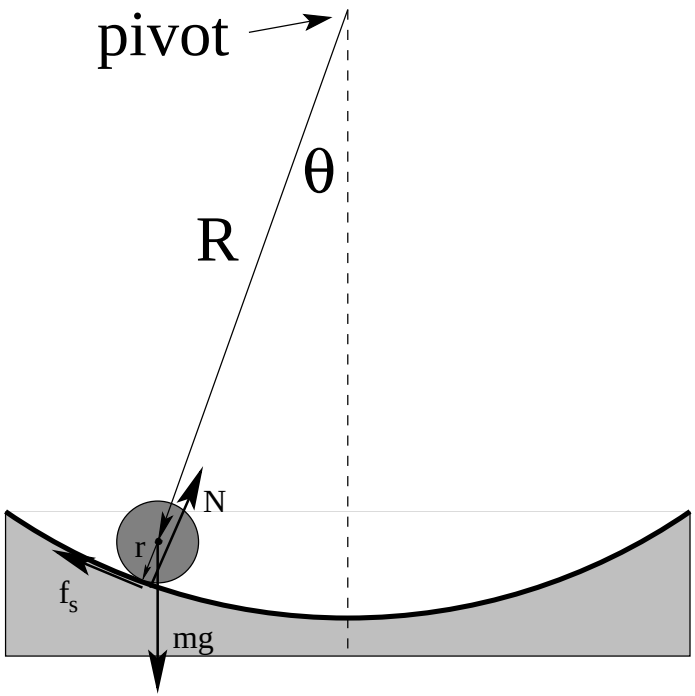

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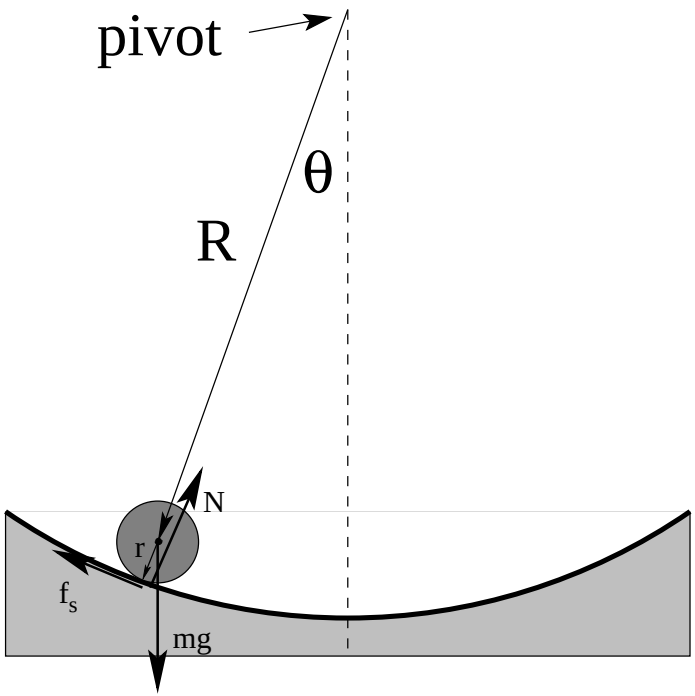

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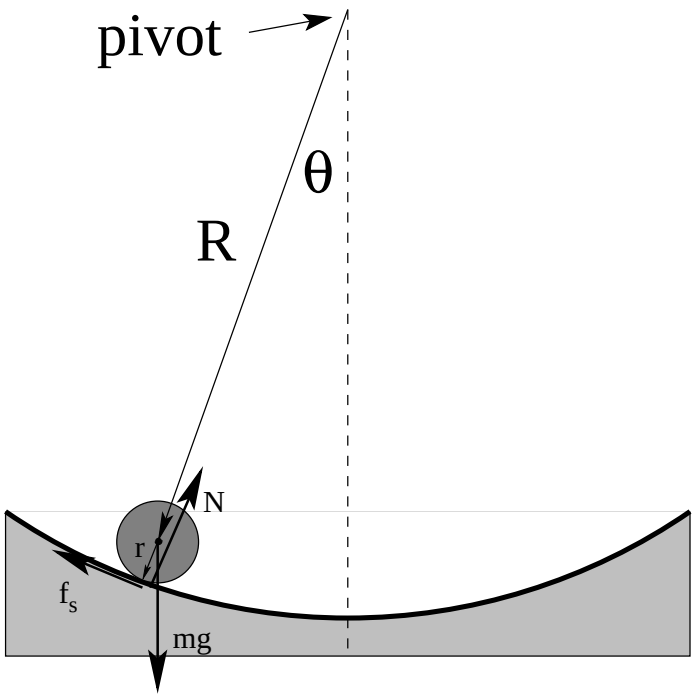

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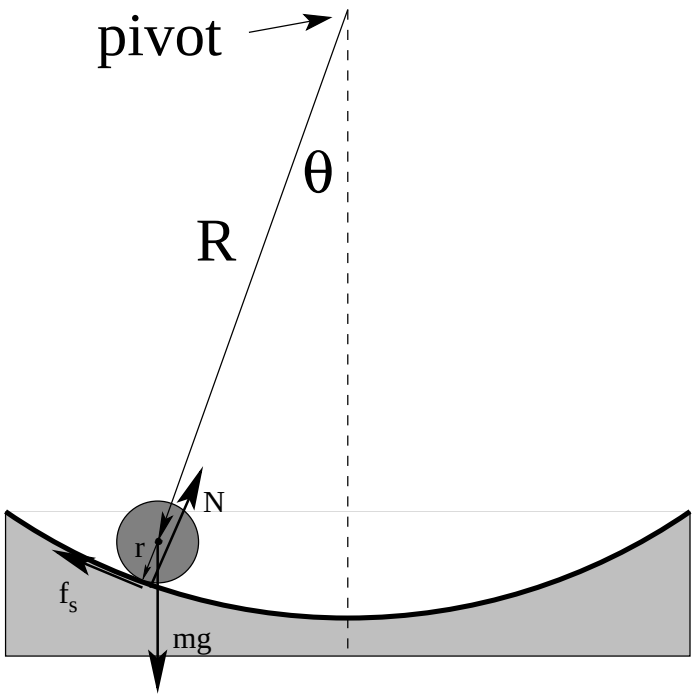

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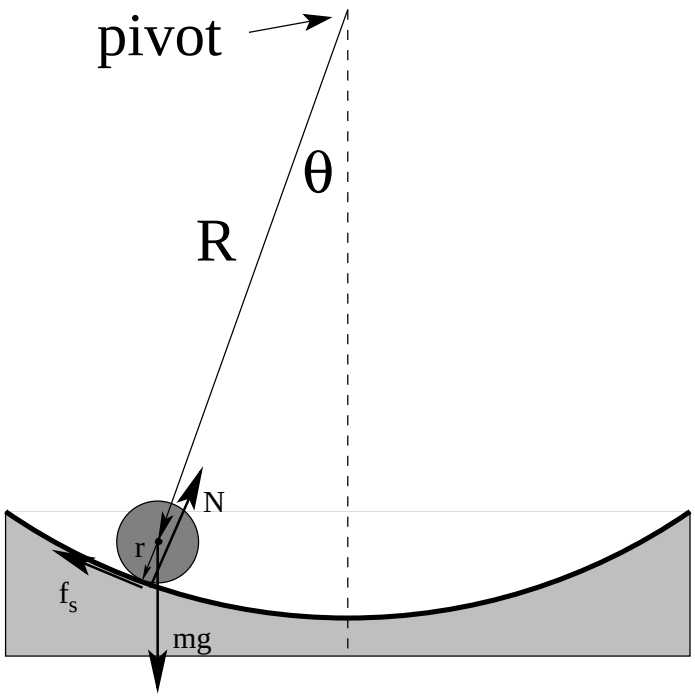

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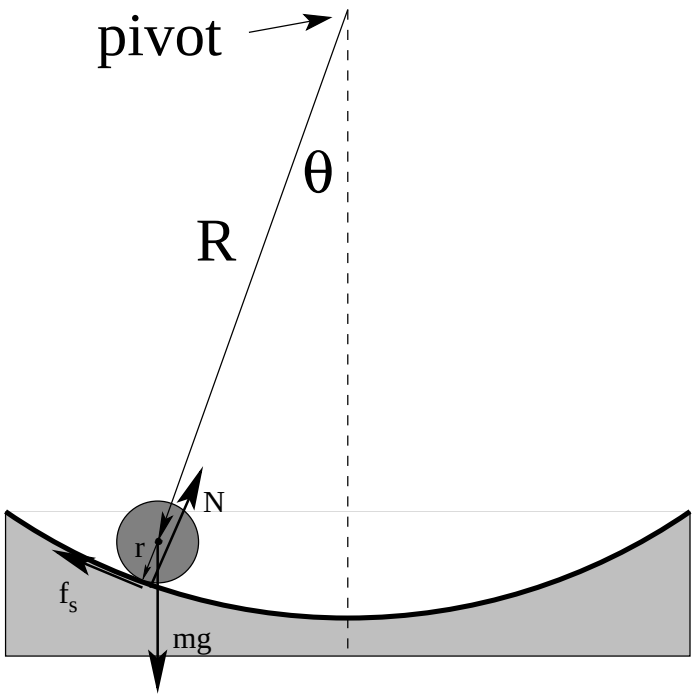

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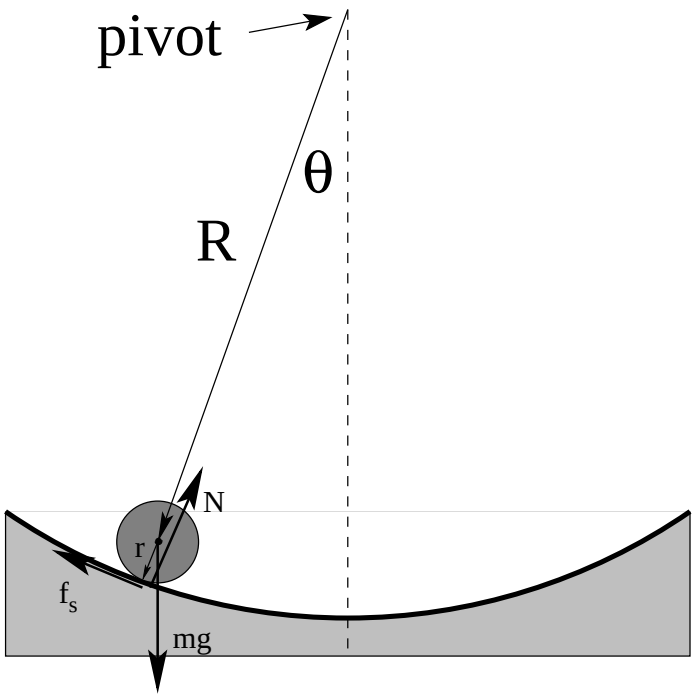

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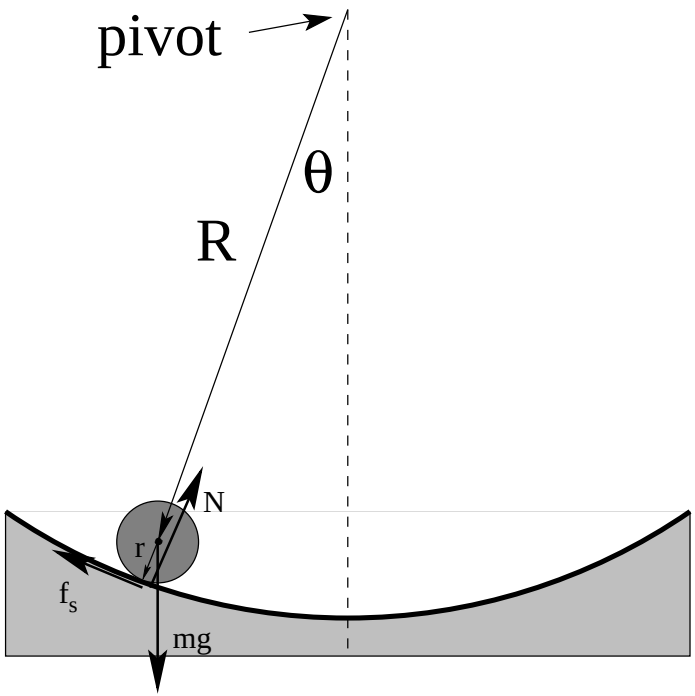

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make the small angle approximation and rearrange into $\frac{d^2\theta}{dt^2} + \frac{2g}{3R}\theta = 0$. Thus:

$$\boxed{\omega = \sqrt{\frac{2g}{3R}}} \qquad \boxed{\theta(t) = \theta_0\cos(\sqrt{\frac{2g}{3R}}t)} \qquad \boxed{f_s = -\frac{1}{2}ma_t = \frac{mg}{3}\theta_0\cos(\sqrt{\frac{2g}{3R}}t)}$$

Oscillations: A Rolling Pendulum-Comments

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This too makes sense! At $t = 0$, the disk starts to roll *down to the right*, so $\Omega_{\rm disk}$ is into the page, positive. You should be able to trace each quarter cycle of its oscillation and see that everything is consistent and correct.

Construct a Travelling Wave

Suppose: Amplitude $A = 1$ cm; Wavelength $\lambda = 0.5$ m; Period $T = 0.001$ sec
Write down the formula for a transverse wave travelling ***in the $-x$ direction***
What is the speed of the wave on the string in terms of the givens? Which of the following changes would ***double the power*** transmitted by the string (changing ***only one of*** A, T, λ and nothing else)?

- ☐ Change the amplitude to $A' = 0.707\ A$
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Use: $y(x,t) = A\sin(\frac{2\pi}{\lambda}x + \frac{2\pi}{T}t)$

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Use: $y(x,t) = A\sin(\frac{2\pi}{\lambda}x + \frac{2\pi}{T}t)$ and $v = \frac{\lambda}{T}$ and $P = \frac{1}{2}\mu\omega^2 A^2 v$ and

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Construct a Travelling Wave-Solution

$$\boxed{y(x,t) = 0.01\sin(4\pi x + 2000\pi t)\ \mathrm{m}}$$

$$\boxed{v = \frac{\lambda}{T} = 500\ \mathrm{m/sec}}$$

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Start with $P = \frac{1}{2}\mu\omega^2 A^2 v$. Then: $P \propto A^2$; also *both* $\omega = \frac{2\pi}{T}$ *and* $v = \frac{\lambda}{T}$ change, so $P \propto \frac{1}{T^3}$; finally if λ changes only v changes, so $P \propto \lambda$. Hence:

- ☐ Change the amplitude to $A' = 0.707\ A$ (No! $P \propto A^2$)
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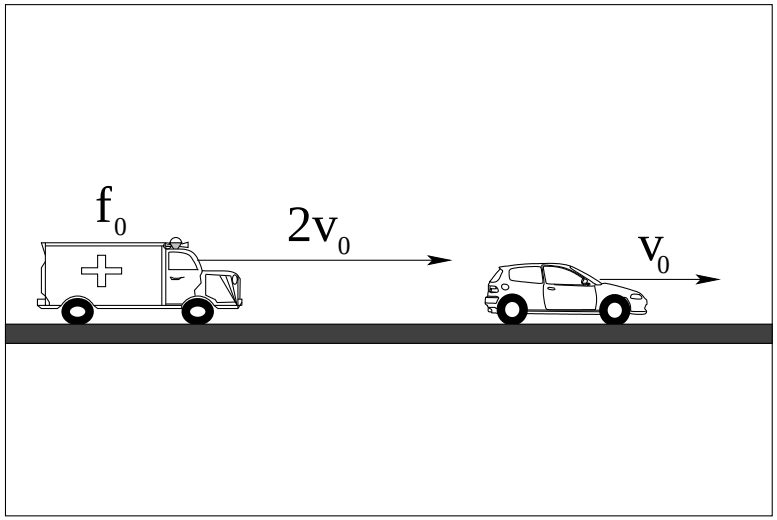
Passed by the Ambulance



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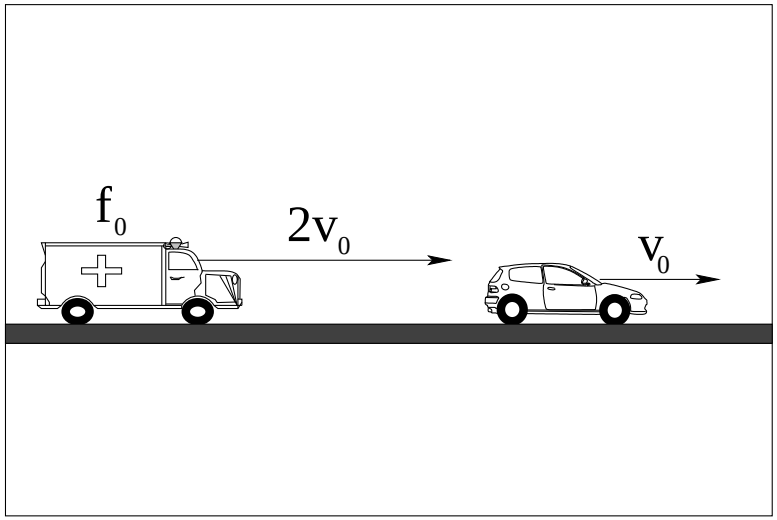

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Solution Map:

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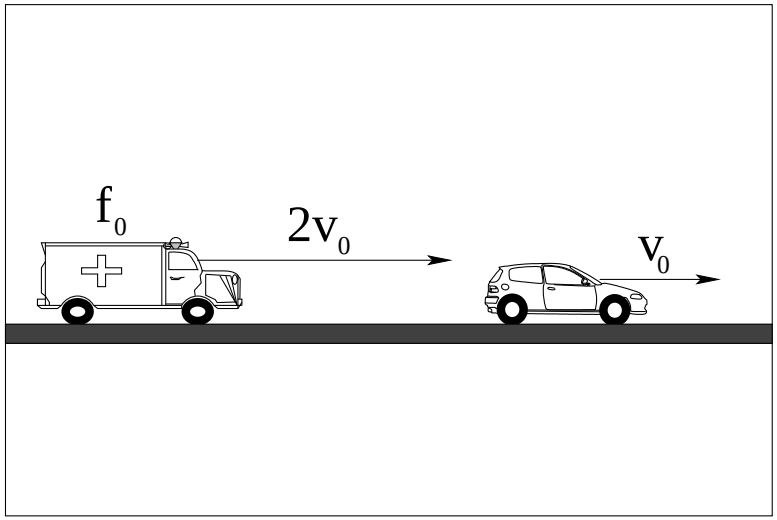

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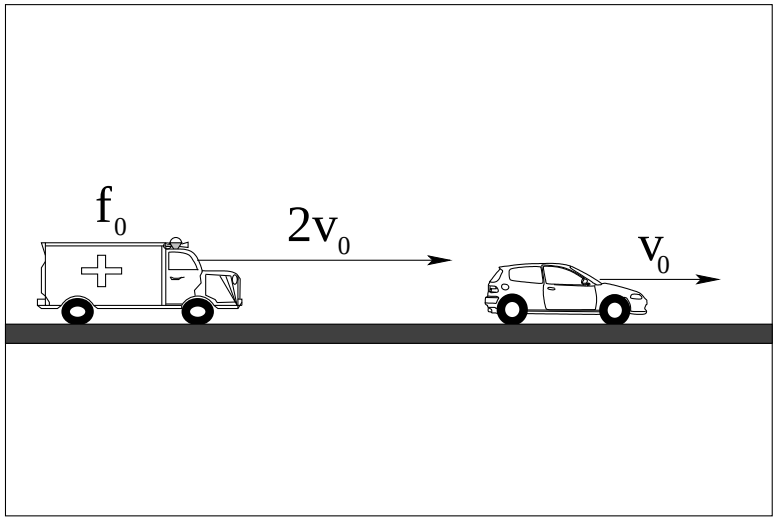

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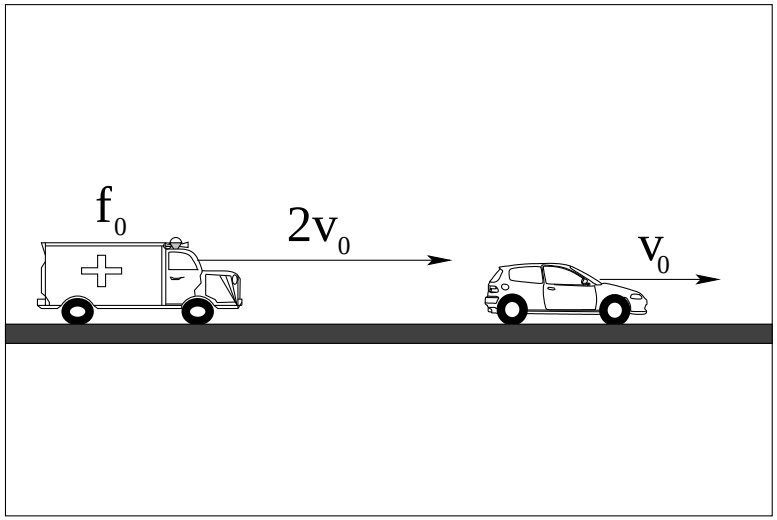

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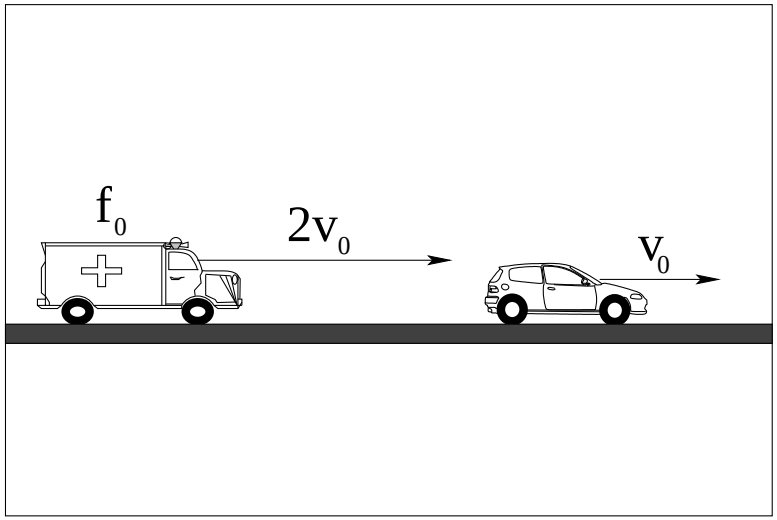

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$$f' = \frac{1 \pm \frac{v_r}{v_a}}{1 \mp \frac{v_s}{v_a}} f_0$$ (upper signs approaching, lower signs receding).

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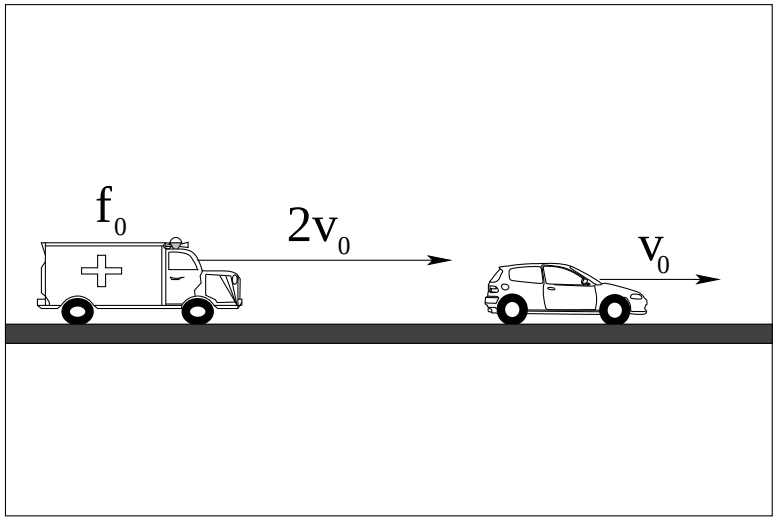

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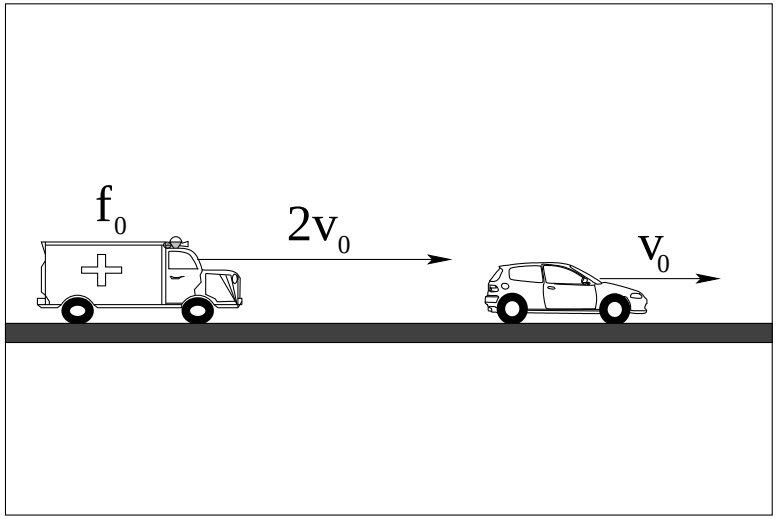

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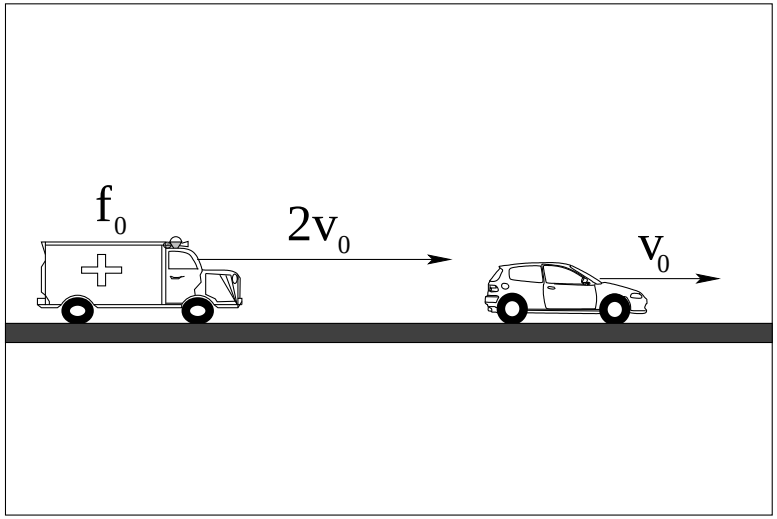

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- Let $x = v_0/v_a$. Then:

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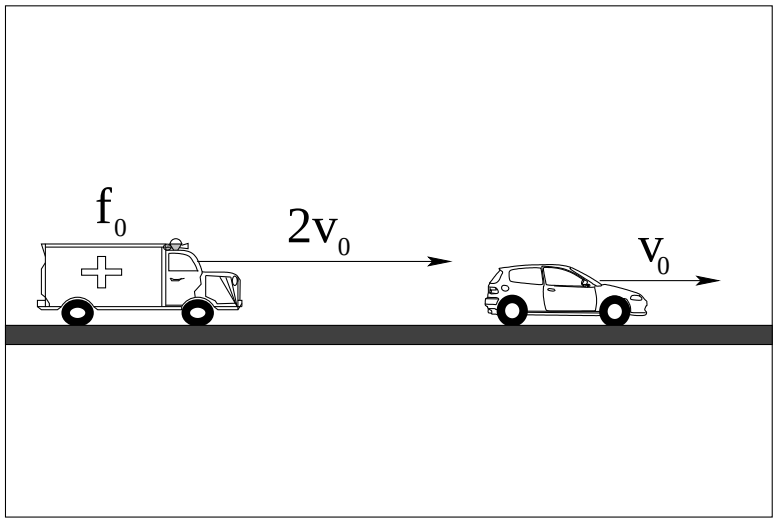

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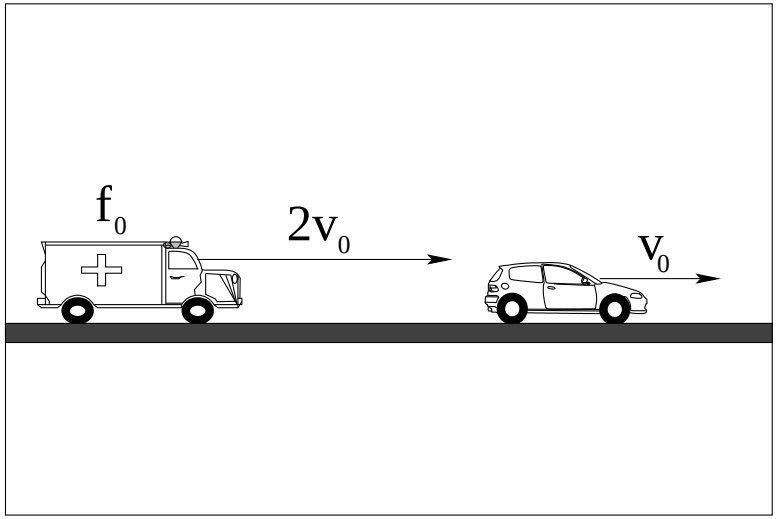

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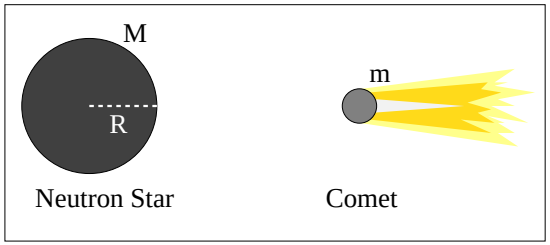
- And:

$$\boxed{v_0 = \frac{f_1 - f_0}{2f_1 - f_0} v_a} \text{ or in SI units, } \boxed{v_0 = \frac{100}{2100} 343 = \frac{343}{21} \approx 16 \text{ m/sec}}$$

If we multiply by 9/4, we get a good estimate of miles per hour: $v_0 = 37$ mph and the ambulance is going around 75 mph.

Collision with a Neutron Star



In the figure, a neutron star with mass $M = 10^{30}$ kg and radius $R = 8000$ m is drawn. Find: a) Escape speed from its surface. First find it algebraically, then evaluate it numerically as a fraction of the speed of light $c = 3 \times 10^8$ m/sec.

b) A comet of mass $m = 10^{14}$ kg falls "from infinity" to its surface. Estimate the energy lost to heat in the collision algebraically, and *then* evaluate it numerically, expressing it as a fraction of the relativistic rest energy of the comet.

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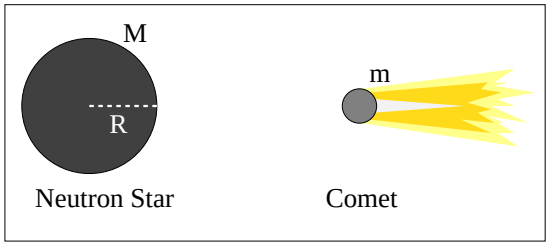

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First, use energy conservation to relate total energy at the surface of the comet to total energy at infinity.

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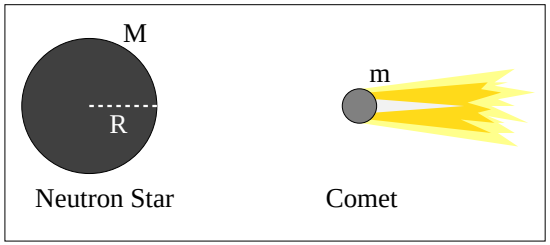

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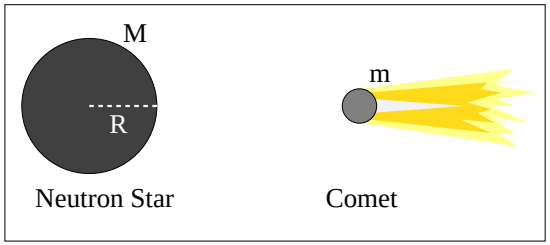

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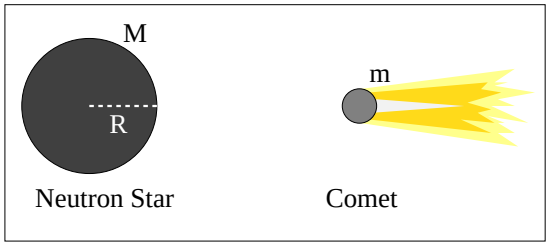

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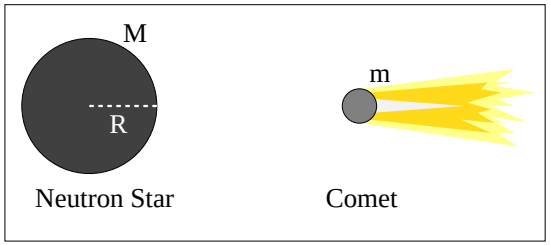
b) A comet of mass $m = 10^{14}$ kg falls "from infinity" to its surface. Estimate the energy lost to heat in the collision algebraically, and *then* evaluate it numerically, expressing it as a fraction of the relativistic rest energy of the comet.

First, use energy conservation to relate total energy at the surface of the comet to total energy at infinity. Remember, $E_\infty = 0$ and then solve for v_e. Falling in, remember it starts with $E_\infty = 0$ so the total energy at the surface will tell you the kinetic energy at the time of impact. It's bb with BB fully inelastic so *all* of this KE goes to heat!

Collision with a Neutron Star



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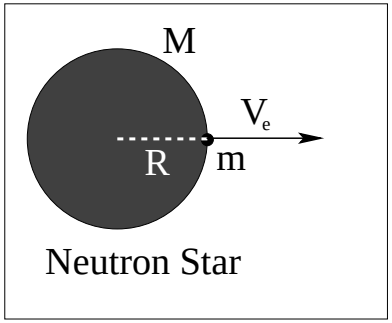
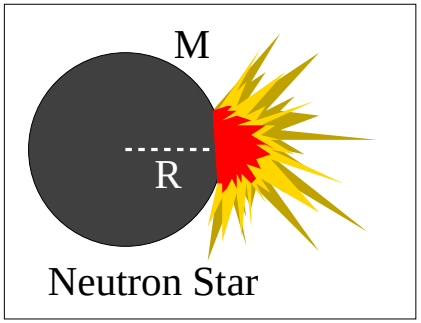
* - The solution is on the next page. Don't advance until you are ready!

Collision with a Neutron Star-Solution



- $E = 0 = -\frac{GMm}{R} + \frac{1}{2}mv_e^2 \Rightarrow \boxed{v_e = \sqrt{\frac{2GM}{R}}}$

Collision with a Neutron Star-Solution



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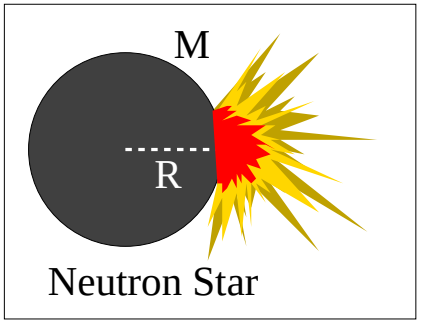

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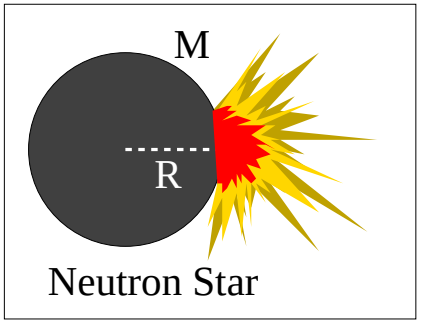
Numerically: $\frac{v_e}{c} = \frac{1}{c}\sqrt{\frac{2GM}{R}} = \frac{1}{3 \times 10^8}\sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 10^{30}}{8000}} \Rightarrow$

Collision with a Neutron Star-Solution



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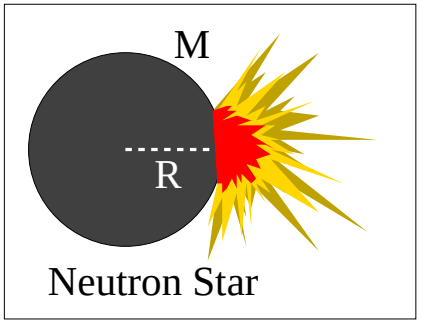

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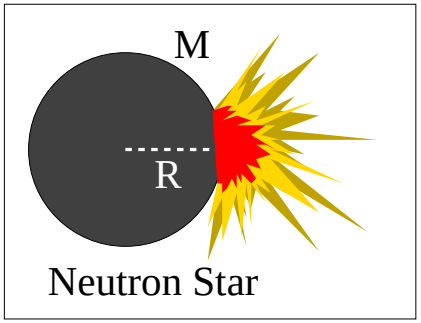
And: $\frac{\Delta E_{\rm heat}}{mc^2} = \frac{GM}{Rc^2} = \frac{6.67\times 10^{-11}\times 10^{30}}{8000\times 9\times 10^{16}} \Rightarrow$

Collision with a Neutron Star-Solution



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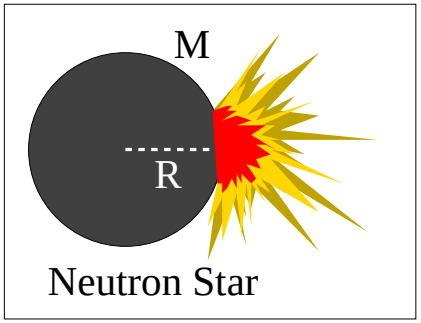
$\boxed{E_{\rm heat} = .093mc^2 = 8.34\times 10^{29}}$ joules.

Collision with a Neutron Star-Solution



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This *classical, non-relativistic* solution isn't quite right! Because v_e is so close to the speed of light, to do it correctly we'd have to use relativistic expressions for things like the total energy and kinetic energy. But it does give us a realistic appreciation for the strangeness of things like neutron stars!

The End

Feedback Welcome

Send Comments To: rgb at duke dot edu