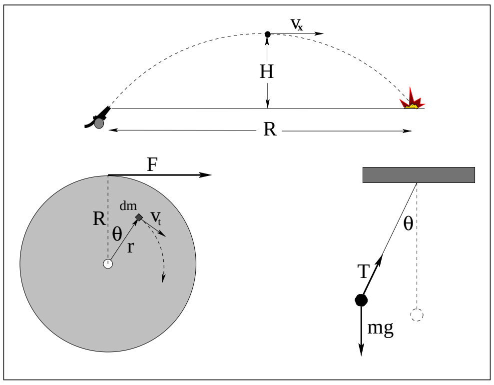
Introductory Physics 141/151/161

Self-Guided Learning Problems

Robert G. Brown



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About These Problems

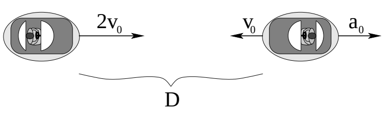
This is an experiment. Beamer allows me to make slides that will successively reveal lines of math-heavy text. This gives me a unique opportunity to build a collection of self-guided learning problems for physics that do what I've fantasized about doing for years now – present a problem, then provide a hint, then another hint, then another (or reveal a step, and then another step) until finally, the entire souution is presented, annotated.

Hopefully these problems will help students everywhere as they struggle to learn physics problems solving techniques and learn to "think like a physicist" as they do so.

To use this resource, pick a problem or topic from the table of contents and go directly to it, or work your way through all the problems systematically. Work on a separate sheet of paper, and when you get stuck, page down through the frames to see (hopefully) where you went wrong.

Remember, the point is to **master these problems**, not just to get through them. Make sure that before you are done, you can do every problem **without looking, without hints, and without remembering the exact solution** but rather, understanding *how* to find it!

Kinematics: Two Bumper Cars



- Two bumper cars are headed straight at one another, one travelling at $2v_0$ to the right, the other at speed v_0 to the left. When they are separated by a distance D, the car on the right slows down with a constant acceleration a_0. Does the right hand car manage to stop before being hit by the left hand car?

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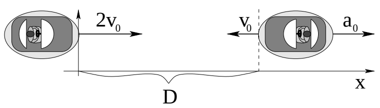

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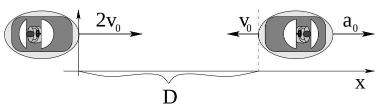

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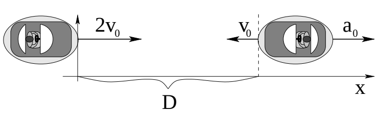

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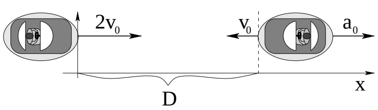

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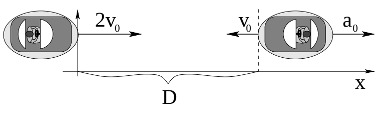

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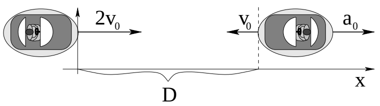

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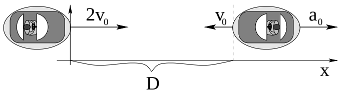
Kinematics: Two Bumper Cars-Solution



Solution: $x_l(t) = 2v_0 t, \quad v_l(t) = 2v_0$

$$x_r(t) = D - v_0 t + \frac{1}{2}a_0 t^2, \quad v_r(t) = -v_0 + a_0 t$$

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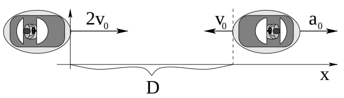

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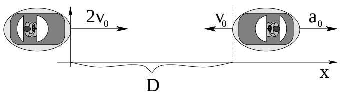

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$$x_r(t) = D - v_0 t + \frac{1}{2}a_0 t^2, \quad v_r(t) = -v_0 + a_0 t$$

Find the time: $t_r = \frac{v_0}{a_0}$. The left hand car is then at $x_l(t_r) = \frac{2v_0^2}{a_0}$ and the right hand car is at $x_r(t_r) = D - \frac{v_0^2}{a_0} + \frac{v_0^2}{2a_0} = D - \frac{v_0^2}{2a_0}$.

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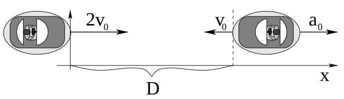

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$$D < \frac{5v_0^2}{2a_0}$$

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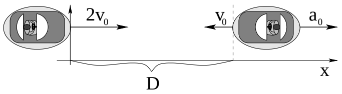

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It makes sense – larger D makes it *less* likely to collide, larger a_0 makes it *less* likely they will collide, larger v_0 makes it *more* likely.

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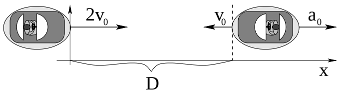

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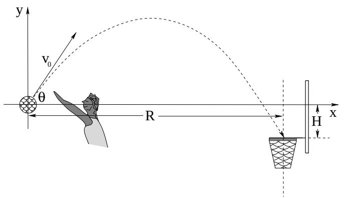
It makes sense – larger D makes it *less* likely to collide, larger a_0 makes it *less* likely they will collide, larger v_0 makes it *more* likely. Knowing they collide, if we write:

$$x_l(t_c) = 2v_0 t_c = D - v_0 t_c + \frac{1}{2}a_0 t_c^2 = x_r(t_c)$$

would let us find the time of collision and answer other questions about e.g. their relative velocity at that time. This is a simple quadratic equation for t_c.

Kinematics: 2D Basketball Trajectory



- A basketball player shoots a jump hook at a (horizontal) distance R from the basket, releasing the ball at a height H above the rim as shown. To shoot over his opponent's outstretched arm, he releases the basketball at an angle θ with respect to the horizontal.

Find v_0, the ***speed*** he must release the basketball with (in terms of H, R, g and θ) for the ball to go through the hoop "perfectly" as shown. Assume that his release is on line and undeflected, at initial speed v_0 and that the acceleration of the basketball is $\vec{a} = -g\hat{j}$, ignoring drag.

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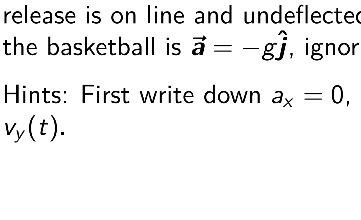

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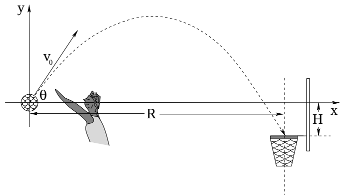

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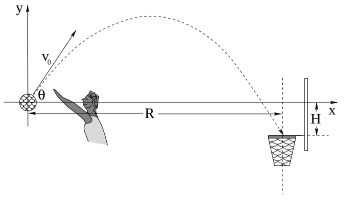

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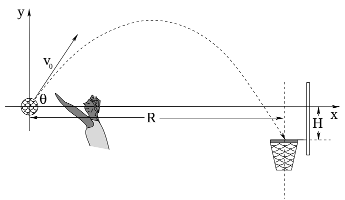

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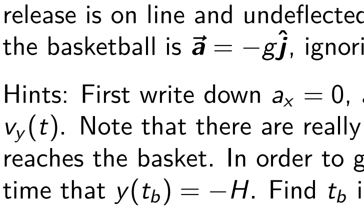

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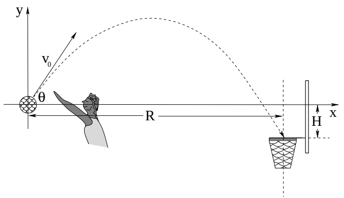

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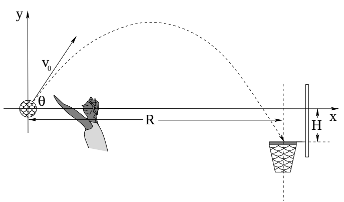

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Kinematics: 2D Basketball Trajectory-Solution

Initial Conditions:

$$a_x = 0, v_{0x} = v_0 \cos\theta, x_0 = 0 \text{ and } a_y = -g, v_{0y} = v_0 \sin\theta, y_0 = 0$$

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Integrate:

$$x(t) = v_0 \cos\theta t \quad v_x(t) = v_0 \cos\theta \quad y(t) = -\frac{1}{2}gt^2 + v_0 \sin\theta t \quad v_y(t) = v_0 \sin\theta - gt$$

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Find the ***time*** t_b that the basketball reaches the horizontal position of the hoop:

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Find the ***time*** t_b that the basketball reaches the horizontal position of the hoop:

$$R = v_0 \cos\theta t_b \Rightarrow t_b = R/(v_0 \cos\theta)$$

This must also be the time that the ball has exactly the height of the hoop:

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Find the ***time*** t_b that the basketball reaches the horizontal position of the hoop:

$$R = v_0 \cos\theta t_b \Rightarrow t_b = R/(v_0 \cos\theta)$$

This must also be the time that the ball has exactly the height of the hoop:

$$-H = -\frac{1}{2}gt_b^2 + v_0 \sin\theta t_b \Rightarrow \frac{gR^2}{2v_0^2 \cos^2\theta} = R\tan\theta + H$$

And finally, we solve for v_0:

Kinematics: 2D Basketball Trajectory-Solution

Initial Conditions:

$$a_x = 0, v_{0x} = v_0 \cos\theta, x_0 = 0 \text{ and } a_y = -g, v_{0y} = v_0 \sin\theta, y_0 = 0$$

Integrate:

$$x(t) = v_0 \cos\theta t \quad v_x(t) = v_0 \cos\theta \quad y(t) = -\frac{1}{2}gt^2 + v_0 \sin\theta t \quad v_y(t) = v_0 \sin\theta - gt$$

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And finally, we solve for v_0:

$$\boxed{v_0 = \sqrt{\frac{gR^2}{2(R\sin\theta\cos\theta + H\cos^2\theta)}}}$$

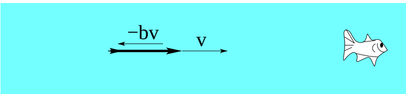
After doing the algebra, check the dimensions. Are they OK?

Firing a Speargun

Hints:

- An underwater fisherman fires her speargun at a distant fish. The neutral-buoyancy spear leaves the gun at initial speed v_0 and experiences a *linear* drag force $F_d = -bv$ opposite to its velocity. Find $v(t)$ and R, the maximum range of the spear.

Firing a Speargun



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Hints:
First, draw a picture of the *general* motion and drag force.

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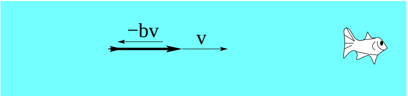

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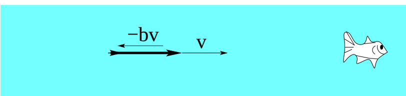
Firing a Speargun



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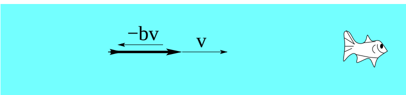

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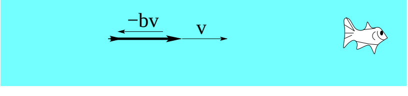
Firing a Speargun



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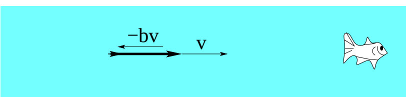
Firing a Speargun



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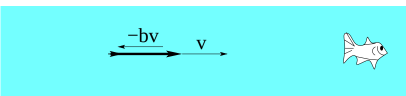
Firing a Speargun



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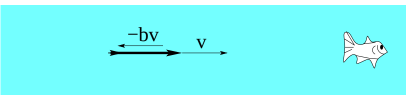
Note that it will come *almost* to rest (and in the real world, completely to rest) long before t equals "infinity". Note also that the "terminal velocity" in this problem is *zero* as drag is the only force acting.

Firing a Speargun



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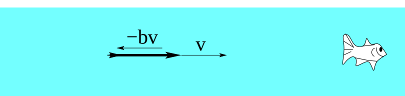
* - The solution is on the next page. Don't advance until you are ready!

Firing a Speargun-Solution



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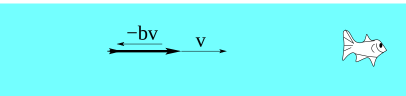
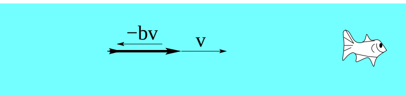


$$F = -bv = m\frac{dv}{dt} = ma \Rightarrow$$

Firing a Speargun-Solution



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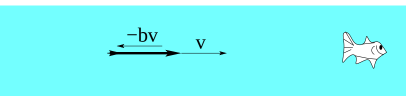

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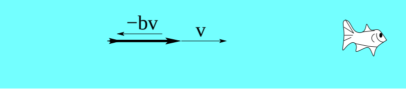

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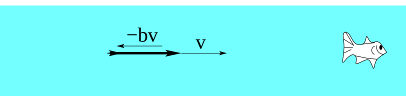

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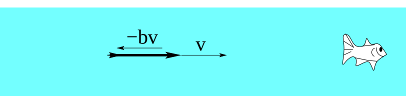

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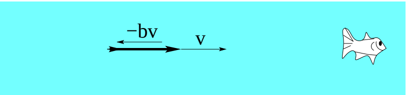

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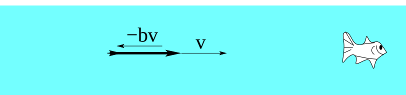

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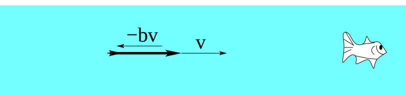

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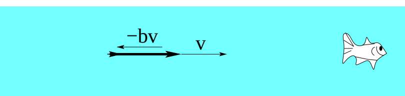

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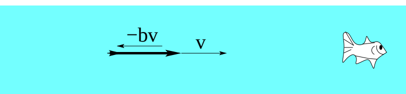
so the range R is:

Firing a Speargun-Solution



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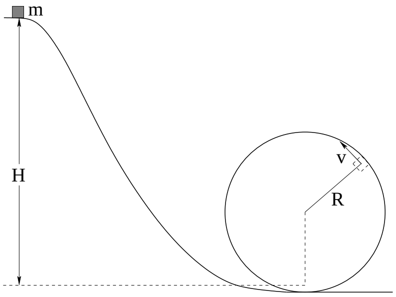
so the range R is:

$$\boxed{R = x(t \to \infty) = \frac{mv_0}{b}}$$

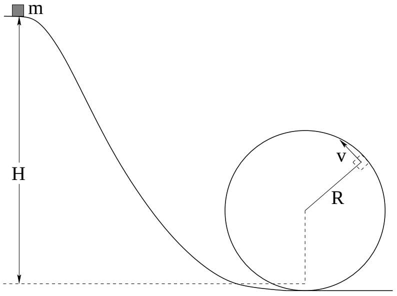
Loop the Loop



- A block of mass M sits at the top of a frictionless hill of height H leading to a circular loop-the-loop of radius R. Find the minimum height $H_{\rm min}$ for which the block *barely* gess around the loop staying on the track at the top. If the block is started at this position, what is the normal force exerted by the track at the *bottom* of the loop, where it is greatest?

Solution Map:

Loop the Loop



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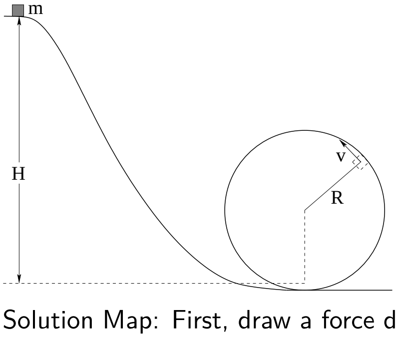
Solution Map: First, draw a force diagram of the mass at the top of the loop (include gravity and the normal force).

Loop the Loop



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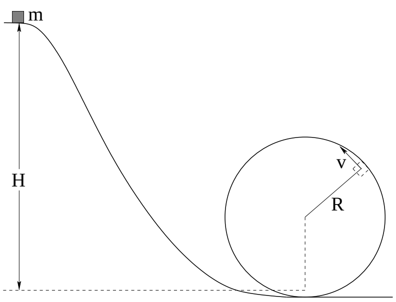
Loop the Loop



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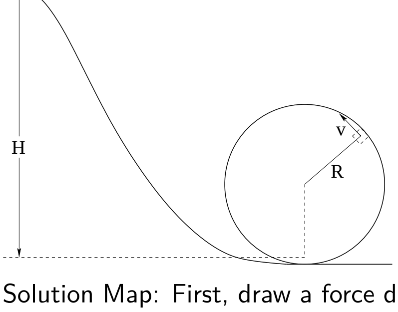
Loop the Loop



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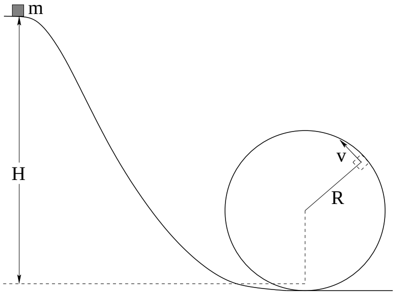
Loop the Loop



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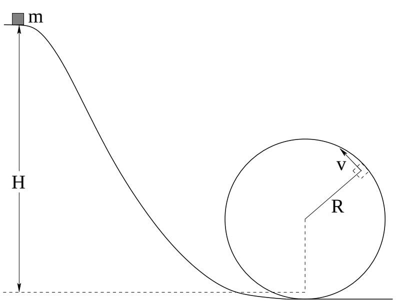
Loop the Loop



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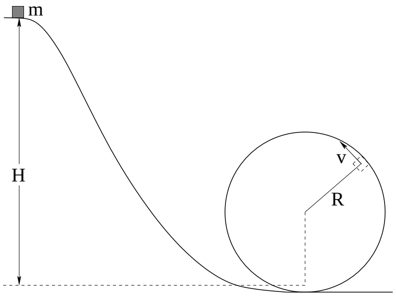
Loop the Loop

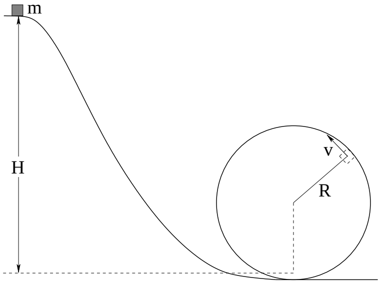


- A block of mass M sits at the top of a frictionless hill of height H leading to a circular loop-the-loop of radius R. Find the minimum height H_{min} for which the block *barely* gess around the loop staying on the track at the top. If the block is started at this position, what is the normal force exerted by the track at the *bottom* of the loop, where it is greatest?

Solution Map: First, draw a force diagram of the mass at the top of the loop (include gravity and the normal force). Use **conservation of energy** to relate H to v at the top of the loop. Write N2 for the block at the top of the loop. Use centripetal acceleration for a in N2 at the top of the loop. Find H_{min} by letting N go to zero and doing algebra. Repeat this process for the bottom of the loop, but solve for N at this point.

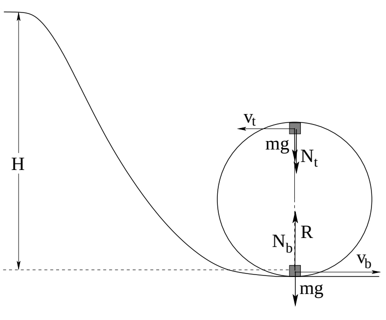
***** - The solution is on the next page. Don't advance until you are ready!

Loop the Loop-Solution



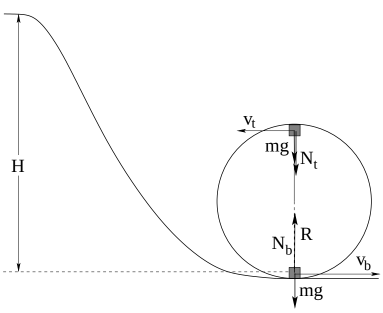
At the top:

Loop the Loop-Solution



At the top: $mgH_{\min} + 0 = mg2R + \dfrac{1}{2}mv_t^2$

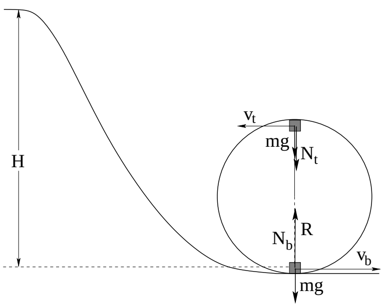
Loop the Loop-Solution



At the top: $mgH_{\min} + 0 = mg2R + \frac{1}{2}mv_t^2$

$mg + \cancel{N_t} = \frac{mv_t^2}{R} \Rightarrow \frac{1}{2}mv_t^2 = \frac{1}{2}mgR$

Loop the Loop-Solution



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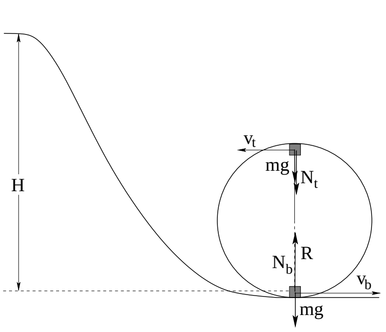
Loop the Loop-Solution



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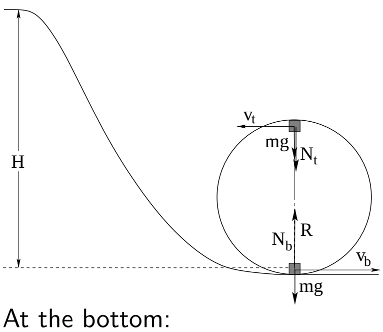
Loop the Loop-Solution



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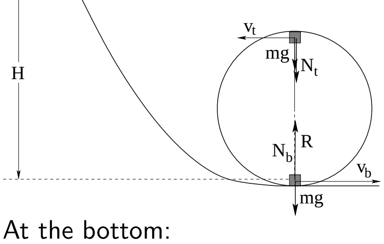
Loop the Loop-Solution



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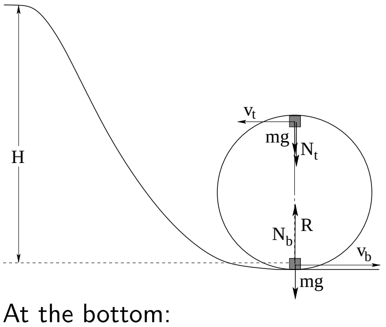
At the bottom:

$$mgH_{\min} + 0 = \frac{5}{2}mgR \quad = \frac{1}{2}mv_b^2 \text{ and}$$

Loop the Loop-Solution



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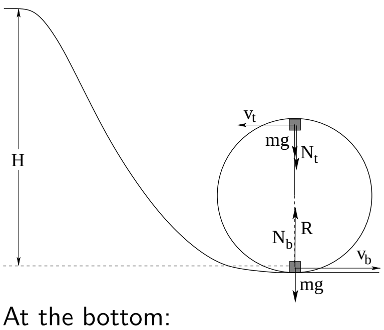
At the bottom:

$$mgH_{\min} + 0 = \frac{5}{2}mgR \quad = \frac{1}{2}mv_b^2 \text{ and } \quad N - mg = \frac{mv_b^2}{R} \Rightarrow$$

Loop the Loop-Solution



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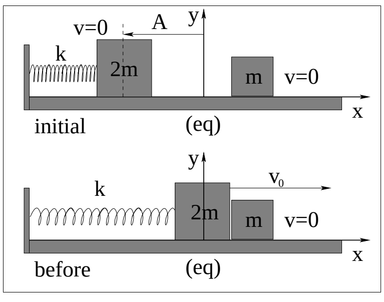
At the bottom:

$$mgH_{\rm min} + 0 = \frac{5}{2}mgR \quad = \frac{1}{2}mv_b^2 \text{ and } \quad N - mg = \frac{mv_b^2}{R} \Rightarrow N = mg + \frac{mv_b^2}{R}$$

or:

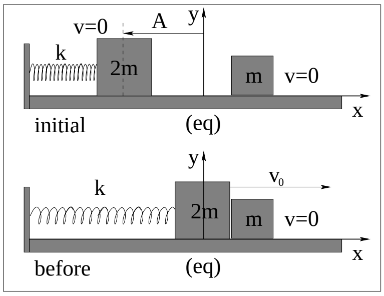
$$mv_b^2 = 5mg \Rightarrow \boxed{N = mg + 5mg = 6mg}$$

Spring-Driven Collision



In the figure above, a mass $2m$ connected to a spring k is released from rest when the spring is compressed to the position $x_0 = -A$ and collides **elastically** with a mass m just as it reaches its equilibrium position moving with velocity $v_0\hat{\boldsymbol{x}}$ as shown. Find: A in terms of v_0, m, and k; v_{2m} and v_m immediately after the collision; the maximum amplitude A' of the big ($2m$) block's oscillation *after* the collision. Ignore friction, and m starts at rest.

Spring-Driven Collision



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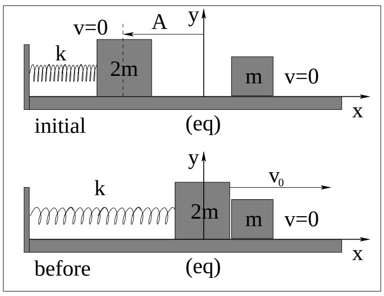
Solution Map:
Use energy conservation to find A.

Spring-Driven Collision



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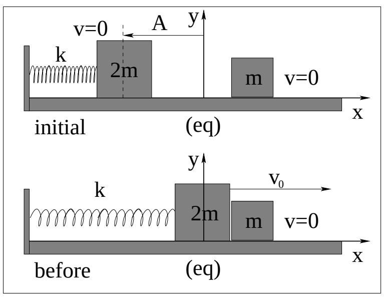
Solution Map:
Use energy conservation to find A. Solve the 1D elastic collision for v_{2m} and v_m.

Spring-Driven Collision



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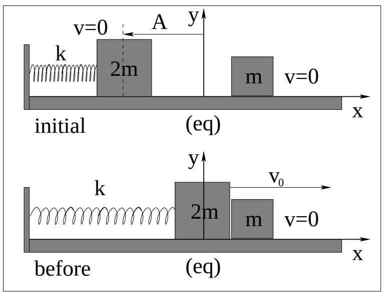
Solution Map:
Use energy conservation to find A. Solve the 1D elastic collision for v_{2m} and v_m. Use energy conservation again to find A'.

Spring-Driven Collision



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Solution Map:

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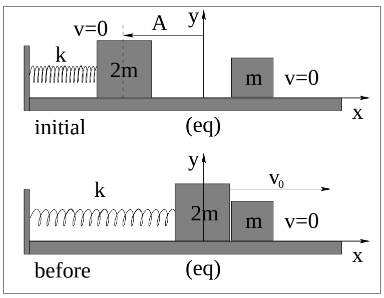

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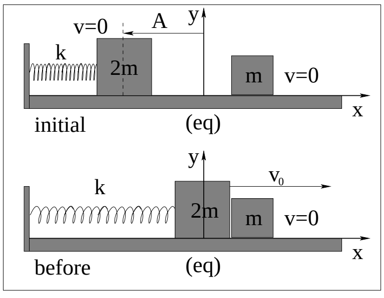
1D Elastic Collision Formula: $v_f = -v_i + 2v_{\rm cm}$ with $v_{\rm cm} = \dfrac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2}$

Spring-Driven Collision



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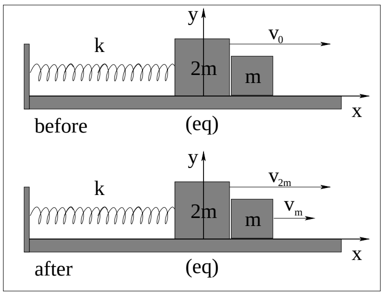
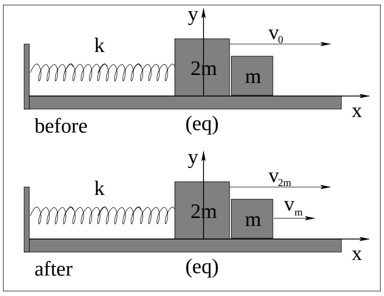
Spring-Driven Collision-Solution



Use mechanical energy conservation before the collision to find A:

$$\frac{1}{2}kA^2 = \frac{1}{2}(2m)v_0^2 \Rightarrow \boxed{A = \sqrt{\frac{2m}{k}}v_0}$$

Spring-Driven Collision-Solution



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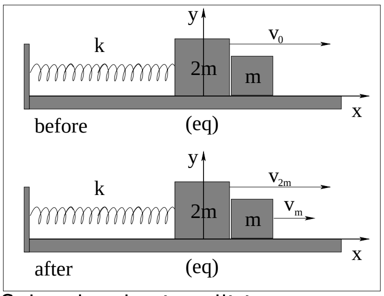
(Note that $A = \dfrac{v_0}{\omega}$ as one expects from the solution obtained in Chapter 9.)

Spring-Driven Collision-Solution



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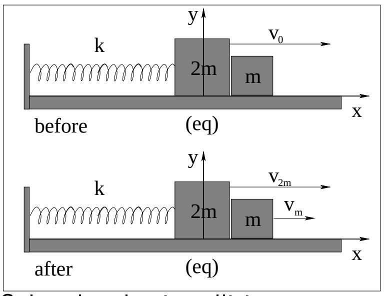
Solve the elastic collision:

$$v_{\rm cm} = \frac{2mv_0}{2m+m} = \frac{2v_0}{3} \Rightarrow \boxed{v_{2m} = \frac{v_0}{3} \qquad v_m = \frac{4v_0}{3}}$$

Spring-Driven Collision-Solution



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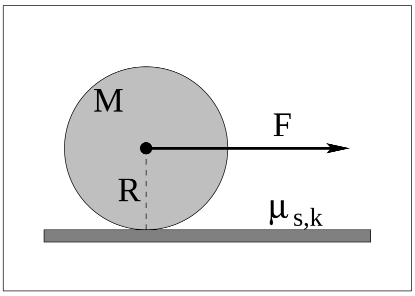
Use mechanical energy conservation again to find A':

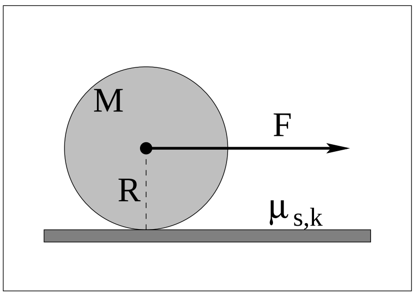
$$\frac{1}{2}(2m)v_{2m}^2 = \frac{1}{9}mv_0^2 = \frac{1}{2}kA'^2 \Rightarrow \boxed{A' = \sqrt{\frac{2m}{9k}}v_0}$$

Rolling with Slipping



- A disk with mass M, radius R, is sitting on a rough floor with coefficients of friction $\mu_{k,s}$ respectively. It is pulled by a force $F(t) = At$ that **increases linearly with time** (A is a constant with units N/sec). Write an expression for $f_s(t)$, the **magnitude** of the force of static friction as a function of time. Find the time t_s that the disk starts to slip instead of roll without slipping.

Rolling with Slipping



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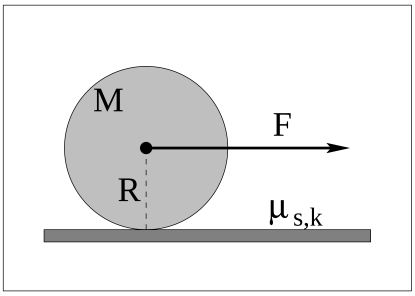
Solution Map:

Rolling with Slipping



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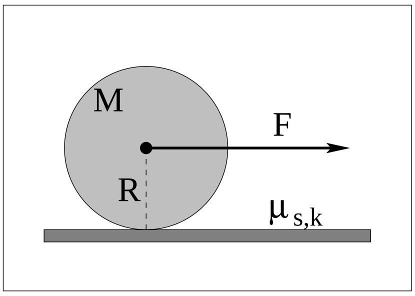
Solution Map:
As always, draw a force diagram and choose good coordinates. Let f_s and N act at the point of contact.

Rolling with Slipping



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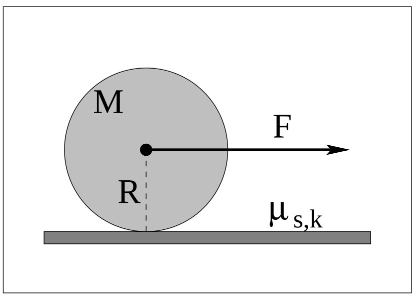
Rolling with Slipping



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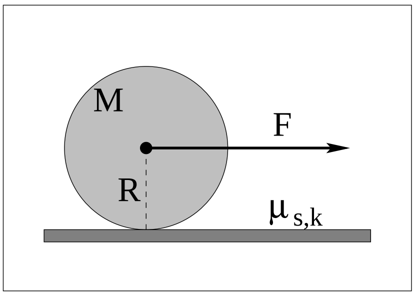
Rolling with Slipping



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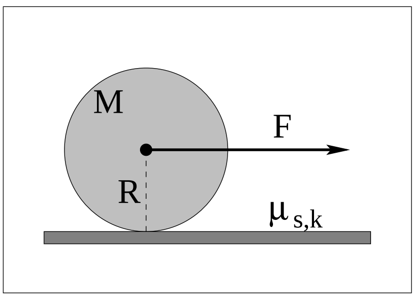
Rolling with Slipping



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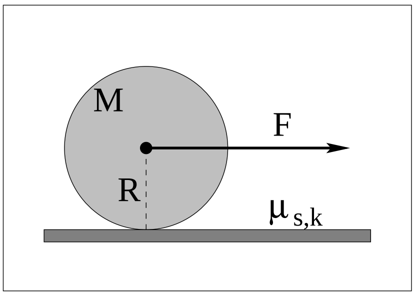
Rolling with Slipping



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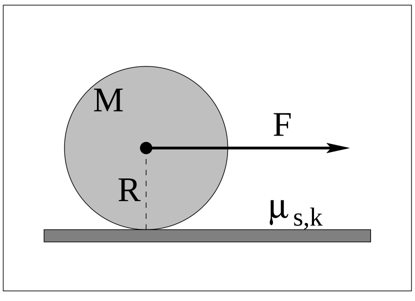
Rolling with Slipping



- A disk with mass M, radius R, is sitting on a rough floor with coefficients of friction $\mu_{k,s}$ respectively. It is pulled by a force $F(t) = At$ that **increases linearly with time** (A is a constant with units N/sec). Write an expression for $f_s(t)$, the **magnitude** of the force of static friction as a function of time. Find the time t_s that the disk starts to slip instead of roll without slipping.

Solution Map:
As always, draw a force diagram and choose good coordinates. Let f_s and N act at the point of contact. Next, write N2 for translation and rotation both. Solve for f_s (which will **not** contain μ_s!). You can solve for the acceleration $a(t)$ if you wish as well (this might have been part of the question, and may be needed in order to most easily find f_s). Use $f_s < \mu_s N$ to determine the time that the disk will start to slip. If you like, find the *independent* linear and angular accelerations after it starts to slip (this could easily have been a question too).

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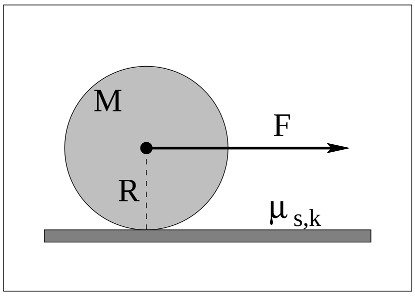

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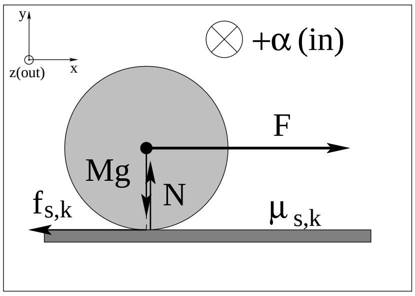

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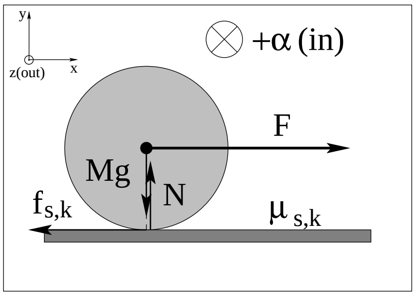

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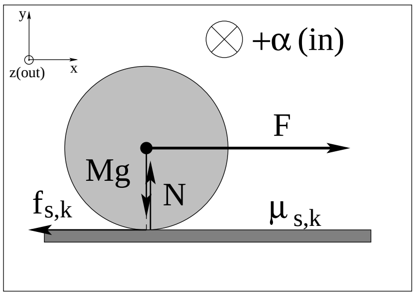

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Next, note that $N - Mg = 0 \Rightarrow N = Mg$ and find the slip time t_s:

$$f_s = \frac{1}{3} At < \mu_s N = \mu_s Mg \Rightarrow \frac{1}{3} At_s = \mu_s Mg \Rightarrow \boxed{t_s = \frac{3\mu_s Mg}{A}}$$

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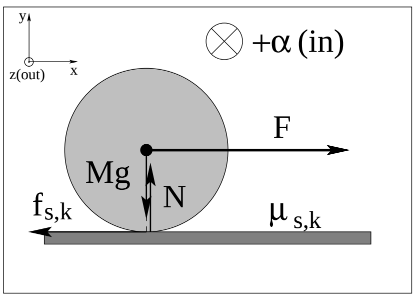

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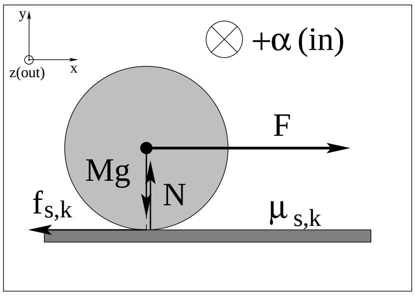

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$$F - \mu_k Mg = Ma' \Rightarrow \boxed{a' = \frac{At}{M} - \mu_k g} \qquad Rf_k = R\mu_k Mg = \frac{1}{2}MR^2\alpha' \Rightarrow \boxed{\alpha' = \frac{\mu_k g}{R}}$$

Rolling with Slipping-Solution Comments

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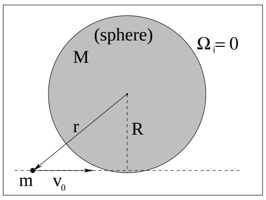
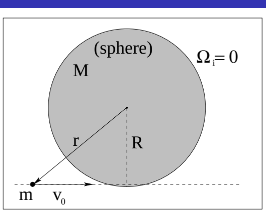
- Note Well! We can solve for everything using just F for the applied force and substitute $F = At$ only at the end when we need it!
- And *note above all* that $\boldsymbol{f_s = \frac{1}{3}At \neq \mu_s N}$. We *only* use $\mu_s N$ **to find the (critical) slipping point, never to find f_s itself!**

A Bullet Grazes a Sphere



A small bullet of mass m fired at an initial speed of v_0 grazes a free (unpivoted!) sphere sphere of mass M and radius R ($I = \frac{2}{5}MR^2$) grazes the surface of the sphere at its equator and emerges travelling along the same line at the reduced speed $v_1 = v_0/2$.

A Bullet Grazes a Sphere



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Find: Ω_f the angular velocity of the sphere after the collision; the velocity v_f of the center of mass of the sphere after the collision; the energy lost during the collision. Is there any value of m (relative to M) for which this collision is elastic?

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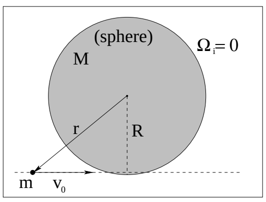

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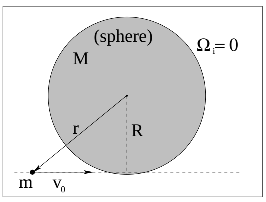

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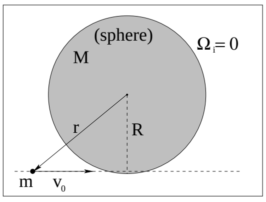

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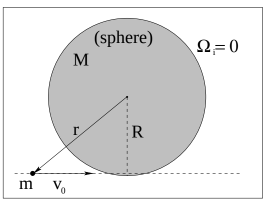

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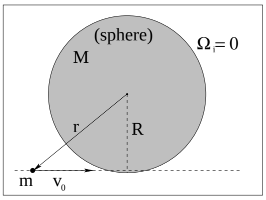

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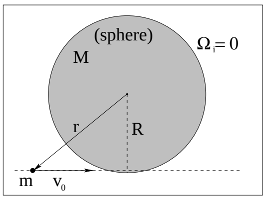

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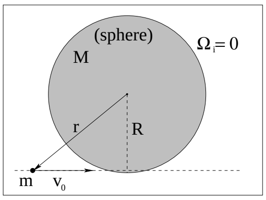

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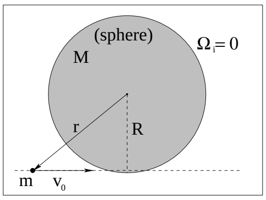

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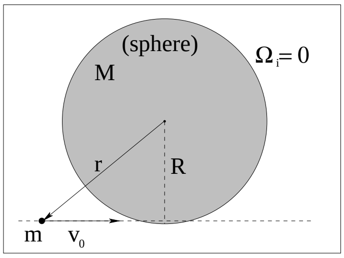

Start by drawing/visualizing the final state of the rotating sphere.

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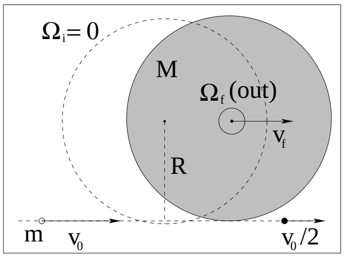

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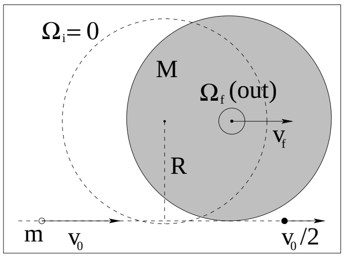

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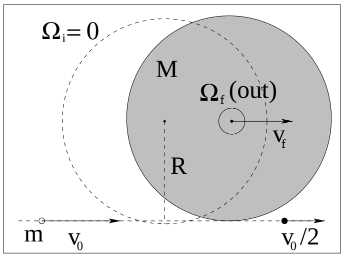

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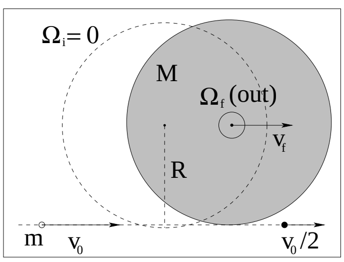

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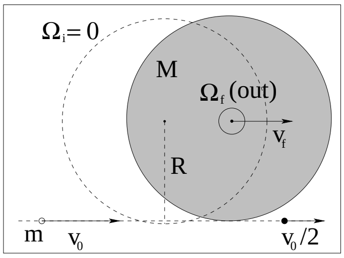

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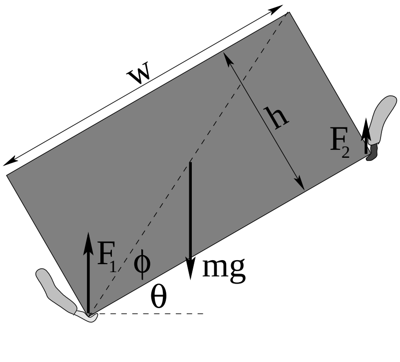
$$\boxed{\Delta E = \left(\left\{\frac{1}{4} + \frac{7m}{8M}\right\} - 1\right)\frac{1}{2} m v_0^2} \text{ and } \Delta E = 0 \text{ if } \boxed{m = \frac{6}{7} M}$$

Static Equilibrium: Carrying A Box Up the Stairs



- Two students are carrying a heavy box of mass m, width w, and height h up the stairs (with center of mass in the middle). The stairs make an angle θ. How is the weight distributed between the two workers?

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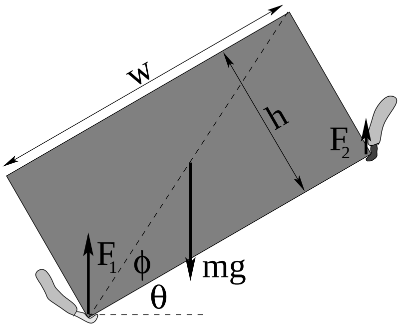

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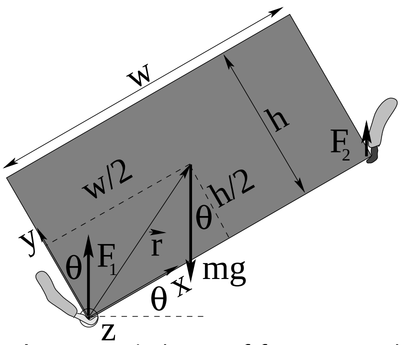
- First, choose a good pivot and coordinate frame for the torque! Avoid the frame needing $\phi = \tan^{-1}(h/w)$ in the trig!

Static Equilibrium: Carrying A Box Up the Stairs



- Two students are carrying a heavy box of mass m, width w, and height h up the stairs (with center of mass in the middle). The stairs make an angle θ. How is the weight distributed between the two workers?
- First, choose a good pivot and coordinate frame for the torque! Avoid the frame needing $\phi = \tan^{-1}(h/w)$ in the trig!

An optimal choice of frame uses the vertical direction for force balance and the *tipped* frame shown for evaluating the torque. I've provided some useful angles in the figure.

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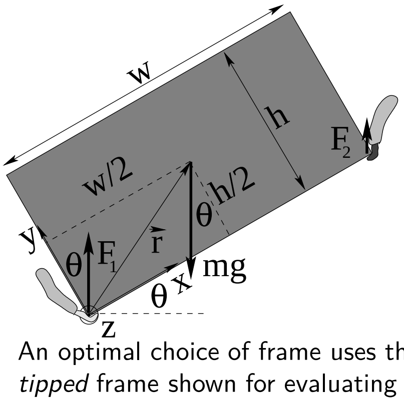

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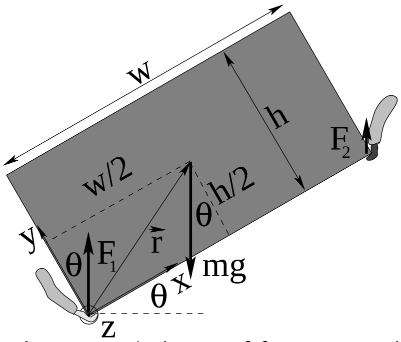

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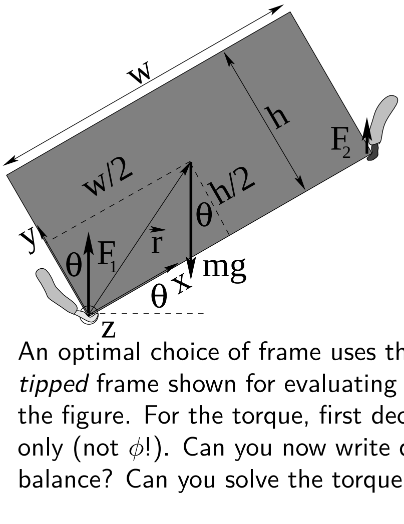

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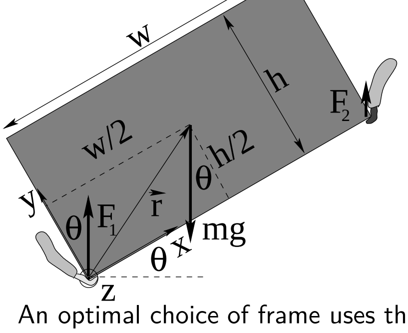

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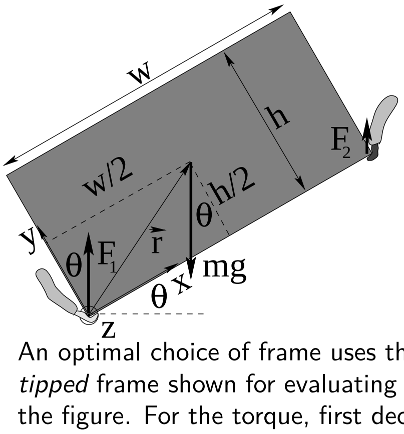

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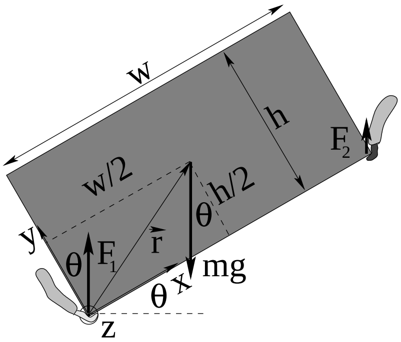
Static Equilibrium: Carrying A Box Up the Stairs-Solution



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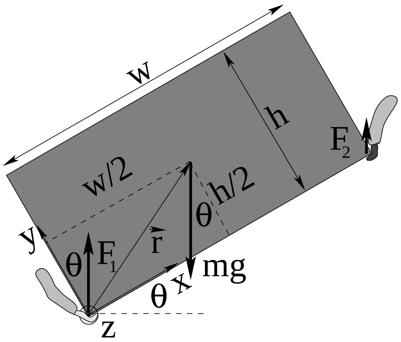

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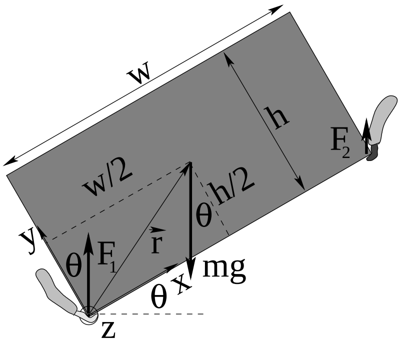
Static Equilibrium: Carrying A Box Up the Stairs-Solution



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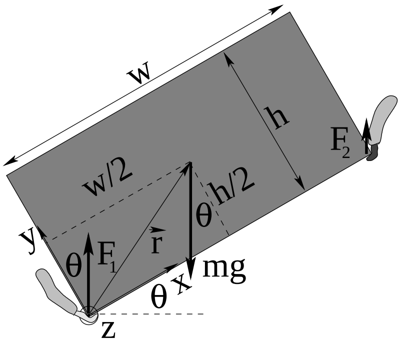

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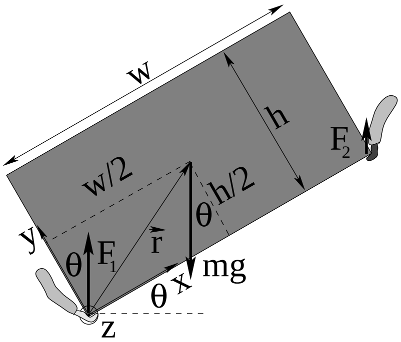

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So:

$$\boxed{F_2 = \frac{mg}{2}\left(1 - \frac{h}{w}\tan\theta\right)} \text{ and } \boxed{F_1 = \frac{mg}{2}\left(1 + \frac{h}{w}\tan\theta\right)}$$

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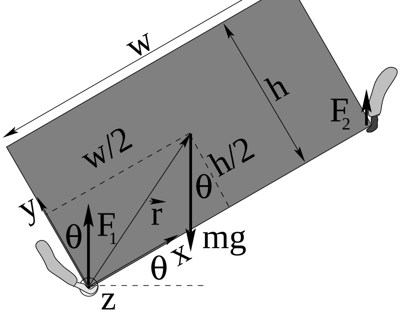

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Note: $F_1 > F_2$ for all $\theta \in (0, \pi/2)$. It's better to be the student at the top!

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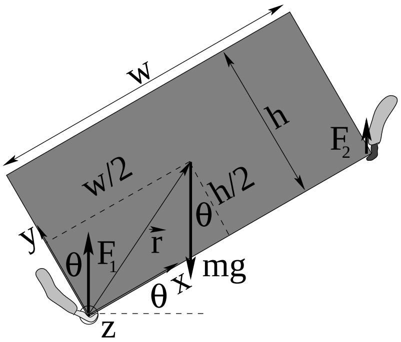

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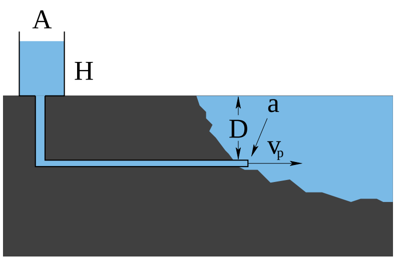
Note: $F_1 > F_2$ for all $\theta \in (0, \pi/2)$. It's better to be the student at the top! Note: $F_1 = mg$ when $\theta = \tan^{-1}\dfrac{w}{h}$. At this point $F_2 = 0$ and the student at the top is just helping balance the load!

Draining into a Lake



- The figure shows a rain-barrel that drains into a nearby lake through an underground pipe. The cross sectional area of the pipe is a, the top of the barrel has area $A \gg a$, the water has a height H in the barrel, and the pipe enters the lake at a depth D. Find v_p, the speed with which the water drains into the lake through the pipe, and the rate of flow through the pipe. Lake and barrel are obviously open to air pressure.

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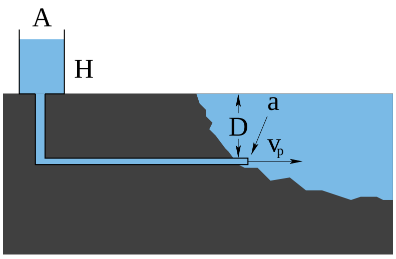

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Pick good coordinates – the y origin at the height of the pipe opening, for example.

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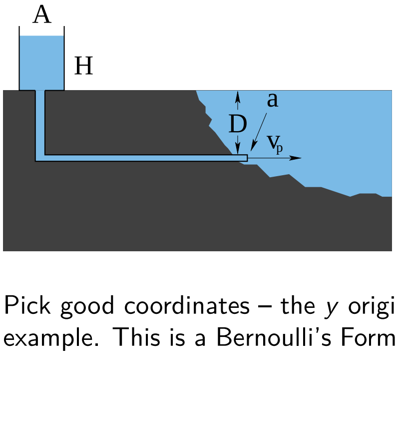

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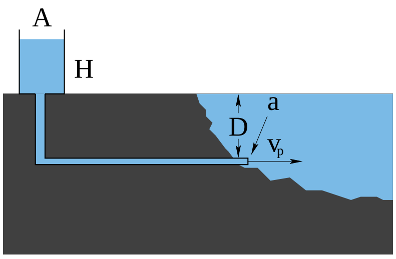

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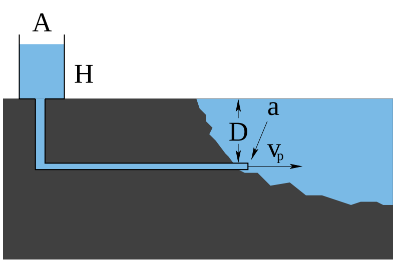

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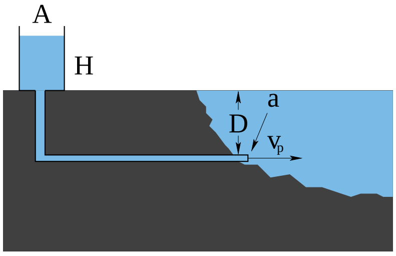

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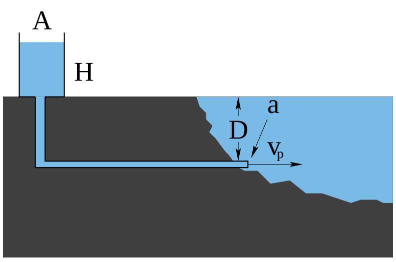

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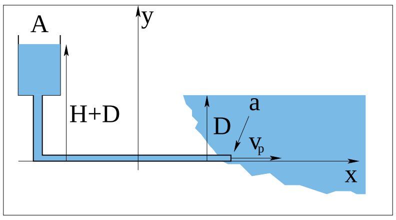
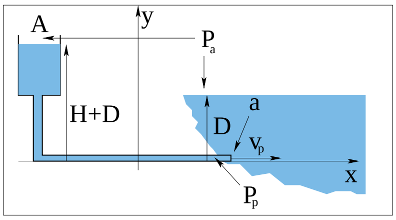


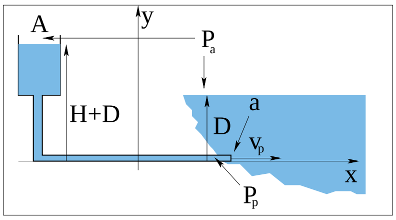
Choose coordinates like those shown.

Draining into a Lake-Solution



Choose coordinates like those shown.
Evaluate $P_p = P_a + \rho g D$.

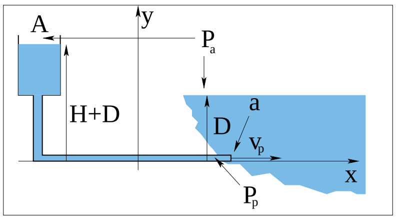
Draining into a Lake-Solution



Choose coordinates like those shown. Evaluate $P_p = P_a + \rho g D$. Apply Bernoulli Formula:

$$P_a + \rho g(H + D) + \frac{1}{2}\rho v_b^2 = P_p + \frac{1}{2}\rho v_p^2$$

Draining into a Lake-Solution



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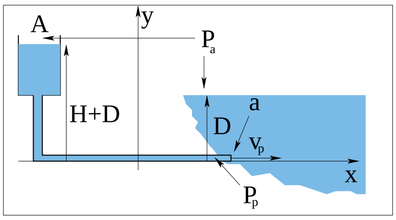
or (neglecting $\frac{1}{2}\rho v_b^2$ because $v_b \ll v_p$ because $A \gg a$ – Torricelli assumption):

$$\frac{1}{2}\rho v_p^2 = P_a + \rho g H + \rho g D - P_a - \rho g D = \rho g H \Rightarrow \boxed{v_p = \sqrt{2gH}}$$

Draining into a Lake-Solution



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This makes sense! If $H = 0$, the water will not flow ($v_p = 0$). The contribution to the pressure from the extra depth D basically cancels out.

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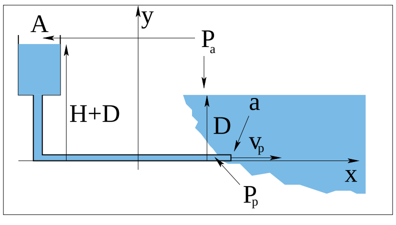

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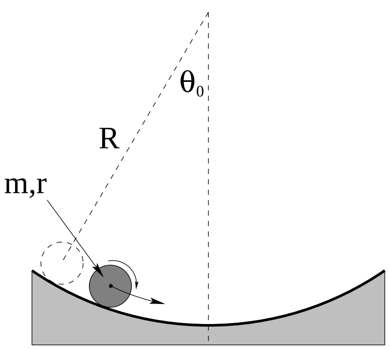
The flow is then easy:

$$\boxed{I_p = v_p a = a\sqrt{2gH}}$$

where the speed in the pipe is much larger than the speed of water in *either* the rainbarrel *or* the body of the lake.

Oscillations: A Rolling Pendulum



- A disk of mass m and radius r is gently set on a rough circular floor so that it makes an angle θ_0 relative to a vertical through the center of curvature of the floor, with its center of mass a distance R from the center of curvature as shown, and is released from rest at $t = 0$ so that it ***rolls without slipping and oscillates***.
- Find: a) ω of the oscillation; b) $\theta(t)$; c) $f_s(t)$ (the force of static friction *as a function of time!*).

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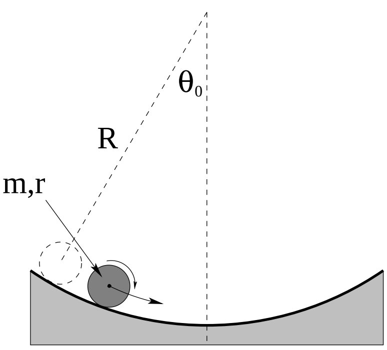

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Draw a force diagram on the rolling mass at a general angle θ.

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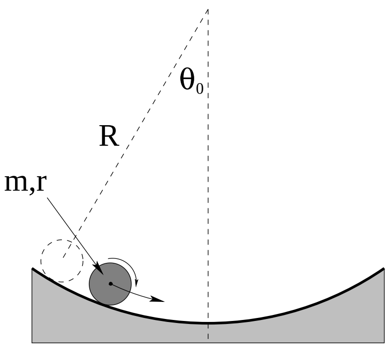

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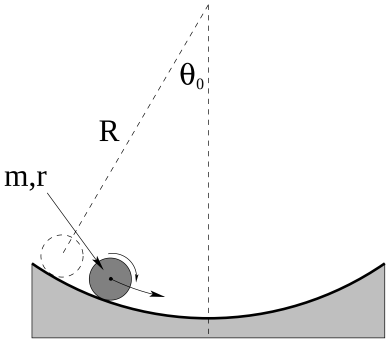
Draw a force diagram on the rolling mass at a general angle θ. Choose a positive direction for θ and (I recommend) choose to use $s = R\theta$ as a linear coordinate and choose a pivot either at the CM of the disk or the point where it touches the floor.

Oscillations: A Rolling Pendulum



- A disk of mass m and radius r is gently set on a rough circular floor so that it makes an angle θ_0 relative to a vertical through the center of curvature of the floor, with its center of mass a distance R from the center of curvature as shown, and is released from rest at $t = 0$ so that it ***rolls without slipping and oscillates***.
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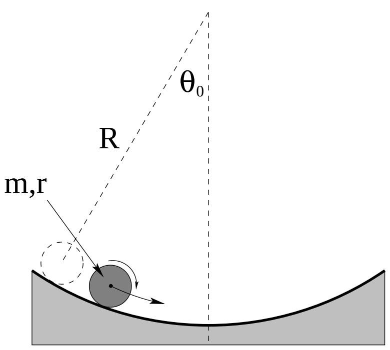

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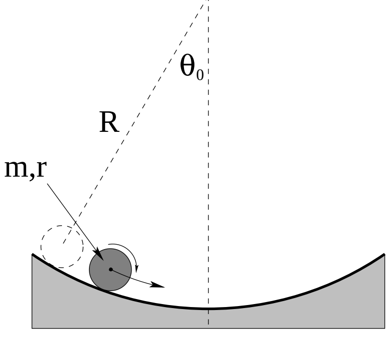

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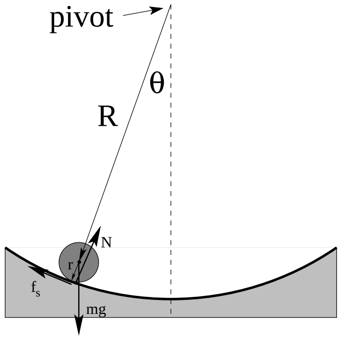

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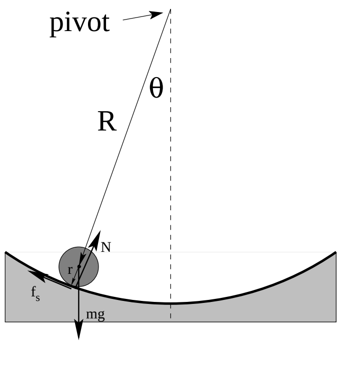

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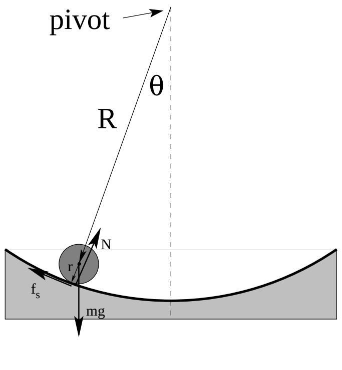

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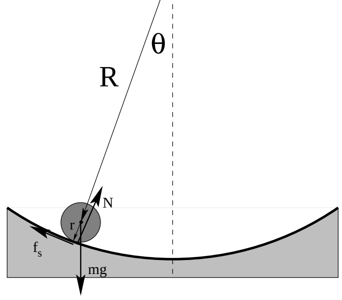

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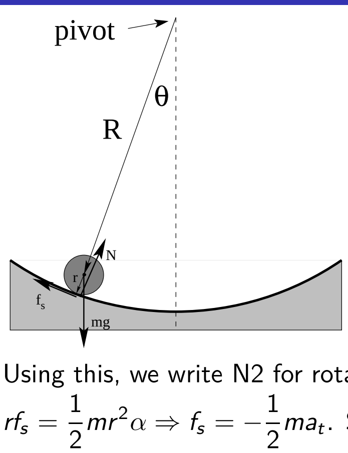

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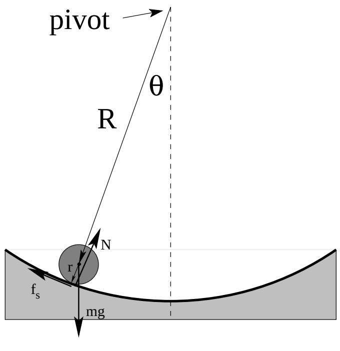

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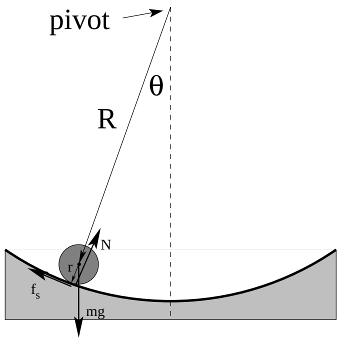

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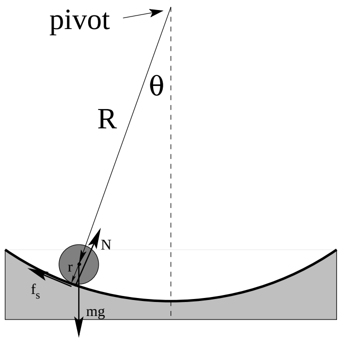

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$$\boxed{\omega = \sqrt{\frac{2g}{3R}}} \qquad \boxed{\theta(t) = \theta_0\cos(\sqrt{\frac{2g}{3R}}t)} \qquad \boxed{f_s = -\frac{1}{2}ma_t = \frac{mg}{3}\theta_0\cos(\sqrt{\frac{2g}{3R}}t)}$$

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  This too makes sense! At $t = 0$, the disk starts to roll *down to the right*, so $\Omega_{\rm disk}$ is into the page, positive. You should be able to trace each quarter cycle of its oscillation and see that everything is consistent and correct.

Construct a Travelling Wave

Suppose: Amplitude $A = 1$ cm; Wavelength $\lambda = 0.5$ m; Period $T = 0.001$ sec
Write down the formula for a transverse wave travelling ***in the $-x$ direction***
What is the speed of the wave on the string in terms of the givens? Which of the following changes would ***double the power*** transmitted by the string (changing ***only one of*** A, T, λ and nothing else)?

- ☐ Change the amplitude to $A' = 0.707\ A$
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Construct a Travelling Wave-Solution

$$\boxed{y(x,t) = 0.01 \sin(4\pi x + 2000\pi t) \text{ m}} \qquad \boxed{v = \frac{\lambda}{T} = 500 \text{ m/sec}}$$

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Start with $P = \frac{1}{2}\mu\omega^2 A^2 v$. Then: $P \propto A^2$; also *both* $\omega = \frac{2\pi}{T}$ *and* $v = \frac{\lambda}{T}$ change, so $P \propto \frac{1}{T^3}$; finally if λ changes only v changes, so $P \propto \lambda$. Hence:

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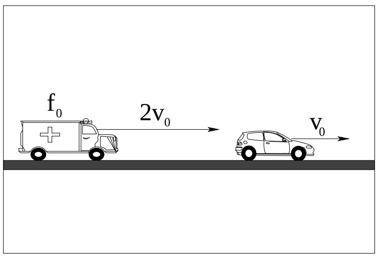
Passed by the Ambulance



You are slowed down, being overtaken by an ambulance that is travelling at twice your speed of v_0. You happen to know from your days working in EMS that the frequency of the siren of the ambulance is f_0, but the frequency detector app on your phone reads $f_1 > f_0$. Using f_0, f_1 and v_a (the speed of sound in air) find an expression for v_0, the speed of your car at the time. Evaluate this for $f_0 = 1900$ Hz, $f_1 = 2000$ Hz.

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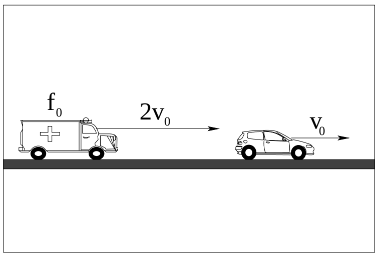

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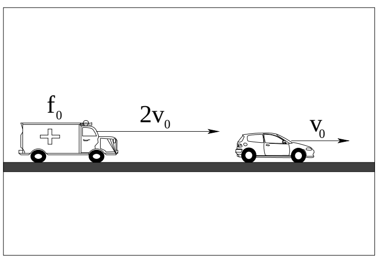

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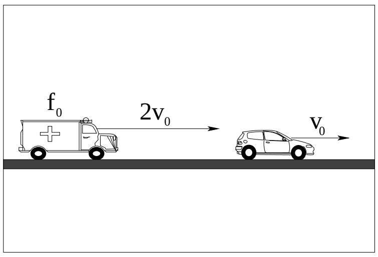

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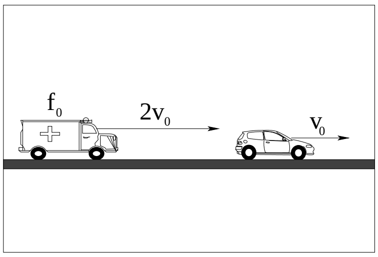
Solution Map:
You are a moving receiver, the ambulance is a moving source. Write down the combined moving source/moving receiver doppler shift formula for f_1, the double shifted f_0.

Passed by the Ambulance



You are slowed down, being overtaken by an ambulance that is travelling at twice your speed of v_0. You happen to know from your days working in EMS that the frequency of the siren of the ambulance is f_0, but the frequency detector app on your phone reads $f_1 > f_0$. Using f_0, f_1 and v_a (the speed of sound in air) find an expression for v_0, the speed of your car at the time. Evaluate this for $f_0 = 1900$ Hz, $f_1 = 2000$ Hz.

Solution Map:
You are a moving receiver, the ambulance is a moving source. Write down the combined moving source/moving receiver doppler shift formula for f_1, the double shifted f_0. Solve this equation algebraically for v_0 in terms of v_a (I like to use $x = v_0/v_a$ in the algebra until the end).

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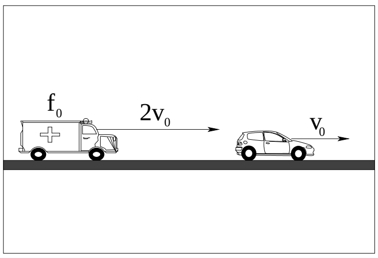

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$$f' = \frac{1 \pm \frac{v_r}{v_a}}{1 \mp \frac{v_s}{v_a}} f_0 \text{ (upper signs approaching, lower signs receding).}$$

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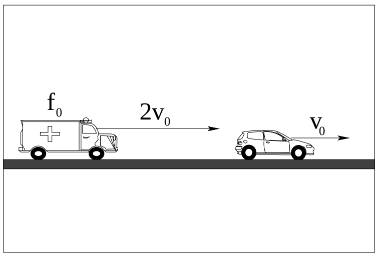

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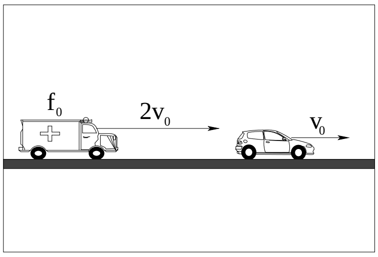
Passed by the Ambulance-Solution



- This one is simple. The car (you, receiver) is receding. The source (ambulance) is approaching. So:

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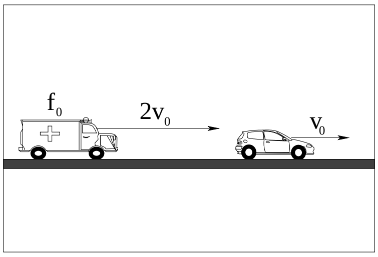

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- Let $x = v_0/v_a$. Then:

$$f_1 = \frac{1-x}{1-2x} f_0 \Rightarrow (1-2x) f_1 = (1-x) f_0$$

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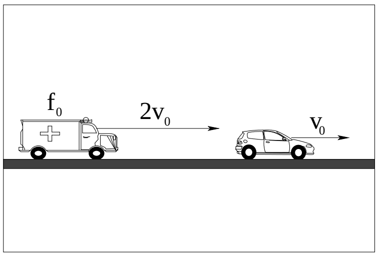

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- Or:

$$f_1 - f_0 = 2xf_1 - xf_0 = x(2f_1 - f_0) \Rightarrow x = \frac{v_0}{v_a} = \frac{f_1 - f_0}{2f_1 - f_0}$$

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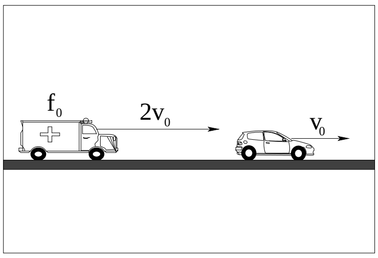

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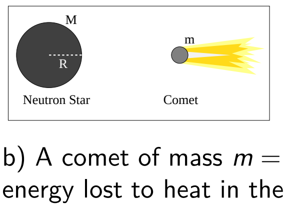
- And:

$$\boxed{v_0 = \frac{f_1 - f_0}{2f_1 - f_0} v_a} \text{ or in SI units, } \boxed{v_0 = \frac{100}{2100} 343 = \frac{343}{21} \approx 16 \text{ m/sec}}$$

If we multiply by 9/4, we get a good estimate of miles per hour: $v_0 = 37$ mph and the ambulance is going around 75 mph.

Collision with a Neutron Star



In the figure, a neutron star with mass $M = 10^{30}$ kg and radius $R = 8000$ m is drawn. Find: a) Escape speed from its surface. First find it algebraically, then evaluate it numerically as a fraction of the speed of light $c = 3 \times 10^8$ m/sec.

b) A comet of mass $m = 10^{14}$ kg falls "from infinity" to its surface. Estimate the energy lost to heat in the collision algebraically, and *then* evaluate it numerically, expressing it as a fraction of the relativistic rest energy of the comet.

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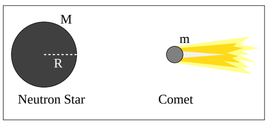

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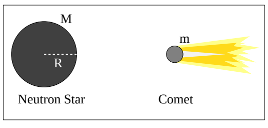

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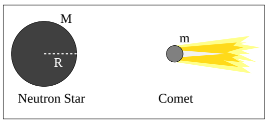

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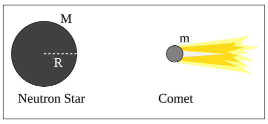

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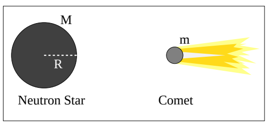

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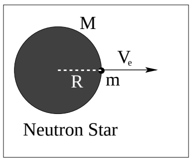

- $E = 0 = -\frac{GMm}{R} + \frac{1}{2}mv_e^2 \Rightarrow \boxed{v_e = \sqrt{\frac{2GM}{R}}}$

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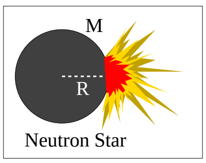

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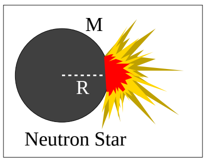

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Numerically: $\frac{v_e}{c} = \frac{1}{c}\sqrt{\frac{2GM}{R}} = \frac{1}{3 \times 10^8}\sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 10^{30}}{8000}} \Rightarrow$

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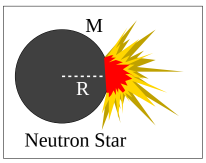

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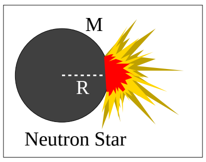

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And: $\frac{\Delta E_{\text{heat}}}{mc^2} = \frac{GM}{Rc^2} = \frac{6.67 \times 10^{-11} \times 10^{30}}{8000 \times 9 \times 10^{16}} \Rightarrow$

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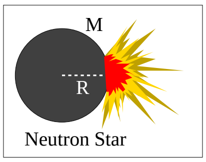

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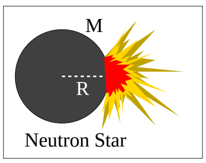

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This *classical, non-relativistic* solution isn't quite right! Because v_e is so close to the speed of light, to do it correctly we'd have to use relativistic expressions for things like the total energy and kinetic energy. But it does give us a realistic appreciation for the strangeness of things like neutron stars!

The End

Feedback Welcome

Send Comments To: rgb at duke dot edu