Introductory Physics III

Thermodynamics, Relativity theory, and Quantum Mechanics

by

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Preface

This introductory text on thermodynamics, relativity theory, and quantum mechanics is intended to be used in the third semester of a three-semester series of courses teaching *introductory physics* at the college level. This course is most often taken only by physics majors, math majors, and/or engineers. The text is intended to support teaching the material at a rapid, but *advanced* level – it was developed to support teaching introductory calculus-based physics to potential physics majors, engineers, and other natural science majors at Duke University over a period of more than twenty-five years.

Students who hope to succeed in learning physics from this text will need, as a minimum prerequisite, a solid grasp of mathematics. It is strongly recommended that all students have mastered mathematics at least through multivariable differential and integral calculus by the time they take this third semester course and hence are familiar with e.g. the concept of the partial derivative and have some knowledge of simple first and second order differential equations.

To help students who are, perhaps, a bit shaky in their math preparation, A separate supplementary text intended *specifically to help students of introductory physics quickly and efficiently review the required math* is being prepared as a companion volume to all semesters of introductory physics. Indeed, it should really be quite useful for any course being taught with any textbook series and not just this one.

This book is located here:

http://www.phy.duke.edu/~rgb/Class/math_for_intro_physics.php

and I *strongly suggest* that all students who are reading these words preparing to begin studying introductory physics pause for a moment, visit this site, and either download the pdf or bookmark the site.

It is also strongly suggested that (new) students using this volume in the series as their first visit and look over Part 1 of *Introductory Physics I*, the first volume of this series. It is devoted not to physics or math but to *how to learn physics* effectively and rapidly, in particular how to work towards *mastery* of the material instead of just “getting through it” a single time. Mastery is important, whether your ultimate goal is to stop after this class or proceed on with a major, minor, or other physics-intensive study in another discipline.

Textbook Layout and Design

This textbook has a design that is just about perfectly backwards compared to most textbooks that currently cover the subject. Here are its primary design features:

- All mathematics required by the student is reviewed in a standalone, cross-referenced (free) work at the *beginning* of the book rather than in an appendix that many students never find.
- There are only *twelve chapters*. The book is organized so that it can be sanely taught in a *single college semester* with at *most* a chapter a week.
- It *begins* each chapter with an "abstract" and chapter summary. Detail, especially lecture-note style mathematical detail, follows the summary rather than the other way around.
- This text does *not* spend page after page trying to explain in English how physics works (prose which to my experience nobody reads anyway). Instead, a terse "lecture note" style presentation outlines the main points and presents considerable mathematical detail to support solving problems.
- Verbal and conceptual understanding *is*, of course, very important. It is expected to come from verbal instruction and discussion in the classroom and recitation and lab. This textbook *relies* on having a committed and competent instructor and a sensible learning process.
- Each chapter ends with a *short* (by modern standards) selection of *challenging* homework problems. A good student might well get through all of the problems in the book, rather than at most 10% of them as is the general rule for other texts.
- The problems are weakly sorted out by level, as this text is intended to support non-physics science and pre-health profession students, engineers, and physics majors all three. The *material* covered is of course the same for all three, but the level of detail and difficulty of the math used and required is a bit different.
- The textbook is entirely algebraic in its presentation and problem solving requirements – with *very few exceptions* no calculators should be required to solve problems. The author assumes that any student taking physics is capable of punching numbers into a calculator, but it is *algebra* that ultimately determines the formula that they should be computing. Numbers are used

in problems only to illustrate what “reasonable” numbers might be for a given real-world physical situation or where the problems cannot reasonably be solved algebraically (e.g. resistance networks).

This layout provides considerable benefits to both instructor and student. This textbook supports a *top-down* style of learning, where one learns each distinct chapter topic by quickly getting the main points onboard via the summary, then derives them or explores them in detail, then applies them to example problems. Finally one uses what one has started to learn working in groups and with direct mentoring and support from the instructors, to solve highly challenging problems that *cannot* be solved without acquiring the deeper level of understanding that is, or should be, the goal one is striving for.

It’s without doubt a lot of work. Nobody said learning physics would be *easy*, and this book certainly doesn’t claim to make it so. However, this approach will (for most students) *work*.

The reward, in the end, is the ability to see the entire world around you through new eyes, understanding much of the “magic” of the causal chain of physical forces that makes all things unfold in time. Natural Law is a strange, beautiful sort of magic; one that is utterly impersonal and mechanical and yet filled with structure and mathematics and light. It *makes sense*, both in and of itself and of the physical world you observe.

Enjoy.

I: Thermodynamics

Let us begin our actual week by week, day by day progress through the course material. For maximal ease of use for you the student and (one hopes) your instructor whether or not that instructor is me, the course is designed to cover *one chapter per week-equivalent*, whether or not the chapter is broken up into a day and a half of lecture (summer school), an hour a day (MWF), or an hour and a half a day (TTh) in a semester based scheme. To emphasize this preferred rhythm, each chapter will be referred to by the *week* it would normally be covered in my own semester-long course.

A week's work in all cases covers just about exactly one "topic" in the course. A very few are spread out over two weeks; one or two compress two related topics into one week, but in all cases the *homework* is assigned on a weekly rhythm to give you ample opportunity to use the *method of three passes* described in the first part of the book, culminating in an expected 2-3 hour *recitation* where you should go over the assigned homework *in a group* of three to six students, with a mentor handy to help you where you get stuck, with a goal of *getting all of the homework perfectly correct by the end of recitation*.

That is, at the end of a week plus its recitation, you *should* be able to do *all* of the week's homework, *perfectly*, and *without looking or outside help*. You will usually *need* all three passes, the last one working in a group, *plus* the mentored recitation to achieve this degree of competence! But without it, surely the entire process is a waste of time. Just *finishing* the homework is not enough, the whole point of the homework is to help you learn the material and it is the latter that is the real goal of the activity not the mere completion of a task.

However, *if* you do this – attempt to really master the material – you are almost certain to do well on a quiz that terminates the recitation period, and you will be very likely to *retain* the material and not have to "cram" it in again for the hour exams and/or final exam later in the course. Once you achieve *understanding* and reinforce it with a fair bit of repetition and practice, most students will naturally transform this experience into remarkably deep and permanent learning.

Note well that each week is organized for *maximal ease of learning* with the week/chapter review *first*. Try to *always look at this review before lecture* even if you skip reading the chapter itself until later, when you start your homework. Skimming the whole week/chapter guided by this summary before lecture is, of course, better still. It is a "first pass" that can often make lecture much easier to follow and help free you from the tyranny of note-taking as you only need to note *differences* in the presentation from this text and perhaps the answers to *questions* that helped you understand something during the discussion. Then read or skim it again right before each homework pass.

Week 1: The 0th Law of Thermodynamics

0th Law of Thermodynamics Summary

- **Thermal Equilibrium**

  A system with many microscopic components (for example, a gas, a liquid, a solid with many molecules) that is isolated from all forms of energy exchange and left alone for a "long time" moves toward a state of *thermal equilibrium*. A system in thermal equilibrium is characterized by a set of macroscopic quantities that depend on the system in question and characterize its "state" (such as pressure, volume, density) that do not change in time.

  Two systems are said to be in (mutual) thermal equilibrium if, when they are placed in "thermal contact" (basically, contact that permits the exchange of energy between them), their state variables do not change.

- **Zeroth Law of Thermodynamics**

  If system A is in thermal equilibrium with system C, and system B is in thermal equilibrium with system C, then system A is in thermal equilibrium with system B.

- **Temperature and Thermometers**

  The point of the Zeroth Law is that it is the basis of the thermometer. A thermometer is a portable device whose thermal state is related linearly to some simple property, for example its density or pressure. Once a suitable temperature scale is defined for the device, one can use it to measure the temperature of a variety of disparate systems in thermal equilibrium. Temperature thus characterizes thermal equilibrium.

- **Temperature Scales**

  a) **Fahrenheit**: This is one of the oldest scales, and is based on the coldest temperature that could be achieved with a mix of ice and alcohol. In it the freezing point of water is at 32° F, the boiling point of water is at 212° F.

b) **Celsius or Centigrade**: This is a very sane system, where the freezing point of water is at 0° C and the boiling point is at 100° C. The degree size is thus $9/5$ as big as the Fahrenheit degree.

c) **Kelvin or Absolute**: 0° K is the lowest possible temperature, where the internal energy of a system is at its absolute minimum. The degree size is the same as that of the Centigrade or Celsius scale. This makes the freezing point of water at atmospheric pressure 273.16° K, the boiling point at 373.16° K.

- **Thermal Expansion**

$$\Delta L = \alpha L \Delta T \tag{1.1}$$

where α is the *coefficient of linear expansion*. If one applies this in three dimensions:

$$\Delta V = \beta V \Delta T \tag{1.2}$$

where $\beta = 3\alpha$.

- **Ideal Gas Law**

$$PV = nRT = NkT \tag{1.3}$$

where $R = 8.315$ J/mol-K, and $k = R/N_A$ = 1.38$\times 10^{-23}$ J/K.

Week 2: The First Law of Thermodynamics

First Law of Thermodynamics Summary

- **Internal Energy**

  Internal energy is all the mechanical energy in all the components of a system. For example, in a monoatomic gas it might be the sum of the kinetic energies of all the gas atoms. In a solid it might be the sum of the kinetic and potential energies of all the particles that make up the solid.

- **Heat**

  Heat is a bit more complicated. It is internal energy as well, but it is internal energy that is *transferred* into or out of a given system. Furthermore, it is in some fundamental sense "disorganized" internal energy – energy with no particular organization, random energy. Heat flows into or out of a system in response to a temperature difference, always flowing from hotter temperature regions (cooling them) to cooler ones (warming them).

  Common units of heat include the ever-popular Joule and the *calorie* (the heat required to raise the temperature of 1 gram of water at 14.5° C to 15.5° C. Note that 1 cal = 4.186 J. Less common and more esoteric ones like the British Thermal Unit (BTU) and erg will be mostly ignored in this course; BTUs raise the temperature of one pound of water by one degree Fahrenheit, for example. Ugly.

- **Heat Capacity**

  If one adds heat to an object, its temperature usually increases (exceptions include at a state boundary, for example when a liquid boils). In many cases the temperature change is *linear* in the amount of heat added. We define the heat capacity C of an object from the relation:

$$\Delta Q = C\Delta T \tag{2.1}$$

  where ΔQ is the heat that flows into a system to increase its temperature by ΔT. Many substances have a known heat capacity per unit mass. This permits us to also write:

$$\Delta Q = mc\Delta T \tag{2.2}$$

where c is the *specific heat* of a substance. The specific heat of liquid water is approximately:

$$c_{\rm water} = 1 calorie/gram -^{\circ} C \tag{2.3}$$

(as one might guess from the definition of the calorie above).

- **Latent Heat** As noted above, there are particular times when one can add heat to a system and not change its temperature. One such time is when the system is changing state from/to solid to/from liquid, or from/to liquid to/from gas. At those times, one adds (or removes) heat when the system is at fixed temperature until the state change is complete. The specific heat may well change across phase boundaries. There are two trivial equations to learn:

$$\Delta Q_f = mL_f \tag{2.4}$$

$$\Delta Q_v = mL_v \tag{2.5}$$

where L_f is the *latent heat of fusion* and L_v is the *latent heat of vaporization*. Two important numbers to keep in mind are $L_f(H_2O) = 333$ kJ/kg, and $L_v(H_2O) = 2260$ kJ/kg. Note the high value of the latter – the reason that "steam burns worse than water".

- **Work Done by a Gas**

$$W = \int_{V_i}^{V_f} PdV \tag{2.6}$$

This is the area under the $P(V)$ curve, suggesting that we draw lots of state diagrams on a P and V coordinate system. Both heat transfer and word depend on the *path* a gas takes $P(V)$ moving from one pressure and volume to another.

- **The First Law of Thermodynamics**

$$\Delta E_{\rm int} = \Delta Q - W \tag{2.7}$$

In words, this is that the change in total mechanical energy of a system is equal to heat put into the system plus the work done *on* the system (which is minus the work done *by* the system, hence the minus above).

This is just, at long last, the fully *generalized* law of conservation of energy. All the cases where mechanical energy was not conserved in previous chapters because of nonconservative forces, the missing energy appeared as *heat*, energy that naturally flows from hotter systems to cooler ones.

- **Cyclic Processes** Most of what we study in these final sections will lead us to an understanding of simple heat engines based on gas expanding in a cylinder and doing work against a piston. In order to build a true engine, the engine has to go around in a repetitive *cycle*. This cycle typically is represented by a closed loop on a state e.g. $P(V)$ curve. A direct consequence of the 1st law is that **the net work done by the system per cycle is the area inside the loop of the $P(V)$ diagram**. Since the internal energy is the same at the beginning and the end of the cycle, it also tells us that:

$$\Delta Q_{\rm cycle} = W_{\rm cycle} \tag{2.8}$$

the heat that flows into the system per cycle must exactly equal the work done by the system per cycle.

- **Adiabatic Processes** are processes (PV curves) such that no heat enters or leaves an (insulated) system.
- **Isothermal Processes** are processes where the temperature T of the system remains constant.
- **Isobaric Processes** are processes that occur at constant pressure.
- **Isovolumetric Processses** are processes that occur at constant volume.
- Work done by an Ideal Gas: Recall,

$$PV = NkT \tag{2.9}$$

where N is the number of gas atoms or molecules. Isothermal work at (fixed) temperature T_0 is thus:

$$\begin{aligned} W &= \int_{V_1}^{V_2} \frac{NkT_0}{V} dV && (2.10) \\ &= NkT \ln(\frac{V_2}{V_1}) && (2.11) \end{aligned}$$

Isobaric work is trivial. $P = P_0$ is a constant, so

$$W = \int_{V_1}^{V_2} P_0 dV = P_0(V_2 - V_1) \tag{2.12}$$

Adiabatic work is a bit tricky and depends on some of the internal properties of the gas (for example, whether it is mono- or diatomic). We'll examine this in the next section.

Week 3: The Second Law of Thermodynamics

Second Law of Thermodynamics Summary

- **Heat Engines**

  A *heat engine* is a cyclic device that takes heat Q_H in from a *hot reservoir*, converts *some* of it to work W, and rejects the rest of it Q_C to a *cold reservoir* so that at the end of a cycle it is in the same state (and has the same internal energy) with which it began. The net work done per cycle is (recall) the area inside the PV curve.

  The *efficiency* of a heat engine is defined to be

  $$\epsilon = \frac{W}{Q_H} = \frac{Q_H - Q_C}{Q_H} = 1 - \frac{Q_C}{Q_H} \tag{3.1}$$

- **Kelvin-Planck statement of the Second Law of Thermodynamics**

  It is impossible to construct a cyclic heat engine that produces no other effect but the absorption of energy from a hot reservoir and the production of an equal amount of work.

- **Refrigerators (and Heat Pumps)**

  A *refrigerator* is basically a cyclic heat engine run backwards. In a cycle it takes heat Q_C in from a cold reservoir, does work W on it, and rejects a heat Q_H to a hot reservoir. Its net effect is thus to make the cold reservoir colder (refrigeration) by removing heat from inside it to the warmer warm reservoir (warming it still further, e.g. as a heat pump). Both of these functions have practical applications – cooling our homes in summer, heating our homes in winter.

  The *coefficient of performance* of a refrigerator is defined to be

  $$\text{COP} = \frac{Q_C}{W} \tag{3.2}$$

  It is not uncommon for heat pumps to have a COP of 3-5 (depending on the temperature differential) giving them a significant economic advantage over resistive heating. The bad side is that they don't work terribly well when the temperature difference is large in degrees K.

- **Clausius Statement of the Second Law of Thermodynamics**

  It is impossible to construct a cyclic refrigerator whose sole effect is the transfer of energy from a cold reservoir to a warm reservoir without the input of energy by work.

- bf Reversible Processes Reversible processes are ones where no friction or turbulence or dissipative forces are present that represent an additional source of energy loss or gain for a given system. For the purposes of this book, both adiabatic and isothermal processes are reversible. Irreversible processes include the transfer of heat energy from a hot to a cold reservoir in general – heat engines and refrigerators can be constructed whose steps in a cycle are all reversible, but the overall effect of transferring heat one way or the other is irreversible.

- **Carnot Engine**

  The *Carnot Cycle* is the archetypical reversible cycle, and a Carnot Cycle-based heat engine is one that does not dissipate any energy internally and uses only reversible steps. *Carnot's Theorem* states that no real heat engine operating between a hot reservoir at temperature T_H and a cold reservoir at temperature T_C can be more efficient than a Carnot engine operating between those two reservoirs.

  The Carnot efficiency is easy to compute (see text and lecture example). A Carnot Cycle consists of four steps:

  a) Isothermal expansion (in contact with the heat reservoir)
  b) Adiabatic expansion (after the heat reservoir is removed)
  c) Isothermal compression (in contact with the cold reservoir)
  d) Adiabatic compression (after the cold reservoir is removed)

  The efficiency of a Carnot Engine is:

$$\epsilon_{\mathrm{Carnot}} = 1 - \frac{T_C}{T_H} \tag{3.3}$$

- **Entropy**

  Entropy S is a measure of disorder. The change in entropy of a system can be evaluated by integrating:

$$dS = \frac{dQ}{T} \tag{3.4}$$

between successive infinitesimally separated equilibrium states (the weasel language is necessary because temperature should be constant in equilibrium, but systems in equilibrium have constant entropy). Thus:

$$\Delta S = \int_{T_i} T_f \frac{dQ}{T} \tag{3.5}$$

has limited utility except for particularly simple processes (like the cooling of a hot piece of metal in a body of cold water.

We extend our definition of reversible processes. A reversible process is one where the entropy of the system does not change. An irreversible process increases the entropy of the system and its surroundings.

- **Entropy Statement of the Second Law of Thermodynamics**

  The entropy of the Universe never decreases. It either increases (for irreversible processes) or remains the same (for reversible processes).

Week 4: Heat Transport

Heat Transport Summary

- **Conduction: Fourier's Law**

$$\vec{q} = -\kappa \vec{\nabla} T$$

where $\vec{q}$ is the local vector *heat flux* in watts per meter squared, κ is the material's *conductivity*, in $\frac{\mathrm{Watts}}{\mathrm{m}^2 K^\circ}$, and T is the local temperature field (temperature as a function of position in the material) in $^\circ$K.

- **The Heat Equation**
- **Convection**
- **Radiation**

II: Relativity

Week 5: The Lorentz Transformation

Here are two very simple derivations of the theory of special relativity in one dimension, also known as the Lorentz transformation in one dimension. The first is given in many places, for example by Einstein in his 1920 book on relativity (as Appendix 1). Let's consider the usual Galilean coordinate frame transformation:

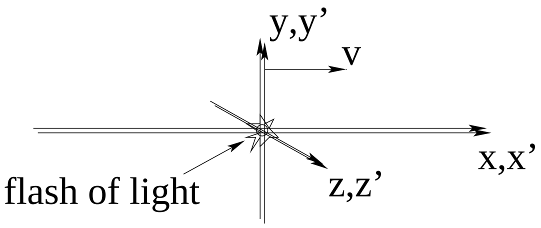

Figure 1: Two coordinate frames, $S = (x, y, z, t)$ and $S' = (x', y', z', t')$ that have ***coincident origins*** at $t = t' = 0$.

In figure 1, two "inertial reference frame" (IRF) coordinate systems are shown, with the system S' in uniform (constant velocity) motion at speed v in the $+x$ direction. As is well known and easy to show, the Galilean (non-relativistic) transformation between the frames that results in their origins coinciding at $t = t' = 0$ is:

$$x' = x - vt \tag{5.1}$$

$$y' = y \tag{5.2}$$

$$z' = z \tag{5.3}$$

$$t' = t \tag{5.4}$$

where the universal assumption is that time itself is invariant in all frames, $t = t'$. This coordinate transformation has the virtue of preserving Newton's Laws of Motion as long as all of the forces that appear in it are one member of Newton's Third Law pairs – no "pseudoforces" lacking a third law partner and resulting from the acceleration of the frame are needed to solve the second law equations of motion consistently, and all of the third-law-consistent force laws are presumed to themselves be invariant under conversion from S to S' coorindates.

However, this last assumption is *violated* by Maxwell's equations and the laws of electrodynamics. As formulated in one IRF, they lead unambiguously to an electromagnetic wave equation. If one changes reference frames according to the Galilean prescription, they do not. One is then faced with a conundrum: No matter what, Newton's Laws are *not* going to be invariant under Galilean inertial reference frame changes, because the force laws associated with the best understood force of nature are themselves not invariant. Either we give on the invariance of the forces of nature under Galilean IRF transformations or give up *Galilean* frame transformations and search for a new IRF transformation that preserves the invariance of the laws of electrodynamics (and hopefully, all the other force laws as well).

One key implication of Maxwell's equations is that the speed of light in a vacuum is:

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = \sqrt{\frac{k_e}{k_m}} \tag{5.5}$$

If this speed is empirically observed to be different when measured in different IRFs, then we should select the first choice, however painful. If not, we should select the second and search for a new IRF transformation that preserves the form of Maxwell's equations (and all of the consequences of preserving that form) as *invariant laws of nature*, among them the very simple consequence that the speed of light in a vacuum itself is invariant under all IRF transformations.

Einstein (according to self-reported legend, if not third-party history) had intuitively figured this out by thinking about whether or not simple electromagnetic experiments performed on a moving light wave could depend on whether or not these experiments were conducted in a frame, like that of a moving train, moving in the direction of the beam. He concluded (in agreement with the general argument above) that the invariance of the speed of light and of Maxwell's equations were tied together all the way back in what amounts to "high school".

This led to him introducing what is recorded as two postulates, but is really only one (plus the pre-existing postulate that Maxwell's equations are laws of nature):

a) ***The laws of nature are invariant with respect to transformations between inertial reference frames***. Note that this postulate is hardly original to Einstein and was the original motivation for the existing Galilean transformation from the time of Newton on.

b) Maxwell's equations and the general equations of electrodynamics are laws of nature, and hence must be invariant with respect to IRF transformations. To keep it simple, he reduced this to the single "postulate" that ***the speed of light is invariant under all inertial reference frame transformations***.

When assigning credit for all this, it is important to realize that all of this had already been worked out and applied to Maxwell's equations by a number of "the giants" of physics of the day years before Einstein published his 1905 paper, which is one reason among many that *Einstein did not win a Nobel prize for relativity!* He actually was not even close to being the first person to *derive* the theory of relativity. The one flaw in the previous treatments and derivations of relativity *from the assumption that Maxwell's equations would be invariant* was they often made the additional assumption of the existence of some sort of *medium* for the propagation of electromagnetic waves – the "luminiferous aether" which was presumed to carry light waves the same way air carries sound waves – which led to consistency issues and which was contradicted by the Michelson-Morley experiment in 1897.

By 1905 (the year Einstein published his theory of special relativity), Henri Poincaré had even realized that the set of *well-known* coordinate transformations that leave Maxwell's equations invariant form a *group*, which he named the ***Lorentz*** transformation, and was hot on the trail of the rest of the kinematical consequences, but he failed to appreciate two things that Einstein's derivation made clear. Einstein's paper, in addition to resulting in a remarkably simple statement of and derivation of the result (the Lorentz transformation), worked out most of the important kinematic and dynamic consequences for simple motions and frame changes, and *from the beginning* established the *symmetry* of frame transformations and the consequent necessary abandonment of the concept of simultaneity and universal time, which Poincaré at that time still preserved. Finally, Einstein's paper *eliminated any need for the concept of the aether* as unnecessary, which meant that it was in agreement with Michelson-Morley from the beginning.

These contributions sufficed to make the resulting derivation of the *Lorentz* transformation "Einstein's" for the rest of all time in the same way that the addition of Maxwell's Displacement Current to Ampere's Law made the set of electrodynamic equations (worked out by Franklin, Gauss, Coulomb, Ampere, and Faraday into "Maxwell's equations" (no doubt to the dismay of Michael Faraday, who had made at least an equal contribution in the form of Faraday's Law).

Let's apply Einstein's second postulate to *both* of the frame transformations implicit in the figure above. Suppose (as shown) a single flash of light is emitted at their coincident origins at time $t = t' = 0$. Then in ***both*** frames, light must spread out in a sphere centered on *each* frame's origin. That is, if we let x be the location(s) of the wavefront emitted at $t = 0$ on the $\pm\hat{\boldsymbol{x}}$-axis of the S frame and x' be the location(s) of the wavefront emitted at $t' = 0$ on the $\pm\hat{\boldsymbol{x}}'$-axis (note that the wave spreads out in a *sphere* in both frames, so there is an obvious symmetry between the distance it has moved in $+\hat{\boldsymbol{x}}$ and $-\hat{\boldsymbol{x}}$, etc) we get:

$$x' - ct' = x - ct = 0 \text{ (along the } +\hat{\boldsymbol{x}} \text{ direction)} \tag{5.6}$$

and:

$$x' + ct' = x + ct = 0 \text{ (along the } -\hat{\boldsymbol{x}} \text{ direction)} \tag{5.7}$$

These waves are illustrated in figure 2.

Note that the wave that is spreading out symmetrically in S is completely offset relative to the (moving in S) origin of S', so that the wave does *not* appear to be

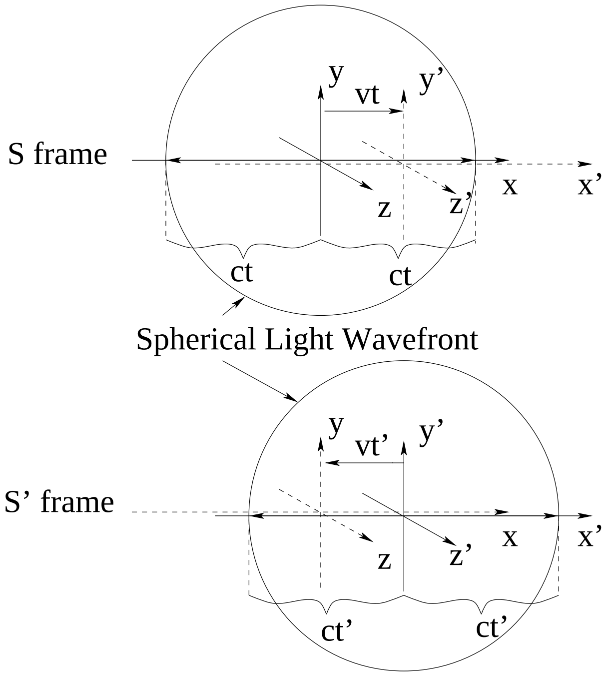

Figure 2: Spherical wavefronts of light emitted at the mutual space-time origin from figure 1, as seen by observers in frame S and S'. Note that these observers *completely disagree* about what the observers in the *other* frame see in this (necesarily) *Galilean* depiction of the frames.

spherically symmetric in S'. But the observer in S' thinks that it is the wavefront in S' that is spreading out in a sphere around the S' origin and this wavefront is obviousl not symmetric around the (moving in S') origin of S! It is this picture that is particularly difficult for a student to initially grasp and visualize, because we live in a low-velocity $v \ll c$ world where the two pictures are practically indistinguishable in either direction.

If you feel like your brain is exploding as you try to imagine both pictures to ***simultaneously be true and correct*** depending on which frame you are sitting in, that is perfectly normal. Relativity is *famous*, after all, for inducing a *paradigm shift* in our view of reality, which basically means that it (like quantum mechanics) makes *everybody's* brain explode when they first encounter it. Our job now is to put the pieces of your brain back together in such a way that you can see that the result, however difficult to visualize, is *consistent* and ultimately is *in agreement with experiment* (the only things that really matter in the long run). Over time and with some hard work and luck, you'll build up a *new and improved* brain that accepts this as just the way things (seem to) work and even learns to visualize what's going on, using tricks I'll illustrate as best I can below.

For the moment, let's completely ignore any better way to visualize this and get on with the algebra inherent in the picture and equations above to deduce the simplest form of the IRF transformation that makes it true. It is completely harmless to multiply one side of the first of the equations above with a dimensionless constant λ, and to multiply the same side of the second with a second dimensionless constant μ. That is:

$$\begin{aligned} x' - ct' &= \lambda(x - ct) = \lambda \times 0 = 0 \qquad (5.8) \\ x' + ct' &= \mu(x - ct) = \mu \times 0 = 0 \qquad (5.9) \end{aligned}$$

Distribute (and lose the zeros, we don't need them any more):

$$\begin{aligned} x' - ct' &= \lambda x - \lambda ct \qquad (5.10) \\ x' + ct' &= \mu x - \mu ct \qquad (5.11) \end{aligned}$$

Add them to eliminate ct', subtract them to eliminate x':

$$\begin{aligned} 2x' &= (\lambda + \mu)x - (\lambda - \mu)ct \qquad (5.12) \\ -2ct' &= (\lambda - \mu)x - (\lambda + \mu)ct \qquad (5.13) \end{aligned}$$

Divide, and rename the resulting combinations of λ and μ to simplify:

$$\begin{aligned} x' &= \frac{\lambda + \mu}{2}x - \frac{\lambda - \mu}{2}ct = \gamma x - \beta ct \qquad (5.14) \\ t' &= -\frac{\lambda - \mu}{2c}x + \frac{\lambda + \mu}{2}t = \gamma t - \frac{\beta}{c}x \qquad (5.15) \end{aligned}$$

where we replace (permanently) the arbitrary λ and μ with the equally arbitrary (but now consistently implemented):

$$\begin{aligned} \gamma &= \frac{\lambda + \mu}{2} \qquad (5.16) \\ \beta &= \frac{\lambda - \mu}{2} \qquad (5.17) \end{aligned}$$

Consider the origin of the S' frame, $x' = 0$. At that point we must always have:

$$x' = \gamma x - \beta ct = \gamma \left(x - \frac{\beta}{\gamma} ct \right) = 0 \qquad (5.18)$$

But $x = vt$ is the *location of the S' origin in S!* Hence, allowing for nontrivial $\gamma \neq 0$:

$$vt - \frac{\beta}{\gamma}ct = 0 \Longrightarrow v = \frac{\beta c}{\gamma} \qquad (5.19)$$

This let's us rewrite our transformation equations in a simpler form:

$$\begin{aligned} x' &= \gamma\,(x - vt) \qquad (5.20) \\ t' &= \gamma \left(t - \frac{v}{c^2}x \right) \qquad (5.21) \end{aligned}$$

Now we implement the "startling" result that *the exact same relationship has to work in the other direction,* that is, going from x', t' back to x, t we simply replace $v \to -v$ as S is moving in the $-\hat{\boldsymbol{x}}'$ direction:

$$x = \gamma\,(x' + vt') \tag{5.22}$$
$$t = \gamma\left(t' + \frac{v}{c^2}x'\right) \tag{5.23}$$

Note that we have to require *both* of these conditions simultaneously to solve for γ, essentially insisting that if we transform from S to S' and then *transform back* from S' to S using the *same* transformation, we get back to where we started! This is essentially a consistency condition. Substituting:

$$x = \gamma\left\{\gamma\,(x - vt) + v\gamma\left(t - \frac{v}{c^2}x\right)\right\} \tag{5.24}$$
$$t = \gamma\left\{\gamma\left(t - \frac{v}{c^2}x\right) + \frac{v}{c^2}\gamma\,(x - vt)\right\} \tag{5.25}$$

Hence:

$$x = \gamma^2\left\{(x - vt) + \left(vt - \frac{v^2}{c^2}x\right)\right\} = \gamma^2\left(1 - \frac{v^2}{c^2}\right)x \tag{5.26}$$

or:

$$\gamma^2\left(1 - \frac{v^2}{c^2}\right) = 1 \Longrightarrow \boxed{\gamma = \frac{+1}{\sqrt{1 - \frac{v^2}{c^2}}}} \tag{5.27}$$

We have ignored propagation in y and z because just requiring $y = y'$ and $z = z'$ suffices to ensure that both waves are spheres in S and S' respectively. One can do a more careful job of this (and not assume *anything* about the direction of $\vec{\boldsymbol{v}}$, for example) but there is no point in doing so using this simple framework in an introductory work – later, in a "real electrodynamics" course you will in all probability derive the special Lorentz group *properly* and such concerns will vanish once and for all, but this is actually sufficient and one can always choose coordinates initially so that x and x' line up with the direction of $\vec{\boldsymbol{v}}$.

We conclude that to go from S to S' coordinates we use:

$$x' = \gamma\,(x - vt) \tag{5.28}$$
$$t' = \gamma\left(t - \frac{v}{c^2}x\right) \tag{5.29}$$
$$y' = y \tag{5.30}$$
$$z' = z \tag{5.31}$$

and to go back from S' to S we use the *exact same transformation* except that (of course) now $\vec{\boldsymbol{v}} \to -\vec{\boldsymbol{v}}$ describes the position of the S origin in S', so we change the sign of v in the transformation:

$$x = \gamma\,(x' + vt') \tag{5.32}$$
$$t = \gamma\left(t' + \frac{v}{c^2}x'\right) \tag{5.33}$$
$$y = y \tag{5.34}$$
$$z = z \tag{5.35}$$

where:

$$\gamma = \frac{+1}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{5.36}$$

There are two very important observations that I omitted in the derivation above but that you should be aware of. First, since we managed quite well with the good old Galilean transformation for well over a century (if not longer, as it was intuitive from experience long before it was quantified) it is worth pointing out that when $v \ll c$, $\gamma \to 1$ and ***the Lorentz transformation asymptotically approaches the Galilean transformation***.

The second is that I cheated just a bit when I wrote the "+" sign in the definition of γ without explaining that I was choosing it *so that this would be the case!* After all, there is nothing in the math above that prohibits the minus sign solution from working! That's an important example of the difference between *pure math* and *physics*. We are *not* just finding the set of *all* transformations (at this time, anyway) that leave the speed of light invariant in all frames, because they include things like simply inverting the coordinate axes, inverting the time coordinate, and for that matter rotations of the coordinates. Right now we want only the subset of all such transformations that reduce in the low velocity limit to *only* the Galilean transformation, the usual transformation that lets you relate what you see throwing a ball up and down in a moving car to what an observer on the ground sees. It would be odd indeed[1] if the ground observer saw a *mirror image* of the motion of the motion they *do* in fact see, but from the point of view of not violating the laws of physics per se both alternatives are possible and correspond to a simple redefinition of the coordinates. Again, all of this will eventually be treated "correctly" in one or more advanced courses, or if you are in a hurry you can visit e.g. Wikipedia: http://www.wikipedia.org/wiki/Lorentz_group and follow links, or look in a good book on Electrodynamics.

[1]That's a complex pun, by the way, based on the fact that picking up a minus sign under coordinate inversion is the definition of odd functions...

III: Quantum Mechanics

Week 6: Blackbody Radiation and the Quantum Hypothesis

Week 7: The Photoelectric Effect

Week 8: Building an Atom: The Bohr Model and Matter Waves

Week 9: The Schrödinger Equation

Week 10: The 1-Dimensional Square Well

Week 11: The Harmonic Oscillator

Week 12: The Atom