

WUN2K FOR LECTURE 9

These are notes summarizing the main concepts you need to understand and be able to apply.

- *Complex frequencies* are useful to understand response of four-terminal networks: one just replaces ω with $\hat{s} = \sigma + j\omega$ in signal and transfer function expressions. A signal $f(t)$ becomes $f(t) = Ae^{\sigma t}e^{j\omega t}$, where the $e^{\sigma t}$ term describes the transient response and the $e^{j\omega t}$ term describes the oscillatory response. Positive σ is associated with exponential growth and negative σ is associated with damping. The complex frequency \hat{s} can be represented on the complex plane.
- A general transfer function describing a linear four-terminal network $\hat{H}(\hat{s})$ can be written as $\hat{H}(\hat{s}) = \frac{A(\hat{s}-\hat{a}_1)(\hat{s}-\hat{a}_2)\dots(\hat{s}-\hat{a}_n)}{(\hat{s}-\hat{b}_1)(\hat{s}-\hat{b}_2)\dots(\hat{s}-\hat{b}_m)}$. The \hat{a}_i are the *zeroes* and the \hat{b}_i are the *poles*; if you know the zeroes and poles, then you can determine the entire system behavior. There must be at least as many reactive (L or C) elements in a network as poles.
- One can infer the frequency-response behavior of a network by plotting the zeroes and poles on the complex \hat{s} plane, and moving up along the j axis ($\hat{s} = j\omega$): $|\hat{H}|$ is proportional to the product of distances between \hat{s} and the zeroes, divided by the product of distances between \hat{s} and the poles.
- The standard RC high-pass and low-pass filters have one pole each. One can arrange sequential high- and low-pass filter “sections” to get multipole filter circuits. Assuming that subsequent sections draw negligible current from previous sections (which can be achieved by giving the subsequent sections high impedance, or separating sections with a buffer), the total transfer function is the product of the individual

section transfer functions: $\hat{H}(\hat{s}) = \hat{H}_1(\hat{s})\hat{H}_2(\hat{s})$. The total transfer function then has the same poles and zeroes as the individual sections. From sequential filters, one can make high and low-pass filters with multiple slopes, as well as *band-pass* filters, which preferentially transmit frequencies in a given range.