Duke University
Department of Physics

Physics 271 Spring Term 2017

WUN2K FOR LECTURE 8

These are notes summarizing the main concepts you need to understand and be able to apply.

- The basic concept of Fourier analysis, very useful for signal processing, is that any periodic function $f(t)$ can be written as a sum of sinusoidal terms with frequencies that are integer multiples of the fundamental frequency, $\omega_1 = \frac{2\pi}{T}$. A useful way of writing this is as follows: $f(t) = a_0 + 2\sum_{n=1}^{\infty}[a_n \cos(n\omega_1 t) - b_n \sin(n\omega_1 t)]$, where the Fourier coefficients are given by $a_n = \frac{1}{T}\int_{t'}^{t'+T} f(t)\cos(n\omega_1 t)dt$ (even contributions) and $b_n = -\frac{1}{T}\int_{t'}^{t'+T} f(t)\sin(n\omega_1 t)dt$ (odd contributions). (Note the integration is over one cycle.) A linear transfer function $\hat{H}(j\omega)$ acts on each component of a signal; then one can sum the components to get the transformed signal. In general, a frequency-dependent transfer function will change the *shape* of an input signal.

- An *amplifier* has a transfer function $\hat{A}(j\omega)$, $\hat{V}_{\rm out}(j\omega) = \hat{A}(j\omega)\hat{V}_{\rm in}(j\omega)$. For $|\hat{A}| > 1$ (in which case power is added to a circuit) this is referred to as "gain".

- Common terminology used to refer to amplitude changes for filters or amplifiers:
  - Decibels are defined as $\mathrm{dB} = 20\log_{10}(\frac{|V_2|}{|V_1|})$.
  - For $|H(j\omega)| \sim \omega^n$, this is $\mathrm{dB} = n20\log_{10}(\frac{\omega_2}{\omega_1})$.
  - For an "octave" $\omega_2 = 2\omega_1$; for a "decade" $\omega_2 = 10\omega_1$. This gives $6.02n$ dB/octave and $20n$ dB/decade of frequency.

- *Filters* are networks with transfer functions that reduce some ranges of frequency but pass others.

- A *low-pass filter*, implemented simply by a resistor R in series with a capacitor C such that input is across both and output is across the capacitor, has the following properties:
  - The transfer function is $\hat{H}(j\omega) = \frac{1}{1+j\omega RC}$.
  - The *corner frequency* is $\omega_c = \frac{1}{RC}$. At this frequency, the response changes.
  - For $\omega << \omega_C$, $H_{\text{low}} \sim 1$ (low frequencies are passed).
  - For $\omega >> \omega_C$, $H_{\text{high}} \sim \frac{1}{\omega RC}$ (high frequencies are attenuated).
  - For $\omega >> \omega_C$, the network acts as an approximate integrator (an ideal integrator has $H_I = \frac{1}{j\omega}$).
- A *high-pass filter*, implemented simply by a resistor R in series with a capacitor C such that input is across both and output is across the resistor, has the following properties:
  - The transfer function is $\hat{H}(j\omega) = \frac{j\omega RC}{1+j\omega RC}$.
  - The *corner frequency* is $\omega_c = \frac{1}{RC}$. At this frequency, the response changes.
  - For $\omega << \omega_C$, $H_{\text{low}} \sim j\omega RC$ (low frequencies are attenuated).
  - For $\omega >> \omega_C$, $H_{\text{high}} \sim 1$ (high frequencies are passed).
  - For $\omega << \omega_C$, the network acts as an approximate differentiator (an ideal differentiator has $H_D = j\omega$).