Duke University
Department of Physics

Physics 271 Spring Term 2017

WUN2K FOR LECTURE 6

These are notes summarizing the main concepts you need to understand and be able to apply.

- The impedance $\hat{Z}$ in the AC form of Ohm's Law $\hat{V} = \hat{Z}_i$, is a complex number that can be written in polar form, $\hat{Z} = Z_0 e^{j\theta}$. The angle θ then determines the phase angle between current and voltage phasors. In general it is a function of the AC driving frequency ω. If $\theta > 0$, the voltage is said to "lead" the current (the peak arrives earlier in time); if $\theta < 0$, the voltage "lags" the current.

- A circuit with R, L and C elements will exhibit behavior called *resonance* when driven by an AC source (a mass on a spring with damping driven by a sinusoidal force is the mechanical analog). For an RLC series circuit, the voltage $\hat{V}_{ab}(j\omega)$ across a circuit element is given by $\hat{V}_{ab}(j\omega) = \hat{H}(j\omega)\hat{V}$, where $\hat{V}$ is the (complex time-dependent) sinusoidal driving voltage and $\hat{H}(j\omega)$ is known as the *transfer function.* For the RLC series circuit, looking at output across the resistor, $\hat{H}(j\omega) = \frac{R}{R+j(\omega L-1/\omega C)}$. The transfer function $\hat{H}$ describes the properties of the circuit independently of the input signal (and a similar quantity can be defined for other circuits). Some relevant quantities for resonant circuits (here, we consider the simple RLC circuit, but similar concepts and quantities apply to other configurations):
  - The *resonant frequency* $\omega_r = \frac{1}{\sqrt{LC}}$ is the frequency at which $|\hat{H}(j\omega)|$ is maximum, and the impedance becomes completely resistive.
  - The "*Q-factor*" or "quality factor" of a circuit is defined as 2π times the ratio of the stored energy in the circuit to the energy lost (e.g., dissipated in a resistor) per cycle. This is given to a good

approximation for situations we're usually interested in (circuits with large Q near resonance) by $Q = \frac{\omega_r L}{R}$. A large Q corresponds to relatively small energy lost per cycle (i.e., a system that "rings" a long time if not driven).

– A "Bode plot" is a plot of $|\hat{H}(j\omega)|$ versus frequency ω (or sometimes ω/ω_r) for a given circuit. It's often accompanied by a plot of θ (impedance phase) versus ω. Such plots are useful for understanding frequency response of a system. Resonant systems show dramatic peaks at $\omega = \omega_r$.