Duke University
Department of Physics

Physics 271 Spring Term 2017

WUN2K FOR LECTURE 5

These are notes summarizing the main concepts you need to understand and be able to apply.

- For AC circuits, in which we have sinusoidally varying currents $I(t)$ and voltages $V(t)$, it's very useful to treat them as complex numbers, $\hat{c}(t) = c_0 e^{j(\omega t+\theta)} = c_0 e^{j\theta} e^{j\omega t}$. The *physical* variables are real, and correspond to the real (or imaginary) parts of the complex quantities. A complex number can be considered to be a vector in the complex plane, with real component $c_0 \cos(\omega t + \theta)$ and imaginary component $c_0 \sin(\omega t + \theta)$. Since angle of this vector with respect to the real axis is $(\omega t + \theta)$, we can think of this vector as sweeping around the complex plane counter-clockwise as t increases. This kind of vector is called a *phasor*. Its initial angle is θ, it rotates with angular frequency ω, and its magnitude is c_0. Phasors are very useful for combining signals by superposition, because they behave just like vectors.

- Another useful concept that applies to AC circuits is that of *complex impedance* $\hat{Z}$, which is a kind of generalized resistance used in an AC-analog of Ohm's Law for sinusoidally time-dependent circuits, $\hat{V}(\omega) = \hat{Z}(\omega)\hat{I}(\omega)$. *Note the dependence on ω, the driving frequency.*

  - Resistors have $\hat{Z}_R = R$ (real impedance).
  - Inductors have inductive impedance $\hat{Z}_L = j\omega L$.
  - Capacitors have capacitive impedance $\hat{Z}_C = \frac{1}{j\omega C}$.

  Impedance can be written $\hat{Z} = R + jX$, where the imaginary part is called the *reactance* and depends on the frequency of oscillation ω in a circuit.

- Complex impedances combine the way resistors do:
  - In series, $\hat{Z}_{eq} = \sum_i \hat{Z}_i$.
  - In parallel, $\frac{1}{\hat{Z}_{eq}} = \sum_i \frac{1}{\hat{Z}_i}$.