FREQUENTLY ASKED QUESTIONS

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Content Questions

Why do low and high-pass filters differ so much when they have the same components?

The simplest low- and high-pass filters both have a capacitor and a resistor in series in a four-terminal network. However in the low-pass case, the output voltage is across the capacitor, and in the high-pass case, the output voltage is across the resistor. Here's a qualitative description of what's going on in each filter case:

- In the low-pass case, you measure voltage across the capacitor, and current comes through the resistor. For low frequencies (slow signals compared to the charging time), the capacitor gets charged up or discharged, reaching the input voltage, or a significant fraction of the input voltage, across it within an oscillation cycle; so the output follows the input signal. For high frequencies, in contrast, the capacitor never gets a chance to fully charge and develop the full input voltage across it: just as it gets started charging, the voltage sign and current direction wiggle back the other way and discharge it. So you never see the input voltage across the capacitor at high frequency.

- In the high-pass filter case, you measure voltage across the resistor, and current has to come through the capacitor. You get more voltage drop for more current going through the capacitor and on through the resistor. If frequency is high (rapidly thrashing voltage on the capacitor), the capacitor transmits current easily (always in the early charging or discharging part of its cycle) and current flows through the resistor. So the output voltage can be a large fraction of the input voltage at high frequency. In contrast, at low frequency, the signal is slow, and the capacitor will discharge or charge up on the timescale of an oscillation cycle. Charged or discharged capacitors don't pass current (capacitors "block DC currents") so there won't be much current through the resistor and not much voltage drop across it. So not much signal gets through to the output in the low frequency case.

What exactly is an "integrator" or a "differentiator"? Why are they called that?

An "integrator" is a circuit which will give you output proportional to the integral of $\hat{V}_{\rm in}(t)$, i.e., $\hat{V}_{\rm out}(t) = \int \hat{V}_{\rm in}(t)dt$. A low-pass filter at high frequency integrates to some approximation, although also attenuates the signal (we'll actually be seeing later different ways to make integrators using active components, so that you don't lose voltage). Similarly, a "differentiator" gives you the derivative of the input, $\hat{V}_{\rm out} = \frac{d\hat{V}_{\rm in}}{dt}$, and a high-pass filter at low frequency does this job. (Why would you want to do this, you may ask? See below.)

What are real integrators and differentiators? What do we want them for?

These are useful in circuits in various applications. For example if you want to make certain waveform shapes, it can be convenient to integrate or differentiate– if you want a sawtooth, say, you could integrate a square wave; if you want a square wave from a sawtooth, you would differentiate.

Integration is useful if you have a circuit in which voltage is proportional to some quantity of interest, and you want to sum that quantity. An application from my own research: I detect photons using sensors for which voltage is proportional to amount of light detected; to measure the total amount of light, I want to integrate the signal.

Similarly, differentiating is useful if you want to determine the rate of change of voltage.

What does $\hat{H}_D$ mean?

This is the transfer function of a differentiator, $\hat{H}_D(j\omega) = j\omega$. You can see it takes the derivative of a complex sinusoidal input voltage, as follows: if $\hat{V}_{\rm in} = \hat{V}e^{j\omega t}$, then the derivative is $\frac{d}{dt}\hat{V}e^{j\omega t} = j\omega\hat{V}e^{j\omega t} = j\omega\hat{V}_{\rm in}$. So taking the derivative is the same as multiplying by $j\omega$.

Why does the low-pass filter act as an integrator?

The low-pass filter acts as an integrator at high frequencies, such that $\omega >> \omega_c = 1/(RC)$. You can look at this in two ways:

- First, mathematically: the transfer function of the low-pass filter is $\hat{H}(j\omega) = \frac{1}{1+j\omega RC}$, and in the $\omega >> \omega_c$ limit this looks like $\hat{H}(j\omega) \sim \frac{1}{j\omega RC}$. Multiplying by $\frac{1}{j\omega RC}$ does exactly the same thing as integration (times a constant) for a sinusoidally-varying signal (or a superposition of sinusoidally-varying signals, which every periodic signal is by Fourier analysis): if $\hat{V}_{\text{in}} = \hat{V}e^{j\omega t}$, then the integral is $\int \hat{V}e^{j\omega t} = \frac{1}{j\omega}\hat{V}e^{j\omega t} = \frac{1}{j\omega}\hat{V}_{\text{in}}$.
- Second, thinking physically: the output is voltage across the capacitor, which is proportional to charge stored in the capacitor. At high frequency, with driving voltage rapidly flipping back and forth, you are always in mode where you have "just started" charging or discharging the capacitor, i.e., in a mode "right after flipping the switch". In this situation, at each instant, charge added to the capacitor in a given time interval is proportional to $\hat{V}_{in}$ at that time, and so total charge stored will be the sum of charge and proportional to integral of the voltage.

Since high-pass filters correspond to differentiators, and low-pass filters correspond to integrators, does this imply that a band-pass has no associated function?

No. Remember that the integration/differentiation functions are frequency-dependent. A low-pass filter integrates at high ω (where it also attenuates) only; a high-pass filter differentiates at low ω (where it also attenuates). A band-pass filter will act like an integrator at high frequency and differentiator at low frequency.

At what points do $\hat{H}_{\text{high}}$ and $\hat{H}_{\text{low}}$ start to deviate from the ideal cases? Are there relatively simple models for these situations? (i.e., when does $\omega_0 << \omega_c$ become $\omega_0 < \omega_c$?) Or when do the phases come into play?

Well, the question "when do you deviate from the ideal case?" is one that doesn't have a single answer; the answer is really "it depends on how good you need the answer to be". The exact filter response is usually possible to calculate, and you often can figure out the difference between the simple approximation and the more complete calculation. For example, if you eyeball the plots in the handouts, you can see the actual filter response with a smoothly curved transition between the $\omega << \omega_c$ and $\omega >> \omega_c$ regimes. For some applications, treating the frequency response with straight lines might be perfectly good enough, even in the corner region. However, if you need to know with precision what your output waveforms will look like at frequencies near ω_c, then you might want to do an analysis using the actual filter response with the curved line and the full $\hat{H}(j\omega)$ transfer function. (Of course, even the "actual filter response" with the curved line is itself an idealization, since it assumes resistors and capacitors are ideal circuit elements– but in real life all abstractions are leaky!)

Whether the phase shift matters or not depends on your application also. Relative phase shifts matter when combining signals.

Why does the V_{out} of a high-pass filter have a discontinuous jump given a square wave input?

For a high-pass filter responding in the low frequency regime, the transfer function is approximately $|H| \sim j\omega RC$, which is equivalent to a (scaled) derivative function. Looking at it mathematically, the output signal will look like the derivative (the slope) of the input square wave. So for the rising edge, the output will be a sharp positive peak. The output will be zero where the square wave is flat, and it will be a sharp negative peak for the square wave's falling edge.

Looking at it physically: in the low frequency regime, the capacitor charges all the way up early in the cycle, and then passes no more current. When the voltage swings down, it discharges fully and charges up in the opposite direction, then stops passing current again. So you'll get only spikes of current (and corresponding spikes of voltage across the resistor) at the square wave rising and falling edges.

Are the rolloffs in filters always (log) linear? Are there cases where it nees a steeper growth/decay?

Very commonly, filters will show power-law frequency response ($\propto \omega^n$) over particular frequency regimes. This looks like a straight line on a log-log plot.

These are approximations—they are not usually exact power laws. In particular, the transitions between the regimes tends to be smooth rather than abrupt "knees". We'll see a bunch of examples.

In general frequency response can be anything, though, and not necessarily a power law.