FREQUENTLY ASKED QUESTIONS

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Content Questions

How does a transformer work? Where do you get $V_2 = \left(\frac{n_2}{n_1}\right) V_1$ from?

Transformers are one of those topics that are nominally covered in first-year E&M but which often get dropped for lack of time... The relation can be derived from Faraday's law: induced voltage is proportional to rate of change of magnetic flux through a coil. In coil 1, $V_1 = -N_1 \frac{d\Phi_B}{dt}$, where Φ_B is the magnetic field through one turn and N_1 is the number of turns (you multiply by the number of turns to get the total magnetic flux threading through the coil). Now we assume that the magnetic field is the same in the other nearby coil, so the flux through one turn is the same on the other side. Then we write $V_2 = -N_2 \frac{d\Phi_B}{dt}$, and combining the two expressions we get $V_2 = \left(\frac{n_2}{n_1}\right) V_1$.

Qualitatively, a changing current on one side of the transformer creates a changing magnetic flux on the other side, which in turn creates an induced voltage there. The relative amount of flux through the secondary coil and hence the induced voltage can be controlled by the turn ratio.

How does the ferromagnet "communicate" the current from one cell to another?

In principle a ferromagnetic core is not needed to transform voltages through adjacent coils– one can pick up a changing magnetic field in a nearby coil without a core. However due to the magnetic properties of a material like iron (it has high magnetic permeability – the physics of this is interesting but going beyond the scope of this course), it can carry a large magnetic field, which increases the magnetic flux and allows the magnetic field lines to penetrate both coils easily.

BTW there are a number of practical considerations in the engineering of transformers that we are ignoring here in our idealized picture of them.

How do you define output and input impedance? Do they change with how you look at the circuit?

Output impedance is the impedance "seen" by a load connected across the output terminals of a circuit. It's determined by Thevenin equivalent impedance found in the usual way: open circuit voltage across the terminals, divided by short circuit current. The input impedance is the impedance "seen" by something (nominally a supply) connected across the input terminals. It's determined operationally by the voltage across the terminals divided by the current drawn.

If we have only reactive elements (i.e., none resistive) in our load, where does the power delivered to it go if it's not dissipated?

If you have a purely reactive load (purely imaginary impedance), the phase shift ϕ between current and voltage will be $\pm\pi/2$ and average power dissipated (proportional to $\cos\phi$) will be zero. This corresponds to a situation where energy is sloshing around between capacitors and/or inductors and never gets dissipated. (In practice, of course, nothing really has *zero* resistance, so energy will always be dissipated, even if slowly. In superconductors, energy dissipation is really tiny though, and energy can be stored for a very long time!)

What does ω_n represent?

This is $\omega_n = 2\pi n/T$, an integer multiple of the fundamental frequency $2\pi/T$ for a given period. Any real function is made up of an infinite sum of sinusoids with frequencies ω_n.

How do I find Fourier components?

For a given function $f(t)$, you can calculate the Fourier coefficients $\hat{c}_n$ from $\hat{c}_n = \frac{1}{T}\int_{t'}^{t'+T} f(t)e^{-j\omega_n t}dt$.

How did you get $\hat{I}_n$ in the last Fourier analysis problem? Why was it negative in $e^{-j\phi_n}$?

The $\hat{I}_n$ for a particular Fourier component is found from $\hat{V}/\hat{Z}$. We wrote $\hat{Z} = R + j\omega_n L + \frac{1}{j\omega_n C}$ in polar form, $\hat{Z} = |\hat{Z}|e^{j\phi_n}$, where the magnitude is

$|\hat{Z}| = \sqrt{R^2 + \left(\omega_n L - \frac{1}{\omega_n C}\right)^2}$ and the phase is $\phi_n = \tan^{-1}\left(\frac{\omega_n L - \frac{1}{\omega_n C}}{R}\right)$. The $e^{-j\phi_n}$ shows up with a negative exponent in the expression for $\hat{I}_n$ because $e^{j\phi_n}$ is in the denominator.