FREQUENTLY ASKED QUESTIONS

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Content Questions

The textbook uses Ω as the unit for impedance $\hat{Z}$. Is it legitimate to use it?

Yes, this is correct. Although impedance has imaginary parts, the physical part is real, and has the same units as resistance. Taking the real part of a complex quantity doesn't change the units.

Is the phase shift just the angle of the impedance vector?

"Phase shift" can actually refer to different things in different contexts. For AC Ohm's law, there is a phase shift between current $\hat{I}$ and voltage $\hat{V}$, and in this case, indeed, the phase shift is the phase angle of the impedance vector $\hat{Z}$.

However "phase shift" can also refer to the relative phase of input and output voltages, in which case it's the phase angle of the transfer function $\hat{H}(j\omega)$.

Where does the definition of θ in an RLC circuit come from. Why is $\theta = \tan^{-1}\left(\frac{1/(\omega C)-\omega L}{R}\right)$?

This θ represents the phase shift between input and output voltages of the four-terminal network. According to $\hat{v}_{ab}(j\omega) = \hat{H}(j\omega)\hat{v}_{\text{in}}(j\omega)$, the relative phase between $\hat{v}_{ab}$ and $\hat{v}_{\text{in}}$ is the phase of the transfer function $\hat{H}$. So to find this phase we write $\hat{H}(j\omega)$ in polar form, $\hat{H}(j\omega) = |\hat{H}|e^{j\theta}$. The phase θ of a complex number $\hat{C} = A + jB$ written as $|\hat{C}|e^{j\theta}$ is $\theta = \tan^{-1}(B/A)$: it's inverse tangent of the imaginary part over the real part. For the RLC network, $\hat{H}(j\omega) = \frac{R}{R+j(\omega L-\frac{1}{\omega C})}$. This can be rewritten (multiply top and bottom by $R - j(\omega L - \frac{1}{\omega C})$) as $\hat{H}(j\omega) = \frac{R^2-j(\omega L-\frac{1}{\omega C})R}{R^2+(\omega L-\frac{1}{\omega C})^2}$. Hence, the phase shift for the network is $\theta = \tan^{-1}\left(-(\omega L - \frac{1}{\omega c})R/R^2\right) = \tan^{-1}\left(\frac{\frac{1}{\omega C}-\omega L}{R}\right)$. We'll see lots of examples like this, so work through the algebra if you are uncomfortable with it.

Can the phase shift be less than $-\pi/2$ or greater than $\pi/2$ at $t = 0$?

For (passive) reactive elements such as the inductors and capacitors we've seen so far, the phase shift between voltage and current is always $-\pi/2 \leq \theta \leq \pi/2$. (We will see later that with *active* circuit elements, i.e., components that add energy to the circuit, this need not be true.)

We should be able to treat DC as an AC case with $\omega = 0$. But then $\hat{Z}_C$ is arbitrarily large, and in an RC circuit we should have zero voltage across the resistor. But in an RC circuit the voltage across the resistor is nonzero for some time. Why is there a disagreement?

The key to resolving this is to realize that for a DC circuit, in *steady state*, there is no current across a capacitor (note: zero *current*, not zero voltage). The capacitor basically acts like an open circuit. For the case of DC battery, capacitor, and resistor in series, at infinite time, there is no current and no voltage drop across the resistor (i.e., both sides of the resistor are at the same potential), so the voltage drop across the capacitor is the same as the drop across the battery. This is consistent with the $\omega \to 0$ limit of AC Ohm's Law: $\hat{Z}_C$ is infinite, and therefore $\hat{I} = \hat{V}/\hat{Z}_C$ is zero.

Now, there are *transient* solutions to the DE's set up using the Loop Rule, as we saw a few lectures ago: for the short period of time while the capacitor is charging up, or discharging, there is a varying current (and voltage) across the capacitor. However the AC limit corresponds to the steady state situation, after transients have died away; for an RC circuit, potential across the capacitor is constant and current is zero after the transients have gone away.

What does $\mathbf{H}(j\omega)$ mean physically?

This is the "transfer function" for a 4-terminal network. It's a complex function that describes how the input signal gets transformed (by whatever stuff is inside the box separating the two input terminals from the two output terminals) into an output signal, according to $\hat{V}_{ab} = \hat{H}(j\omega)\hat{V}$, where $\hat{V}$ is the sinusoidal power supply voltage at the input and $\hat{V}_{ab}$ is what you see at the output. It's a function that describes the *frequency response* of the network: what you get out depends on the driving frequency ω.

Where did the Q approximation for RLC come from?

I didn't do this explicitly in class– it's a bit lengthy. But the derivation is given in, e.g. Fortney, pp. 64-65. The idea is to write down a damped oscillatory solution to the RLC circuit differential equation for $V(t)$; since energy dissipated is proportional to the square of V, you can then plug this in to the definition of Q (ratio of total energy to energy loss per cycle). Using some Taylor series approximations, the expression $Q = \frac{\omega L}{R}$ follows. Near resonance, we replace ω with ω_r.

If $Q = \frac{\omega L}{R}$, why does the term $-\frac{j}{\omega C}$ become $-\frac{j}{\omega_r}\omega$?

The expression $H(j\omega) = \frac{1}{1+jQ(\omega/\omega_r)[1-(\omega_r/\omega)^2]}$ results from several algebraic steps following $H(j\omega) = \frac{R}{R+j(\omega L-1/\omega C)}$. First, pull R out from numerator and denominator. For $Q \sim \frac{\omega_r L}{R}$, we can write $\frac{wL}{R} = \frac{Q\omega}{\omega_r}$. Substitute in and do a bit of manipulation to get the desired form.

How do we determine Q for other circuits?

I think you can use the same approach as for the RLC series circuit (see above question) to find the energy dissipated per cycle. However I don't think we'll be computing this for many other circuits in this class. For this course, the qualitative concept is WUN2K.

How does a 4-terminal network work?

So, "4-terminal network" is a generic thing representing a bunch of circuit elements in some network (it could be pretty much any configuration) with two "input" terminals and two "output" terminals. Often one plugs in a power supply to the input, and the output is the "business end", where one is interested in the voltage. However you could arrange more than one network together, or reverse input and output. We looked at one specific example today and we'll see lots more examples. We'll usually be considering these in an AC context, i.e., assuming some sinusoidal steady-state driving, rather than with transients.

What is so powerful in the 4-terminal circuit concept compared to other types of circuits we've discussed?

It's very useful to characterize networks as "boxes" with input and output—you can describe the response of some quite complicated arrangement of circuit elements with a single transfer function.

Are there cases when you can't use a transfer function?

Yes, there are non-linear systems for which transfer functions cannot be properly defined. An example is a relaxation oscillator. However they can be defined for most of the circuits we will be considering.

How do you intuitively see how the $\hat{Z}_1$ and $\hat{Z}_2$ construction is a voltage divider?

You can apply the voltage divider equation anywhere you are looking at a voltage in between two circuit elements, if you know the overall voltage drop across the whole network. It doesn't matter if there are additional wires or terminals sticking out at the places you're looking at the voltage (if they are just open circuit). Sometimes it takes a bit of practice to acquire the intuition about where one can apply the voltage divider equation, but it will soon seem more obvious.