FREQUENTLY ASKED QUESTIONS

February 2, 2017

Content Questions

How are phasors useful?

Phasors are a useful visualization of what's going on in an AC circuit (and in fact for many situations involving sinusoidal waves). Although in this class we won't see too many examples, they are quite useful for looking at sums and differences of waves (*interference*) since you can make vector sums and differences out of the rotating phasors. As we'll see shortly, they are also useful when thinking about phase shifts between sinusoidal quantities.

If the real component of $e^{j\theta}$ corresponds to voltage or current, what does the imaginary component correspond to? Do we ever use the imaginary parts of these complex quantities?

The oscillatory $V(t)$ and $I(t)$ quantities we deal with are typically solutions to linear differential equations. The *physically* real quantities are the (mathematically) real parts of the solutions, but general complex solutions to the equations have imaginary parts too; as we discussed, if $C_0 \cos(\omega t + \theta)$ is a solution, then so is $jC_0 \sin(\omega t + \theta)$ and so is the sum. You can think of the imaginary part of the complex $e^{j\theta}$ quantity as an orthogonal solution to the DE. Keeping the general solution intact is very helpful for doing calculations; it works for manipulations involving addition and subtraction. You keep the imaginary part around during intermediate steps (and for visualizations, like the phasor spinning) but then convert to physical solutions at the end of the calculation.

In some situations the imaginary part will correspond to something physical, but I think for our case we'll mostly be using real parts for voltages and currents.

Is the same imaginary mathematical method used if the signal is a sawtooth, step or some other non-sinusoidal function?

Aha, yes, as it turns out! This is where Fourier analysis comes in: *every periodic waveform can be written as a sum of sinusoids*, so that we can use the same complex number methods on the waveform. This is an extremely powerful idea. We'll see more on this soon.

How did you go from $\omega RC + j$ to $\tan^{-1}\left(\frac{1}{\omega RC}\right)$?

You can write any complex number $\hat{z} = A + jB$ as a vector in the complex plane, where A is the real axis component and B is the imaginary axis component (here, in electronics world, $j = \sqrt{-1}$). The angle the vector makes with the real axis, by trigonometry, is $\theta = \tan^{-1}(B/A)$. Here, $\hat{z} = \omega RC + j$, and A, the real part is ωRC; the imaginary part is $B = 1$. Hence, θ, the phase angle, is $\theta = \tan^{-1}\left(\frac{1}{\omega RC}\right)$.

Why does high capacitance correspond to low impedance and vice versa? Physically, why do capacitors increase in impedance when you lower frequency, and vice versa?

Capacitive impedance is $\hat{Z}_C = \frac{1}{j\omega C}$, which means that, for a given AC frequency ω, a large capacitance means a smaller impedance (i.e., more current for a given voltage according to the AC Ohm's Law, $\hat{I}(\omega) = \hat{V}(\omega)/\hat{Z}(\omega)$). Here's the qualitative explanation: if you have a very large capacitance, that means a lot of charge can be stored for a given potential difference, and the capacitor takes a long time to charge up. In an AC circuit, current only passes through a capacitor during the time a capacitor is either charging or discharging. If a capacitor is fully charged or discharged, it acts like an open circuit and does not pass current— its impedance is infinite. A large capacitance means that (for a given ω of AC driving voltage) the capacitor will spend more of its time in a charging or discharging mode. A small capacitance means that the capacitor will charge up quickly and spend most of the cycle behaving like an open circuit and so not passing current.

Similarly, for a given C, a fast driving wiggle will mean that the capacitor is constantly charging-discharging-charging-..., and so passes more current on average (so has low impedance). In contrast, a small ω will mean that the capacitor has a chance to charge all the way up or discharge all the way down, and so will be open-circuit-esque during much of the cycle, and hence won't pass much current (so has high impedance).

Both the size of C and ω matter; impedance is related to the product of them.

Can you clarify reactance? What is it exactly?

Reactance is just the imaginary part of the impedance. A circuit's equivalent impedance $\hat{Z}$ can have contributions from resistive, inductive and capacitive components (which combine according to the resistor rules). Since in general $\hat{Z}$ has a real and imaginary part, it can be written $\hat{Z} = R + jX$. The real part R is the "resistance", and X is called the "reactance", since it results from having imaginary impedances from inductors and capacitors as part of the circuit. Inductors and capacitors are not like stodgy resistors in that they "react" to time-dependent sources.

Can you add impedances from different circuit elements, i.e. inductors and capacitors?

Yes. AC impedances act like resistors for the purpose of adding them in series and parallel, and determining Thevenin equivalents.

In an RLC circuit, will the voltages at different points between the different elements have different phases? Shouldn't the voltages across the resistor have the same phase? Should the phase change around the loop be a multiple of 2π?

Note that the phase shift we're talking about in "AC Ohm's Law" is a relative phase shift between the voltage across a circuit element and the current through a circuit element— it's not a phase shift between voltages at different points in a circuit. Although at any given point in time, by Kirchoff's Loop Rule, the sum of the voltages around a loop must be zero, the phase shift need not sum to a multiple of 2π around the loop.