FREQUENTLY ASKED QUESTIONS

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Content Questions

What are the values of κ and ϵ_0 in the capacitance equation?

This was the expression giving the capacitance of a parallel plate capacitor, $C = \kappa\epsilon_0 A/d$. A is the cross-sectional area, d is the distance between the plates, ϵ_0 is the permittivity of free space ($\epsilon_0 = 8.854 \times 10^{-12}$F/m, a constant of nature), and κ is the dielectric constant of the material between the plates. The value of κ is 1 for vacuum, and > 1 for media (so capacitance increases when there's a dielectric between the plates).

For a current coming into the inductor the magnetic field is circular, but in the inductor solenoid, the current is circular. So the induced $\vec{B}$ field is either up or down [along the inductor]). So how exactly can the inductor resist the change?

Right, you get a circular magnetic field around a straight current-carrying wire and a magnetic field along the axis of an ideal solenoid, in both cases described by the right hand rule (in the former case, the fingers curl in the direction of the field; in the second case, the fingers curl in the direction of the current.) This is an idealization; in real life, the solutions to Maxwell's equations describing the field induced by a current will have continuous transition regions going from one geometry to another.

As for resistance to change: what matters is *change* in current and magnetic field, not the current and magnetic fields themselves. The back-emf results from the *change* in current, and opposes that change. In either case, if there's a dI/dt, there will be a potential $V = LdI/dt$ which resists the change, e.g., if the current is increasing, the inductance will be responsible for a back-emf that opposes that increase (works to decrease the current). If the current is decreasing, the inductance will be responsible for a back-emf that works to increase the current. This happens for either a wire or a solenoid (or anything), but we treat the idealized wire as having zero inductance. We consider only the solenoid, which will develop a back-emf to decrease an increasing current, and increase a decreasing current.

How do inductors store and discharge energy?

In an inductor, the energy is stored *in the magnetic field* when there is current through the coil. A current creates an induced magnetic field along the axis of a coil, and you may remember from E&M that energy is stored in a magnetic field according to $U = \frac{1}{2\mu_0}\int \vec{B}^2 dV$, where the integral is over space. When the current is maximum, the stored energy in the coil is maximum. Energy is released from the inductor as the current through it decreases. Where exactly the energy goes depends on the rest of the circuit. The energy might be dissipated in a resistor, or stored in the electric field of a capacitor. (Even if the inductor is not a coil— remember, everything has self-inductance— energy is stored in the magnetic field induced by the current through the inductor.)

I remember that capacitors in series combine similarly to resistors in parallel, but how do inductors combine?

Inductances combine in the same way that resistors do, i.e., they add in series. In parallel, the reciprocals add. See if you can show this!

How does energy get transferred from the inductor to capacitor in the LC circuit?

Just as energy is stored in the *magnetic field* induced by a current through an inductor (see question above, $U = \frac{1}{2\mu_0}\int \vec{B}^2 dV$), energy is stored in the *electric field* between a capacitor's plates, $U = \frac{1}{2}\epsilon\int \vec{E}^2 dV$. The energy stored in the magnetic field induced by the current through the inductor gets transformed into energy stored in the electric field of the capactor, as the capacitor charges up and the current decreases (capacitor electric field increases as inductor magnetic field decreases, and vice versa).

If energy sloshes back and forth between the capacitor and inductor, does this mean that current never goes past the inductor?

Yes, current flows past the inductor, and through the whole circuit. However the faster the rate of change of current through the inductor, the larger the back-emf, which opposes the change in current. Here's what happens in the LC circuit:

- We start with all the energy stored in the capacitor. The capacitor starts to discharge, creating a current. This sets up a magnetic field in the inductor, so energy gets transferred to the inductor. Since the current is *changing* (initially increasing very fast) the magnetic field is changing and there's a back-emf in the inductor which attempts to decrease the current. Just as the capacitor starts to discharge, the rate of change of current is largest. [Mechanical analogy: this is like the point at which a mass on an extended spring is released. Position ($x \leftrightarrow q$) is maximum; velocity ($v \leftrightarrow i$) is zero; acceleration is maximum ($a \leftrightarrow di/dt$).]

- The current increases, but not beyond a maximum value, due to the opposing back-emf. The maximum current is when the capactor is completely discharged; current is flowing all the way through the circuit at this point in time, including the inductor. At this point, all the energy is in the inductor, stored in the magnetic field. [Mechanical analogy: this is like the point at which the mass is zooming through equilibrium. Position is zero; velocity is maximum; acceleration is zero.]

- Eventually the capacitor is fully charged up with opposite charge sign with respect to the initial condition, and current is zero. [Mechanical analogy: this is like the point at which the mass on the spring is at maximum compression. Position is maximum with opposite sign; velocity is zero; acceleration is maximum.]

- The cycle continues. Charge, current, and stored energy slosh back and forth (forever, if no energy is dissipated).

This animation visualizes the oscillation. See if you can draw the sinusoids for $q(t)$ and $i(t)$ corresponding to this circuit.

Are there other factors besides resistance that would stop an LC circuit from being ideal? (How do they lose energy?)

In a real circuit, generally ohmic energy loss, i.e., from an iR voltage drop (for which mechanical analog is a force proportional to velocity, like drag), is actually a pretty good model, and is associated with energy loss via heat (usually the dominant form), light, sound, etc.. In a real circuit, the capacitor and inductor might not be ideal— there can be stray capacitances and inductances, effectively in parallel as well as in series, which can change the circuit behavior... or there can current leakages in or out of the system, environmental electromagnetic noise, vibrations changing the properties of the system, and all kinds of other time-dependent environmental effects... the real world is a messy place!

For the LRC loop rule, why is the $-L\frac{di}{dt}$ term negative?

The signs can be a bit confusing in these loops, as currents can flow in either direction and the quantities can be increasing or decreasing, so the derivatives of charges and currents can be positive or negative. The important thing is that the relation between the signs in the resulting differential equation is right — the solution should then give you the right actual signs of currents and voltages. I wrote down $-L\frac{di}{dt}-iR+q/C=0$, and then $L\frac{d^2q}{dt^2}+R\frac{dq}{dt}+\frac{q}{C}=0$, as follows:

Imagine the capacitor is has just been charged up (say, $+q$ on the top plate, $-q$ on the bottom) and take the moment at which the current has just started flowing counter clockwise (the loop rule will still be true at any moment). We'll take q to be the charge on the top plate. Step around the loop in the direction of the current. The voltage change across the capacitor is $+q/C$, and the voltage change across the resistor in the direction of the current is $-iR$. The inductor will be developing an EMF proportional to $L\frac{di}{dt}$. If i is CCW as the discharge starts, the inductor (by Lenz's Law) is creating a back-emf to counteract the increase in current, so an emf that is negative, $-L\frac{di}{dt}$. So we have $-L\frac{di}{dt}-iR+q/C=0$. But then, since q is the charge on the top plate, and it's decreasing, we should write $i=-\frac{dq}{dt}$ and so $L\frac{d^2q}{dt^2}+R\frac{dq}{dt}+\frac{q}{C}=0$.

Similarly, for the LC circuit we write $-L\frac{di}{dt}+\frac{q}{C}=0$, then take $i=-\frac{dq}{dt}$ to get $L\frac{d^2q}{dt^2}+\frac{q}{C}=0$.

You mentioned there is a mechanical analogy to resonance– does that mean that inductors have a resonant frequency that is a property of the material?

Yes, with the spring mechanical analogy to the RLC circuit, an applied sinusoidal force is analogous to an oscillating power supply, and you will get a resonance for certain frequencies. However the resonant frequency is a property of the whole circuit, rather than just the inductor. We will discuss this further soon.