FREQUENTLY ASKED QUESTIONS

January 19, 2017

Can the homework be done based on what we learned in this lecture?

Yes, I think so. If you are stuck, you can email me, as I won't have office hours next week.

When finding R_{eq} between two terminals, are there some good ways to find the "topologically equivalent" circuit?

Well, I'm not sure there's a systematic method... this mostly comes with practice... the key thing to remember is that all points connected by a wire are at the same potential, and you can bend and morph the wire, and rotate circuit elements, in any way so long as you keep the relationships between the potentials and elements intact. After a while you will find you will easily recognize equivalent patterns. You'll get more practice as we go along.

Why do the (V, I) points for various R_L lie on a line?

This follows from Thevenin's theorem– if the circuit contains linear elements (resistors, voltage sources and current sources), it will behave just like a voltage source in series with the Thevenin-equivalent resistance R_{Th}. The behavior of such a circuit will follow Ohm's Law for the Thevenin-equivalent circuit, with R_L in series with R_{Th}. For a given R_L, the current across the load resistor is $I_L = V_{\text{Th}}/(R_{\text{Th}} + R_L)$ and the voltage across the load resistor V_L will be $V_L = I_L R_L = R_L(V_L/(R_{\text{Th}} + R_L))$. After a bit of algebra, you can show that $I_L = (V_{\text{Th}} - V_L)/R_{\text{Th}}$, which is exactly an expression of a straight I vs. V load line with slope $-1/R_{\text{Th}}$ and y-intercept $I_N = V_{\text{Th}}/R_{\text{Th}}$.

How do you know when to use a Thevenin equivalent instead of a Norton equivalent, or vice versa, when both a voltage source and a current source are present?

If a circuit overall behaves more like an ideal current source than like an ideal voltage source, you might use a Norton equivalent instead of a Thevenin equivalent. (An ideal current source can only have a Norton equivalent, and an ideal voltage source can only have a Thevenin equivalent. Why?) In some cases, either might be useful. Use of Thevenin equivalent tends to be more common.

How do you conceptually find the voltage difference across open terminals vs shorted terminals?

The idea is that you write down the circuit with the terminals open, and then find the voltage across them in this configuration, using whatever circuit-solving methods you have at your disposal (i.e., Kirchoff's Laws). To "short the terminals", you draw the circuit with a zero-resistance wire across the terminals, and then find the current in this configuration, again using Kirchoff's Laws, etc. Please note that "opening" and "shorting" supplies and terminals is an imaginary, abstract action— you write it down to find what the current or voltage *would* be if you did this.

Why can you short voltage sources and open current sources and not vice versa?

This method of finding R_{Th} follows from the linearity of all the Kirchoff's Law equations (from which Thevenin's theorem is derived.) The value is $R_{\mathrm{Th}} = V_{AB}(\mathrm{open})/I_{AB}(\mathrm{short})$. If you multiply all of the equations by a constant, you get the same answer when calculating V_{AB} and I_{AB} by Kirchoff's Laws. All of the terms in the circuit equations with current in them are proportional to I and the voltage source terms are proportional to ε. If you multiply all equations by an arbitrarily small constant (still getting the same answer), that's like setting the EMF's to zero, which is like shorting them out (no voltage drop across them). At the same time it's also like making all the currents approach zero, which is equivalent to removing the current sources from the circuit as well. So you get the same equivalent resistance when you short the voltage sources and open the current sources.

In contrast, if you opened the EMF's, that would give you a fundamentally different circuit– there could then be any potential difference across a voltage source's terminals, not an arbitrarily small one. Similarly, shorting the current source changes the nature of the network.

In the Norton equivalent example, why did shorting A and B result in the resistance R_1 being ignored?

Shorting A and B in this example would result in the two sides of R_1 being at the same potential. So there would be no current through it according to Ohm's Law, and so it is irrelevant to the circuit. Alternatively, you can think the shorted circuit as having R_1 in parallel with a zero-resistance wire. All the current is going to go through the wire and nothing is going to go through R_1.

Note that you can't ignore the branch with the voltage source and R_2 when A and B are shorted. The reason is that the voltage source will be pumping charge to maintain a fixed voltage across it. To keep the two ends of the branch at the same potential, there must then be a voltage across the resistor R_2 and hence some current flowing through that branch.

What is the difference between resistance and impedance?

"Impedance" refers to a kind of generalized resistance which can be applied to non-resistor circuit elements like capacitors, inductors and others. We will go into some detail on this in the coming weeks.

Will we talk about more complicated uncertainty propagation in the future?

Well, a lot more detail would be getting a bit off topic... but here are some references and links if you want more information:

- A basic tutorial
- Another link with introductory error analysis
- *Data Reduction and Error Analysis for the Physical Sciences*, by P. R. Bevington and D. K. Robinson: this one is a classic, and I recommend it as a general reference. If you get one book on error analysis, get this one. Watch out for typos, though, even in the second edition.
- *An Introduction to Error Analysis*, by John R. Taylor: Another classic, at a somewhat more elementary level than Bevington & Robinson. This is a good one to go through if you feel you lack background.

- I teach a mini-course on error analysis occasionally.

For the purpose of this course, the basic no-correlation error propagation formula is the most important thing to know.

How do you take into account correlation in error propagation?

In the error propagation formula I wrote down, we assume there are no *correlated systematic uncertainties*. An example of a correlated systematic would be for the case of a ruler that has some error. A ruler will measure lengths with some fractional error of $\Delta L/L$, but if its ticks are a little too long, or too short, then its measurements of the height and width of a rectangle will *both* be a little too short, or *both* too long. The amounts the values differ from the "true" values will be correlated. You will have to take into account the correlation of the measurements when you estimate the uncertainty on the area. If you calculate area from the product of height and width, your answer will be more likely to be too small or too large than if you used different, randomly-off ruler errors for height and width. Quantitatively, the correlation is taken into account in error propagation using a quantity called *covariance*, often denoted σ_{xy}.
Here is the more general error propagation formula:

For some quantity $w = f(x, y, ...)$, the variance of w, σ_w^2 (what we've been calling $(\Delta w)^2$), is given (to first order) by

$$\sigma_w^2 = \sigma_x^2 \left(\frac{\partial f}{\partial x}\right)^2 + \sigma_y^2 \left(\frac{\partial f}{\partial y}\right)^2 + ... + 2\sigma_{xy}^2 \left(\frac{\partial f}{\partial x}\right)\left(\frac{\partial f}{\partial y}\right) + ...$$

More generally, any two variables can be correlated, like heights and weights of people in a population, and one can describe the spreads of the distributions with variances and covariances. (There's also such a thing as *anti*-correlation, when a large value of one variable means that the value of the other variable is more likely to be small.)

But this is a whole subject in itself. This stuff comes up a lot all over science and engineering, and you will probably bump into it if you haven't already. In this class we will pretty much assume uncorrelated uncertainties (independent variables) though.