This is a closed book exam, with one side of one page cheat sheet allowed. Calculators are allowed, but only for basic calculations: you may not use special memory, graphing etc. functions. You must always show your work for credit; all answers must be justified. You must hand in your cheat sheet with your test.

Problem 1: (10 points)

Using only capacitors and resistors, make a sketch of a two-pole bandpass filter. Roughly sketch the resulting Bode plot, labeling the slopes of any lines. Don’t use specific values for \( R \) and \( C \) but indicate relative sizes of them.

\[
\begin{align*}
\hat{H}(w) &\sim \hat{H}_1(w) \hat{H}_2(w) = \left( \frac{\omega C_1}{\omega R_1 + R_1} \right) \left( \frac{R_2}{\omega C_2 + R_2} \right), \\
\text{Bode plot} &\quad (\log_{10} \omega) \\
\text{slope} &\quad \Delta \omega
\end{align*}
\]
Problem 2: (10 points)

Design a diode clipper circuit that limits the output to no more than +2.3 V and -1.6 V. Sketch the output for a 100 Hz sine wave with 6 V amplitude, and for a 50 Hz square wave with 2 V amplitude.

Choose $V_a = 1.7$ V, $V_b = -1$ V

Then the output will never exceed 2.3 V or be less than -1.6 V

Input 100 Hz

6 V

Output (same T)

2.3 V

-1.6 V

clipped

Input 50 Hz

2 V

Output (same T)

only negative part is clipped

T=0.01 s

T=0.02 s
Problem 3: (10 points)

Determine the gain of the given op-amp circuit, assuming that the op-amp’s open-loop DC gain is \( A \).

\[
V_{in} \quad + \quad A \quad - \quad V_{out}
\]

\[
V_{h} = \left( \frac{R_g}{R_f + R_g} \right) V_{3}
\]

\[
|V| = \left| \frac{A}{1 + \frac{A F}{1 + \frac{R_g}{R_f + R_g}}} \right| = \left| \frac{\hat{A}}{1 + \frac{R_g}{R_f + R_g}} \right|
\]

Assuming \( |\hat{A} F| >> 1 \), \( |\hat{A}| = G = \frac{1}{F} = \frac{R_f + R_g}{R_g} \)
Problem 4: (10 points)

Determine the locations of the poles and zeroes of the transfer function $H(\omega) = \frac{\omega^3}{\omega^2 - 4\omega - 15}$. Express your answers in the form of values of $\hat{s}$, and indicate their positions (in standard notation) on the complex frequency plane.

$\hat{s} = j\omega$

Zero for $\omega = 3$

$\text{so } \hat{s} = 3j$ → ○

$\omega^2 = -\hat{s}^2$

Pole for $-\hat{s}^2 - 4\hat{s} - 13 = 0$

$\hat{s}^2 + 4\hat{s} + 13 = 0$

$\hat{s} = -4 \pm \sqrt{16 - 4 \cdot 13}$

$\hat{s} = -4 \pm 2\sqrt{4 - 13}$

$\hat{s} = -2 \pm 3j$ → ×
Problem 5:  (20 points)

For the given circuit

a. Determine the Thevenin resistance $R_T$.  \( \rightarrow \text{Open source, short supply.} \)

b. Find the Norton equivalent current $I_N$.

c. Find the Thevenin equivalent voltage $V_T$.

\[
R_{Th} = \frac{R_3(R_1+R_2+R_4)}{R_1 + R_2 + R_3 + R_4} \\
= \frac{5(2\Omega)}{2.5} \\
R_{Th} = 4 \Omega
\]

b) $I_N$ is for AB shunted (ignore $R_3$)

\[
\begin{align*}
\text{Use Kirchhoff:} \quad (I_z - I_N) \\
I_s + I_1 = I_z \\
\epsilon - I_1R_1 - I_z(R_z + R_4) = 0
\end{align*}
\]

\[
\begin{align*}
\epsilon - I_1R_1 - (I_1+I_s)(R_2+R_4) &= 0 \\
I_1(R_1 + R_2 + R_4) &= \epsilon - I_s(R_2 + R_4) \\
\Rightarrow I_1 &= \frac{12 - 2(1b)}{20} = -1 \text{ A}
\end{align*}
\]

\[
I_N = (2 - 1) A = \frac{1}{A}
\]

c) $V_{Th} = I_N R_{Th} = (1)(4A) = 4V$
Problem 6: (20 points)

For the given circuit, show that the maximum power will be dissipated in $R_{\text{load}}$ if $R = R_{\text{load}}$. (This is known as impedance matching, and is a useful result.)

\[ P = V I \]

\[ V = I^2 R \]

\[ \Rightarrow P = I^2 R = \frac{V^2}{R} \]

Same $I$ through each $R$

Total power dissipated:

\[ P_{\text{tot}} = I^2 R + I^2 R_{\text{load}} \]

**Loop Rule**

\[ V_{\text{in}} - I R - I R_{\text{load}} = 0 \]

\[ I = \frac{V_{\text{in}}}{R + R_{\text{load}}} \]

Power dissipated in $R_{\text{load}}$

\[ P_{\text{load}} = \frac{V_{\text{in}}^2}{R + R_{\text{load}}} \]

Maximize

\[ \frac{dP}{dR_{\text{load}}} = V_{\text{in}}^2 \left[ \frac{2}{(R + R_{\text{load}})^2} + R_{\text{load}} \left[ - \frac{2}{(R + R_{\text{load}})^3} \right] \right] = 0 \]

\[ 1 - \frac{2 R_{\text{load}}}{R + R_{\text{load}}} = 0 \]

\[ (R + R_{\text{load}}) - 2 R_{\text{load}} = 0 \]

\[ \Rightarrow R_{\text{load}} = R \] at maximum
Problem 7: (25 points)

For the given filter

\[ V_{in}(\omega) \quad \Rightarrow \quad V_{out}(\omega) \]

\[ \hat{Z}_{eq} = \frac{j\omega L_1 R}{j\omega L_1 R + R} \]

a) \[ \hat{H}(\omega) = \frac{j\omega L_1 R}{j\omega L_2 + \frac{j\omega L_1 R}{j\omega L_1 R + R}} = \frac{j\omega L_1 R}{j\omega L_2 \left( j\omega L_1 R + R \right) + j\omega L_1 R} \]

a. What is \( \hat{H}(\omega) \)? Answer in terms of \( L_1, L_2, R \) and \( \omega \).

b. Is this a low-pass, high-pass, band-pass or band-rejection filter? Explain. Sketch the Bode plot.

c. Determine the corner frequency or frequencies.

d. Determine the values of the zeroes and poles and sketch them on the complex frequency plane.

e. Write down \( |\hat{H}(\omega)| \) using the results of part d.

\[ \hat{H}(\omega) = \frac{j L_1 R}{j R \left( L_1 + L_2 \right) - \omega^2 L_1 L_2} \]

\[ \Rightarrow \]
b) Low $\omega$ behavior: neglect 2nd term in denominator

$$|H| \propto \frac{j\omega l_1 R}{j\omega R(l_1 + l_2)} = \frac{l_1}{l_1 + l_2}$$

High $\omega$ behavior:

2nd term in denominator dominates

$$|H| \propto \frac{j\omega l_1 R}{-\omega^2 l_1 l_2} \propto \omega^{-1}$$

Bode plot: $|H| \propto \text{const} \sqrt{\omega} \propto \omega^{-1}\text{low pass}$

c) Corner frequency:

$$\frac{L_1}{L_1 + L_2} = \frac{l_1 R}{w_c l_1 l_2}$$

$$w_c = \frac{(l_1 + l_2) R}{l_1 l_2}$$

d) $H(s) = \frac{l_1 R}{s l_1 l_2 + R(l_1 + l_2)}$

No zeros

Poles: $s = -\frac{R(l_1 + l_2)}{l_1 l_2}$

\[ s = \frac{-R(l_1 + l_2) R}{l_1 l_2} = -w_c \]

\[ \text{more} \]

\[ \text{more} \]

\[ \frac{|H|}{\text{distance to zeros}} = \frac{1}{\sqrt{w^2 + w_c^2}} \]