Content Questions

What’s the difference between motional emf and induced emf?

“Induced emf” is the more general term. By Faraday’s Law, you get an induced emf whenever there’s a changing magnetic flux through a loop. If the changing emf is due to some kind motion of a conductor in a magnetic field, you would call it a “motional emf”. For example, if a loop moves into or out of a region of field, or rotates, or a bar rolls along a rail, you’d get a “motional” induced emf. But if the changing magnetic flux were due to, say, an increasing current in a wire, you wouldn’t call it a “motional” emf.

Why was Ampère’s Law modified?

Maxwell modified it in order to include the effect of time-varying electric fields. Time-varying electric flux creates a magnetic field, just as does conduction current. One can take this effect into account by adding an extra term, $\mu_0 I_d$, to the RHS of Ampère’s Law (making it the “Ampère-Maxwell” Law. Here $I_d$ is the “displacement current”, $I_d = \epsilon_0 \frac{d\Phi_E}{dt}$, a kind of equivalent current associated with a changing electric flux. Section 34.1 has a detailed discussion, and shows how you need this term in order to apply Ampère’s Law to the case of a charging capacitor.

When do you use the modified form of Ampère’s Law?

You need it when you have a case of changing electric flux, but not necessarily an actual (conduction) current. The classic case is a capacitor being charged: there’s no actual, physical, current between the plates, but there’s an “effective” current (the displacement current) due to the changing electric field between the plates.

What’s the difference between displacement current and actual current?

The actual current (or “conduction current”) is current due to moving charges, $I = \frac{dQ}{dt}$. The displacement current is a kind of “effective” or “equivalent”
current that is related to the rate of change of electric flux, \( I_d = \epsilon_0 \frac{d\Phi_E}{dt} \). Both kinds of current create induced magnetic fields.

**How do you find \( I_d \) enclosed?**

To find the magnetic field due to displacement current using the Ampère-Maxwell law, the RHS \( \mu_0 I_d \) includes only the displacement current enclosed within the Ampèrian loop you’ve chosen. So when finding the electric flux \( \Phi_E = \int \vec{E} \cdot d\vec{A} \) (which = \( EA \) for electric field constant over area, like for a capacitor), include *only* the area enclosed within the Ampèrian loop.

**How do you deal with \( I_d \) for a discharging capacitor?**

The direction of the displacement current \( I_d \) is in the direction of change of electric field. So for a charging capacitor, for which the magnitude of \( \vec{E} \) is increasing, the direction of \( I_d \) is in the direction of the increase of field. For a discharging capacitor, the magnitude of \( \vec{E} \) is *decreasing* in time, so the direction of *change* of field is opposite to the direction of the field (it’s the direction of \( \Delta \vec{E} \)). So the direction of displacement current is opposite to the direction of the field. The direction of the induced \( \vec{B} \) field is found using the RHR: point your thumb in the direction of \( I_d \) and fingers curl with the \( \vec{B} \) field.

**How do you get the direction of \( \vec{B} \) between the capacitor plates?**

Use the RHR: point your thumb in the direction of the displacement current \( I_d \) and your fingers curl in the direction of the induced \( \vec{B} \). (See above question for how to find the direction of \( I_d \).)

**How does this material relate to our previous usage of capacitors?**

The displacement current between the plates of a capacitor is another aspect of capacitors— all of the material you learned about capacitors previously is still valid.
Where does the the wave equation come from? How do I show it’s true?

We did not derive this wave equation (although one can... and the text derives wave equations for $E$ and $B$ from Maxwell’s equations). You will not need to “show the wave equation is true”, but you do need to understand that the solution of the wave equation is a wave function, i.e. a sinusoid. You could also easily show that a plane wave of the form $D(x,y) = A \sin(2\pi x/\lambda - \omega t)$ is a solution to the wave equation $\frac{\partial^2 D}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 D}{\partial t^2}$. To do this, just plug $D$ into both sides, and show that the LHS equals the RHS.

**If you have an em wave $E = E_{\text{max}} \cos(kz - \omega t)$, how do you find $\omega$? How do you get it out of the cosine?**

In the kind of problems we’ll be seeing, usually you can just read off the $\omega$ from the equation – it’s just the thing that’s the coefficient of $t$.

In general, the $\omega$ can be anything; it’s determined by the thing that’s causing the excitation of the fields.

**If you have an em wave, how do you find the direction of the $\vec{B}$ field?**

You know that for a plane wave, $\vec{B}$ must be perpendicular to both the direction of propagation, and to $\vec{E}$. The direction of $\vec{B}$ must be such that $\vec{E} \times \vec{B}$ is the direction of propagation (note that the cross-product direction follows the right hand rule).

**How do you tell what direction an electromagnetic wave is moving in?**

If you know the $\vec{E}$ and $\vec{B}$ directions at any time, the wave is traveling in the direction $\vec{E} \times \vec{B}$ (the direction of the Poynting vector $\vec{S}$).

But actually you can figure it out just from the form of a given plane wave equation. For instance consider $E_{\text{max}} \cos(kz - \omega t)$.

This equation shows that the wiggling is a function of $z$, so it must be moving in either the $+z$ or $-z$ direction. The rule is that if the $kz$ and $\omega t$ parts in the argument of cosine have opposite signs (either $kz - \omega t$ or $\omega t - kz$),
then the wave is traveling in the +z direction (to see these are equivalent: remember \( \cos x = \cos(-x) \).)

On the other hand, if \( kz \) and \( \omega t \) have the same sign, (argument of cosine is \( kz + \omega t \)) then the wave is in the \(-z\) dir.

To see how this works: take the case of the wave

\[
E_{\text{max}} \cos(kx - \omega t) = E_{\text{max}} \cos(2\pi x/\lambda - \omega t) = E_{\text{max}} \cos\left(\frac{2\pi}{\lambda}(x - vt)\right)
\]

using that \( \omega = 2\pi v/\lambda \). Now suppose you are riding along on the crest of the wave to the right (+x dir, increasing \( x \)). The argument of the cosine function must remain the same as \( t \) increases, so that you always sit at the same amplitude as you move along with speed \( v \). As \( t \) increases, \( x \) increases as \( vt \); but the argument of the cosine must be constant, so it must have \( x - vt \) in it. So the argument of cosine if you are moving to the right with speed \( v \) is \( \frac{2\pi}{\lambda}(x - vt) \). Similarly, if you are moving to the left, \( x \) decreases as \(-vt \), so to make the argument of the cosine constant, it must have \( x + vt \).

**Where did \( c = 1/\sqrt{\mu_0\epsilon_0} \) come from?**

This came from the wave equation that can be derived from Maxwell’s equations. The \( \epsilon_0 \) and \( \mu_0 \) show up in Maxwell’s equations, and the derivation in section 34.3 uses both Faraday’s Law and the Maxwell-Ampère Law to get two wave equations: \( \frac{\partial^2 E}{\partial x^2} = \mu_0\epsilon_0\frac{\partial E}{\partial t} \) for electric field, and \( \frac{\partial^2 B}{\partial x^2} = \mu_0\epsilon_0\frac{\partial B}{\partial t} \). Comparing these to the general wave equation, \( \frac{\partial^2 D}{\partial x^2} = \frac{1}{v^2}\frac{\partial D}{\partial t} \), we know that the solutions are sinusoidal traveling waves for \( E \) and \( B \) that propagate with speed \( v = \frac{1}{\sqrt{\mu_0\epsilon_0}} \).

This is one of those truly amazing things in physics: Maxwell’s equations have constants in them measured in completely independent ways: \( \epsilon_0 \) is known from measurements of forces on charges, and \( \mu_0 \) from experiments with currents. Maxwell’s equations predict the speed of electromagnetic waves. And this speed, \( c \), is measured, and it matches the prediction! It seems too good to be true, but it is true!

**What is the physical meaning of intensity?**

The intensity of electromagnetic waves, \( I = S_{\text{avg}} \), is the rate of energy transfer through a unit area: it’s energy transferred per area per time.