Duke University
Department of Physics

Physics 391 Fall Term 2010

WUN2K FOR LECTURE 8

These are notes summarizing the main concepts you need to understand and be able to apply.

- A "confidence interval" refers to a range of parameters $x_- \leq X \leq x_+$ from a measurement chosen such that $\text{Prob}(x_- \leq X \leq x_+) = \int_{x_-}^{x_+} P(x)dx = C$. For non-Gaussian probability distributions, the interval can satisfy this definition and be chosen in various ways, *e.g.* $x_\pm$ equidistant from the mean, or representing the shortest interval in X or (most commonly) chosen as a "central" interval such that $\int_{-\infty}^{x_-} P(x)dx = \int_{x_+}^{\infty} P(x)dx = (1-C)/2$.
- For a given measured parameter $\hat{a}$ (this could be a best-fit or something directly measured) to find the confidence interval for the true value A, one uses a "Neyman construction":
  - For each possible value of A, find the range of a values representing a confidence interval C, going from a_- to a_+. The sets of a_- and a_+ points form curves in an A vs a space.
  - For a given measurement $\hat{a}$, one can read off the confidence interval from A_- to A_+ for A. This confidence interval for A is usually stated as "A is between A_- and A_+ a fraction C of the time": what this means is "for a fraction C of identical experiments that you do, the true value A will be straddled by the limits you find".
  - Constraints on allowed values of the parameters can be incorporated by normalizing the likelihoods within their allowed ranges (note there are some subtleties for small numbers of events, that we do not have time to discuss).

- "Hypothesis testing" refers to methods that attempt to decide whether some model fits the data or not. The "null hypothesis" refers to some statement that you want to reject or accept as consistent with the data.
  - The χ^2 quantity, $\chi^2 \equiv \sum_{i=1}^{N} \frac{[y_i - f(x_i)]^2}{\sigma_i^2}$, can be used to try to decide whether or not to accept the "null hypothesis" that the data are well described by some function f.
    * If the data are well described by the function, then we should have $\chi^2 \sim N$. To judge the quality of the description, one looks up $\mathrm{Prob}(\chi^2, N) = \int_{\chi^2}^{\infty} P(\chi'^2; N) d\chi'^2$.
    * If the function is one parameterized by m quantities (including possibly a normalization parameter), and one has determined these parameters from the data from a minimization fit, then the χ^2 probability to use is $\mathrm{Prob}(\chi^2, N - m) = \int_{\chi^2}^{\infty} P(\chi'^2; N) d\chi'^2$. The quantity $N - m$ is referred to as "the number of degrees of freedom". For a good fit, the "reduced χ^2", $\chi^2/(N - m)$ will have a value of approximately unity.