Duke University
Department of Physics

Physics 391 Fall Term 2010

WUN2K FOR LECTURE 6

These are notes summarizing the main concepts you need to understand and be able to apply.

- A common problem is that of determining the best parameters of some function describing a data set, $f(x_i; a_1, ..., a_m)$. Here the x_i are the N data points and the a_j are the m parameters.
- For "method of maximum likelihood", we write a normalized probability density function $P = P_i(x_i; a_1, ..., a_m)$ for each measurement; it's the probability of the measurement x_i, given the parameters. Then we define a "likelihood function" $\mathcal{L}(a_1, ..., a_m) \equiv \Pi_{i=1}^{N} P_i$.
  - The best choice of the parameters a_j corresponds to the maximum value of $\mathcal{L}$; so the problem of finding the parameters becomes the problem of solving the set of simultaneous equations $\frac{\partial \mathcal{L}}{\partial a_j} = 0$.
  - It's often easier to maximize $\log \mathcal{L}$ than to maximize $\mathcal{L}$.
- The "method of least squares" is a special case of the maximum likelihood method. For measurement pairs (x_i, y_i), assuming Gaussian errors σ_i on y_i, to find the parameters fitting $y = f(x; a_1, ..., a_m)$, we define the "chi-squared" $\chi^2 \equiv \sum_{i=1}^{N} \left[\frac{y_i - f(x_i; a_1, ..., a_m)}{\sigma_i}\right]^2$. The best-fit parameters can be obtained from the solution to the coupled differential equations $\frac{\partial \chi^2}{\partial a_j} = 0$.