Duke University
Department of Physics

Physics 391 Fall Term 2010

WUN2K FOR LECTURE 3

These are notes summarizing the main concepts you need to understand and be able to apply.

- For a Gaussian distribution (very often the case in practice), if you make N measurements x_i, assuming that each measurement has the same σ, the best estimate of the mean is $\mu' = \frac{1}{N}\sum_{i=1}^{N} x_i$. The variance can be estimated as $\sigma^2 = s^2$ (see lecture 1) or by some other method. However the *uncertainty* on the mean is not σ: rather, it's $\sigma_\mu = \frac{\sigma}{\sqrt{N}}$, the "error on the mean". Don't forget the factor of $\frac{1}{\sqrt{N}}$!
- If the individual measurement uncertainties are unequal, σ_i, the best estimate of the mean is weighted according to these uncertainties, $\mu' = \frac{\sum_i \left(\frac{x_i}{\sigma_i^2}\right)}{\sum_i \left(\frac{1}{\sigma_i^2}\right)}$; the best estimate of the error on the mean is $\sigma_\mu^2 = \frac{1}{\sum_i \left(\frac{1}{\sigma_i^2}\right)}$. In this formula, the measurements with smaller uncertainties have higher weight.
- For a Poisson distribution (reasonably Gaussian except for very small μ), which is appropriate for counting events in some time window, this analysis leads to the estimate of error on the mean number of counts in time t of $\sigma_t \sim \sqrt{\frac{\bar{x}_t}{N}}$.
- The "error matrix" is a compact way of specifying uncertainties and correlations for multiple variables $(u_1, u_2, ...)$, useful for changes of variables: the elements of the error matrix are $M_{ij} \equiv \langle (u_i - \bar{u}_i)(u_j - \bar{u}_j) \rangle$. (More on this next lecture).